CE 506 Fall 2004 Homework #1 Solution

1. Regression

(a)

x	y
1	3
3	4
4	5
7	5

$n = 4$

$n_0 = 2$

$r = 2$

solve by indirect observations

- need $n_0 = 2$ parameters: m, slope, and b, intercept
- need $n = 4$ condition equations, one observation per equation, of the form $Y_i + V_i = mx_i + b$

x: constant, y: observation

1. $3 + V_1 = m \times 1 + b$ $\quad V_1 = m + b - 3$
2. $4 + V_2 = m \times 3 + b$ $\quad V_2 = 3m + b - 4$
3. $5 + V_3 = m \times 4 + b$ $\quad V_3 = 4m + b - 5$
4. $5 + V_4 = m \times 7 + b$ $\quad V_4 = 7m + b - 5$

} plug these into objective function

$$\Phi = V_1^2 + V_2^2 + V_3^2 + V_4^2 = (m+b-3)^2 + (3m+b-4)^2 + (4m+b-5)^2 + (7m+b-5)^2$$

$$\frac{\partial \Phi}{\partial m} = 2(m+b-3) + 2(3m+b-4)3 + 2(4m+b-5)4 + 2(7m+b-5)7 = 0$$

$$\frac{\partial \Phi}{\partial b} = 2(m+b-3) + 2(3m+b-4) + 2(4m+b-5) + 2(7m+b-5) = 0$$

$m + 9m + 16m + 49m = 75m$, $b + 3b + 4b + 7b = 15b$, $-3 - 12 - 20 - 35 = -70$

$m + 3m + 4m + 7m = 15m$, $b + b + b + b = 4b$, $-3 - 4 - 5 - 5 = -17$

$$75m + 15b - 70 = 0$$
$$15m + 4b - 17 = 0$$

$$\begin{bmatrix} 75 & 15 \\ 15 & 4 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 70 \\ 17 \end{bmatrix}, \quad \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 0.3333 \\ 3.0000 \end{bmatrix}$$ from matlab

plug into condition equations,

$V_1 = .3333$

$V_2 = 0$

$V_3 = -.6667$

$V_4 = .3333$

adjusted observations $Y_i + V_i = \hat{Y}_i$

$\hat{Y}_1 = 3.3333$

$\hat{Y}_2 = 4.0000$

$\hat{Y}_3 = 4.3333$

$\hat{Y}_4 = 5.3333$

See plot of results

(b) Same regression problem, but use point #3 twice

X	Y
1	3
3	4
4	5
4	5
7	5

$n = 5$, $n_0 = 2$, $r = 3$

Solve by indirect observations

$n_0 = 2$ parameters m, b

$n = 5$ condition equations $\quad y_i + v_i = m x_i + b$

1. $3 + v_1 = m \cdot 1 + b \qquad v_1 = m + b - 3$
2. $4 + v_2 = m \cdot 3 + b \qquad v_2 = 3m + b - 4$
3. $5 + v_3 = m \cdot 4 + b \qquad v_3 = 4m + b - 5$
4. $5 + v_4 = m \cdot 4 + b \qquad v_4 = 4m + b - 5$
5. $5 + v_5 = m \cdot 7 + b \qquad v_5 = 7m + b - 5$

$$\Phi = v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 = (m+b-3)^2 + (3m+b-4)^2 + (4m+b-5)^2 + (4m+b-5)^2 + (7m+b-5)^2$$

$$\frac{\partial \Phi}{\partial m} = 2(m+b-3) + 2(3m+b-4)3 + 2(4m+b-5)4 + 2(4m+b-5)4 + 2(7m+b-5)7 = 0$$

$$\frac{\partial \Phi}{\partial b} = 2(m+b-3) + 2(3m+b-4) + 2(4m+b-5) + 2(4m+b-5) + 2(7m+b-5) = 0$$

$m + 9m + 16m + 16m + 49m = 91m$, $\quad b + 3b + 4b + 4b + 7b = 19b$,

$-3 - 12 - 20 - 20 - 35 = -90$

$m + 3m + 4m + 4m + 7m = 19m$, $\quad b + b + b + b + b = 5b$,

$-3 - 4 - 5 - 5 - 5 = -22$

$$\begin{aligned} 91m + 19b - 90 &= 0 \\ 19m + 5b - 22 &= 0 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} 91 & 19 \\ 19 & 5 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 90 \\ 22 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 0.3404 \\ 3.1064 \end{bmatrix}$$

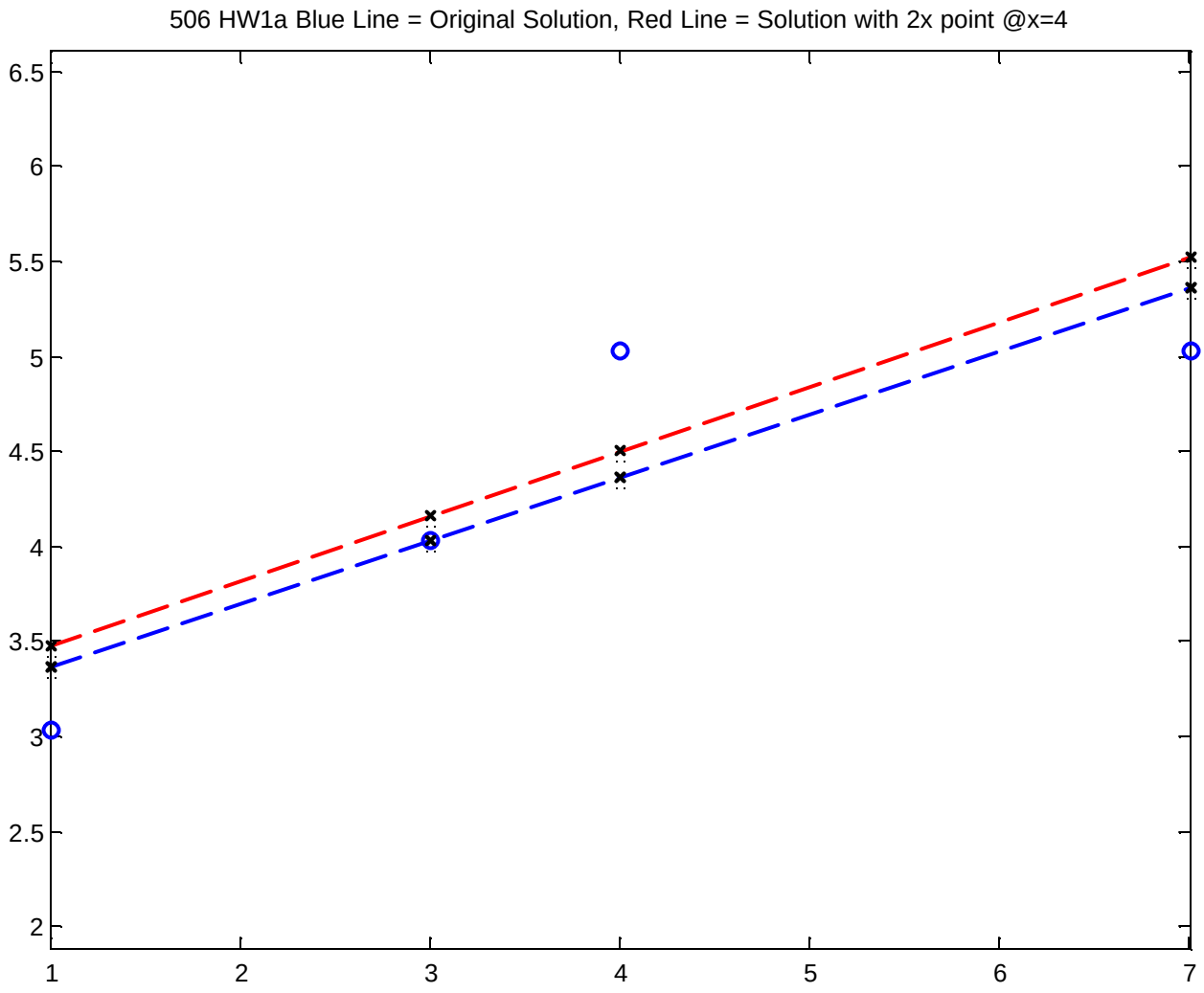
plug into condition equations,

$v_1 = .4468$
$v_2 = .1277$
$v_3 = -.5319$
$v_4 = -.5319$
$v_5 = .4894$

$\hat{y}_i = y_i + v_i$

$\hat{y}_1 = 3.4468$
$\hat{y}_2 = 4.1277$
$\hat{y}_3 = 4.4681$
$\hat{y}_4 = 4.4681$
$\hat{y}_5 = 5.4894$

fitted line moves toward the point used twice. See the plot.


506 HW1a Blue Line = Original Solution, Red Line = Solution with 2x point @x=4
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
1
2
3
4
5
6
7