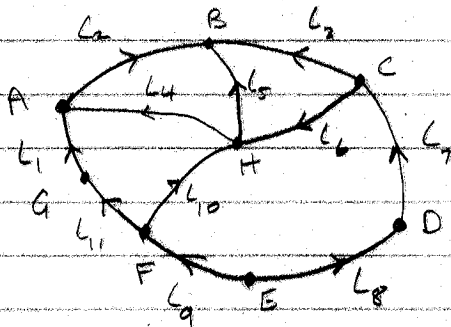


# CE Homework #1

## Level network

(20)



$$n = 4 \quad (2)$$

$$n_0 = 7 \quad (2)$$

$$r = 4$$

10

Condition Equations (by observation only)

$$\begin{aligned} \hat{L}_4 + \hat{L}_2 &= \hat{L}_5 \\ \hat{L}_6 + \hat{L}_5 &= \hat{L}_3 \\ \hat{L}_9 + \hat{L}_{10} &= \hat{L}_8 + \hat{L}_7 + \hat{L}_6 \\ \hat{L}_{11} + \hat{L}_1 &= \hat{L}_{10} + \hat{L}_4 \end{aligned}$$

(2)

In Matrix Notation

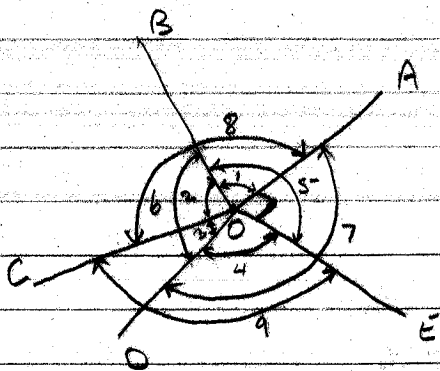
$$\begin{bmatrix} 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \hat{L}_1 \\ \hat{L}_2 \\ \hat{L}_3 \\ \hat{L}_4 \\ \hat{L}_5 \\ \hat{L}_6 \\ \hat{L}_7 \\ \hat{L}_8 \\ \hat{L}_9 \\ \hat{L}_{10} \\ \hat{L}_{11} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(1) Coefficient matrix  $(4 \times 11)$   
rank = 4 (full rank) (3)

(20)

(2)

Angles



$$n = 9 \quad (2)$$

$$n_0 = 3 \quad (2)$$

$$r = 6$$

10

Condition Equations (by observations only)

$$\begin{aligned} \hat{\alpha}_1 &= \hat{\alpha}_5 - 90^\circ \\ \hat{\alpha}_2 + \hat{\alpha}_3 &= \hat{\alpha}_6 \\ \hat{\alpha}_3 + \hat{\alpha}_4 &= \hat{\alpha}_9 \quad (2) \\ \hat{\alpha}_4 &= \hat{\alpha}_7 - 90^\circ \\ \hat{\alpha}_1 + \hat{\alpha}_2 &= \hat{\alpha}_8 \\ \hat{\alpha}_1 + \hat{\alpha}_2 + \hat{\alpha}_3 &= 270^\circ \end{aligned}$$

In Matrix notation

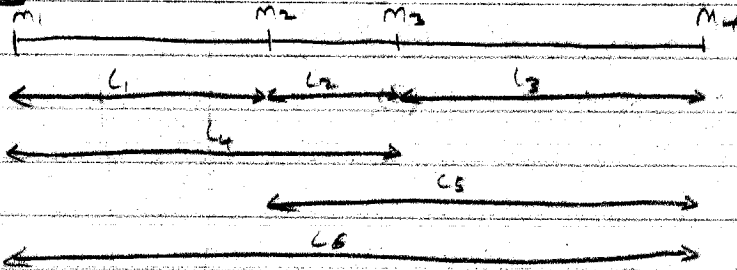
$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \\ \hat{\alpha}_3 \\ \hat{\alpha}_4 \\ \hat{\alpha}_5 \\ \hat{\alpha}_6 \\ \hat{\alpha}_7 \\ \hat{\alpha}_8 \\ \hat{\alpha}_9 \end{bmatrix} = \begin{bmatrix} -90^\circ \\ 0 \\ 0 \\ -90^\circ \\ 0 \\ 270^\circ \end{bmatrix}$$

Coefficient matrix (1)  
(6x9)

rank = 6 (full rank) (3)

(60) (3)

Truss



$$n = 6 \quad (1)$$

$$n_0 = 3 \quad (1)$$

$$r = 3$$

Condition Equations

$$\begin{aligned} \hat{L}_1 + \hat{L}_2 &= \hat{L}_4 \\ \hat{L}_2 + \hat{L}_3 &= \hat{L}_5 \\ \hat{L}_1 + \hat{L}_2 + \hat{L}_3 &= \hat{L}_6 \end{aligned}$$

$$\begin{aligned} \Rightarrow (L_1 + U_1) + (L_2 + U_2) &= L_4 + U_4 \\ (L_2 + U_2) + (L_3 + U_3) &= L_5 + U_5 \\ (L_1 + U_1) + (L_2 + U_2) + (L_3 + U_3) &= L_6 + U_6 \end{aligned}$$

$$\Rightarrow 5.00 + U_1 + 2.00 + U_2 - 7.20 - U_4 = 0$$

$$2.00 + U_2 + 7.00 + U_3 - 8.90 - U_5 = 0$$

$$5.00 + U_1 + 2.00 + U_2 + 7.00 + U_3 - 14.10 - U_6 = 0$$

⇒ (rewriting in terms of  $U_1, U_2, U_3$ )

$$\Rightarrow U_4 = -(U_1 + U_2) - 0.20$$

$$U_5 = U_2 + U_3 + 0.10$$

$$U_6 = U_1 + U_2 + U_3 - 0.10$$

Objective function:  $\Phi = \sum U_i^2 = U_1^2 + U_2^2 + U_3^2 + U_4^2 + U_5^2 + U_6^2$

$$\Rightarrow \Phi = U_1^2 + U_2^2 + U_3^2 + (U_1 + U_2 - 0.20)^2 + (U_2 + U_3 + 0.10)^2 + (U_1 + U_2 + U_3 - 0.10)^2$$

$$\frac{\partial \Phi}{\partial U_1} = 2U_1 + 2(U_1 + U_2 - 0.20) + 2(U_1 + U_2 + U_3 - 0.10) = 0$$

$$\frac{\partial \Phi}{\partial U_2} = 2U_2 + 2(U_1 + U_2 - 0.20) + 2(U_2 + U_3 + 0.10) + 2(U_1 + U_2 + U_3 - 0.10) = 0$$

$$\frac{\partial \Phi}{\partial U_3} = 2U_3 + 2(U_2 + U_3 + 0.10) + 2(U_1 + U_2 + U_3 - 0.10) = 0$$

$$\Rightarrow 3U_1 + 2U_2 + U_3 = 0.30$$

$$2U_1 + 4U_2 + 2U_3 = 0.20$$

(3)

$$U_1 + 2U_2 + 3U_3 = 0.00$$

In Matrix Notation

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0.30 \\ 0.20 \\ 0.00 \end{bmatrix}$$

$A \qquad \qquad \qquad V \qquad \qquad \qquad f$

$$\Rightarrow V = A^{-1}f = \begin{bmatrix} 0.100 \\ 0.025 \\ -0.050 \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

(1)

$$\Rightarrow U_4 = 0.100 + 0.025 - 0.20 = \cancel{0.075} - 0.075$$

$$U_5 = 0.025 + 0.050 + 0.10 = 0.075$$

$$U_6 = 0.100 + 0.025 - 0.050 - 0.10 = -0.025$$

$$\Rightarrow V = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{bmatrix} = \begin{bmatrix} 0.100 \\ 0.025 \\ -0.050 \\ -0.075 \\ 0.075 \\ -0.025 \end{bmatrix}$$

$$\begin{array}{l} \Rightarrow \hat{L}_1 = L_1 + U_1 = 5 + 0.100 = 5.100 = 5.10 \\ \hat{L}_2 = L_2 + U_2 = 2 + 0.025 = 2.025 = 2.02 \\ \hat{L}_3 = L_3 + U_3 = 7.0 + 0.05 = \cancel{7.050} 6.95 = 6.95 \\ \hat{L}_4 = L_4 + U_4 = 7.2 + 0.075 = \cancel{7.075} 7.125 = 7.12 \\ \hat{L}_5 = L_5 + U_5 = 8.9 + 0.075 = 8.975 = 8.98 \\ \hat{L}_6 = L_6 + U_6 = 14.1 - 0.025 = \cancel{14.075} = 14.08 \end{array} \quad \begin{array}{l} (3) \\ \\ \\ \\ \\ \\ (1) \end{array}$$