CE 570: Advanced Structural Mechanics

HOMEWORK 4

Part 1: Due ONLINE on blackboard on at 11:30am Saturday, Sep 30, 2016

Part 2: Due ONLINE on blackboard **and** in class at 11:30am Monday, Oct 2, 2016

Part 1 guidelines:

- Work your solution **independently** and **neatly,** on **one side** only on college-rule / engineering paper.
- You may use any combination of mix of **black / blue / green** pens or pencils (but not red).
- Start every problem on a **new** page.
- All **diagrams** must be drawn **neatly** using a straight edge.
- All work should be presented in a **logical sequence**.
- **Scan & submit your homework online** on Blackboard as a **single pdf-file**.
- **Do not email** your homework to the instructor.
- Make sure that your **scan is good quality** and your pdf-file is **clearly readable.** Cell-phone / camera pictures of your homework will **not** be accepted / graded. Illegible or light scans will **not** be graded.
- All the scans must be in a **single pdf-file**. To edit, combine or create pdf-files you may use any of the following freely software programs:
  - *PDF Architect* and/or *PDF Creator* (http://www.pdfforge.org/)
  - *Primo-pdf* (http://www.primopdf.com)

  Try to make sure that your pdf-file size is not more than 5MB (Maximum 10MB).
- The **file name** of your scan must be in the format "`HW??-FirstLast-1.pdf`" where "`??`" is the HW number, "`First`" and "`Last`" are your first and last names, and the "`-1`" denotes Part 1. e.g. `HW01-ArunPrakash-1.pdf`.

Part 2 guidelines: (Work in red pen only)

- The solutions will be posted online at 5pm on Friday (on the due date for Part-1).
- Based on the posted solutions:
  - Correct any errors in your work and revise your solution. If you made any errors, comment why you think you made the error(s) and how you will avoid such error(s) in the future.
  - For each problem, list the most important concepts that you learned.
  - Briefly comment how you may be able to verify / cross-check your revised solution and the posted solution. Also comment, if you think that the posted solution is incorrect.
- You may add pages if necessary, but do **not** submit an entirely new homework file for Part 2.
- **Scan & submit your revised homework online** on Blackboard as a **single pdf-file**.
- The **file name** of your scan must be in the format "`HW??-FirstLast-2.pdf`"

Grading & Solutions:

- **Part 1**: 10 points = 3 problems x 3 points each + 1 presentation point
  - For Part-1, we will grade based only on your effort: You can get full 3 points for a problem, if you made an **honest independent effort** (even if your solution was incorrect!).
- **Part 2**: 5 points (for revisions and comments)
- Total: **15 points**

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HW Guidelines:

- Read Chapter 2 (very carefully!) from *Fundamentals of Structural Mechanics* by KD Hjelmstad.
- Work your solution neatly, starting all the problems on a new page.
- Be **very precise** with notation. You will lose ½ point for every notational error that you make. So, if you make 10 notational errors in 1 question, you will receive a *zero* score even though your solution may have the right idea.

Problem 1: (5 points)
Solve Problem 49 from the textbook.

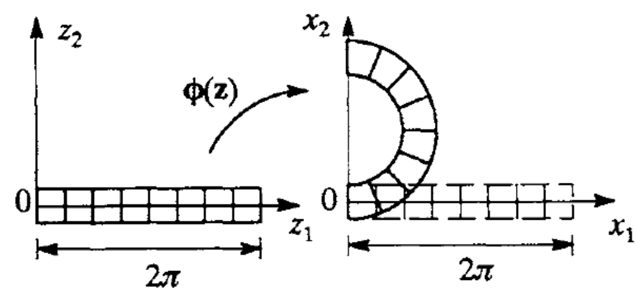
49. Consider a unit cube in the positive octant with a vertex positioned at the origin of coordinates subjected to the following deformation map

$$\boldsymbol{\phi}(\mathbf{z}) = (z_1 + \epsilon z_2 z_3)\mathbf{e}_1 + (z_2 + \epsilon z_1 z_3)\mathbf{e}_2 + (z_3 + \epsilon z_1 z_2)\mathbf{e}_3$$

where ϵ is a constant. Compute the deformation gradient $\mathbf{F}$, the Green deformation tensor $\mathbf{C}$, and the Lagrangian strain tensor $\mathbf{E}$ for the given deformation. Using graph paper, plot the deformed position of a square in the $x_1 - x_2$ plane by locating the positions of a grid of points. (Select a value of ϵ to execute the plot.)

Problem 2: (5 points)
Solve Problem 58 from the textbook.

58. Find the mathematical expression for the map that takes a strip of length 2π and deforms it into a semicircular arc without changing the depth of the strip. The deformation map is illustrated in the sketch. Compute the deformation gradient $\mathbf{F}$, the Green deformation tensor $\mathbf{C}$, and the Lagrangian strain tensor $\mathbf{E}$ for the map.



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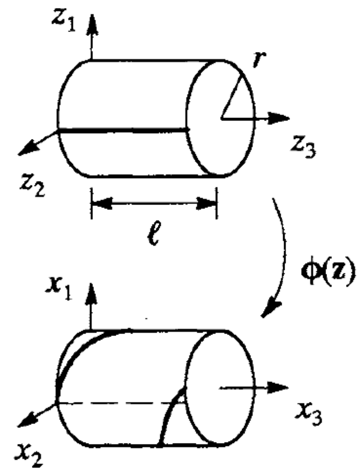
Problem 3: (5 points)
Solve Problem 61 from the textbook (except as modified below).

61. The deformation map for the pure twist of a circular shaft of length ℓ and radius r can be expressed in terms of the rate of twist β (a constant) as follows

$$\boldsymbol{\phi}(\mathbf{z}) = \left(z_1 \cos(\beta z_3) - z_2 \sin(\beta z_3)\right) \mathbf{e}_1 + \left(z_1 \sin(\beta z_3) + z_2 \cos(\beta z_3)\right) \mathbf{e}_2 + z_3\, \mathbf{e}_3$$



Compute the deformation gradient $\mathbf{F}(\mathbf{z})$. Find the displacement of the point initially located at the position $\mathbf{z} = (r, 0, \ell)$ in the undeformed configuration. ~~Find the volume of the deformed shaft in terms of the angle of twist β~~. A horizontal line is etched on the surface of the undeformed shaft, parallel to the z_3 axis as shown. Find the length of the line in the deformed configuration.

Compare your result with HW-1 Problem 2 for $\beta = 2n\pi/l$.
(*Note*: β is the **rate** of twist = angle of twist / length)