CE 570: Advanced Structural Mechanics

HOMEWORK 10

Part 1: Due ONLINE on blackboard on at 11:30am Thursday, Dec 7, 2017
Part 2: Due ONLINE on blackboard **AND** IN CLASS at 11:30am Friday, Dec 8, 2017

Part 1 guidelines:

- Work your solution **independently** and **neatly,** on **one side** only on college-rule / engineering paper.
- You may use any combination of mix of **black / blue / green** pens or pencils (but not red).
- Start every problem on a **new** page.
- All **diagrams** must be drawn **neatly** using a straight edge.
- All work should be presented in a **logical sequence**.
- **Scan & submit your homework online** on Blackboard as a **single pdf-file**.
- **Do not email** your homework to the instructor.
- Make sure that your **scan is good quality** and your pdf-file is **clearly readable.** Cell-phone / camera pictures of your homework will **not** be accepted / graded. Illegible or light scans will **not** be graded**.**
- All the scans must be in a **single pdf-file**. To edit, combine or create pdf-files you may use any of the following freely software programs:
  - *PDF Architect* and/or *PDF Creator* (http://www.pdfforge.org/)
  - *Primo-pdf* (http://www.primopdf.com)

  Try to make sure that your pdf-file size is not more than 5MB (Maximum 10MB).
- The **file name** of your scan must be in the format "`HW??-FirstLast-1.pdf`" where "`??`" is the HW number, "`First`" and "`Last`" are your first and last names, and the "`-1`" denotes Part 1. e.g. `HW01-ArunPrakash-1.pdf`.

Part 2 guidelines: (Work in red pen only)

- The solutions will be posted online at 5pm on Friday (on the due date for Part-1).
- Based on the posted solutions:
  - Correct any errors in your work and revise your solution. If you made any errors, comment why you think you made the error(s) and how you will avoid such error(s) in the future.
  - For each problem, list the most important concepts that you learned.
  - Briefly comment how you may be able to verify / cross-check your revised solution and the posted solution. Also comment, if you think that the posted solution is incorrect.
- You may add pages if necessary, but do **not** submit an entirely new homework file for Part 2.
- **Scan & submit your revised homework online** on Blackboard as a **single pdf-file and also submit the hard paper copy in class** on the following Monday.
- The **file name** of your scan must be in the format "`HW??-FirstLast-2.pdf`"

Grading & Solutions:

- **Part 1**: 9 points = 3 problems x 3 points each
  - For Part-1, we will grade based only on your effort: You can get full 3 points for a problem, if you made an **honest independent effort** (even if your solution was incorrect!).
- **Part 2**: 6 points = 3 problems x 2 points each (for revisions and comments)
- Total: **15 points**

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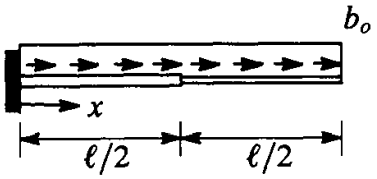
HW Guidelines:

- Read Chapters 6 from *Fundamentals of Structural Mechanics* by KD Hjelmstad.
- Work your solution neatly, starting all the problems on a new page.
- Be **very precise** with notation. You will lose ½ point for every notational error that you make. So, if you make 10 notational errors in 1 question, you will receive a *zero* score even though your solution may have the right idea.

Problem 1: (5 points)

Solve problem 147 from the textbook:

147. The uniaxial rod shown has unit area and length ℓ. It is fixed at the left end, is free at the right end, and is subjected to a constant body force field $b(x) = b_o$ along its length. The elastic modulus $C(x)$ is characterized by



$$C(x) = \begin{cases} 2C & 0 < x \le \ell/2 \\ C & \ell/2 < x \le \ell \end{cases}$$

Find the classical solution to the governing differential equation. Using the principle of virtual work, compute a stress field $\sigma(x)$ assuming a Ritz approximation as follows

$$u(\xi) = a_0 + a_1\xi + a_2\xi^2 \qquad\qquad \overline{u}(x) = \overline{a}_0 + \overline{a}_1\xi + \overline{a}_2\xi^2$$

where $\xi \equiv x/\ell$. Because the modulus changes abruptly at $x = \ell/2$, the stresses and strains are discontinuous at that point. Why is this discontinuity a problem for the polynomial base functions suggested? What happens if you increase the order of the approximation?

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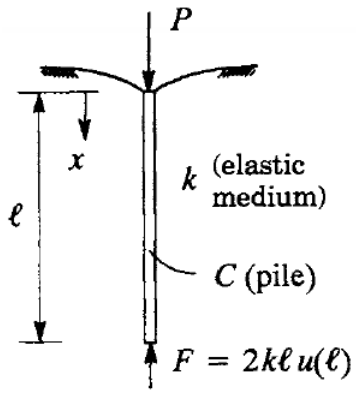
Problem 2: (5 points)



Following up on Problem 2 from HW-9:
Assuming the values of $l = 10$, $C = 100$, $k = 5$, $P = 20$ units,
and using the PVW weak form derived in HW-9, Problem 2, part (a):

a) Divide the length of the pile into equal (N-1) elements *i.e.* N nodes, assume that the approximating functions $h_i(x)$ are given by the finite element basis: $h_i(x) = \begin{cases} \frac{x - x_{i-1}}{x_i - x_{i-1}} & \text{for } x_{i-1} \le x \le x_i \\ \frac{x_{i+1} - x}{x_{i+1} - x_i} & \text{for } x_i \le x \le x_{i+1} \end{cases}$
and derive the expressions for the stiffness matrix $\boldsymbol{K}$ and forcing vector $\boldsymbol{f}$ for a any number of nodes N (similar to pages 14-15) of the Chapter 6 lectures notes).

b) Modify the given MATLAB program from the lecture notes to find approximate solutions for this problem using 2 elements ($N = 3$), 4 elements ($N = 5$), and 8 elements ($N = 9$).

c) Plot the solution for displacements and stresses and compare with the exact solution from HW-9 Problem 1, part (c).

Problem 3: (5 points)

Complete the online course feedback. Your input will be helpful in strengthening and improving the course. In particular, I'd appreciate if you leave comments about what you liked and disliked about the course including, lectures, handouts, homeworks, quizzes, review sessions, Blackboard webpage etc.

Attach a printout of the final confirmation page that you have completed the survey. Please do *not* attach/print your actual feedback/comments (I will receive that anyway, after the semester is over).
If you have already completed the survey, simply log back in and take a print of the summary page showing that you have completed the online survey for CE-570.