CE 570: Advanced Structural Mechanics

HOMEWORK 2

Part 1: Due ONLINE on blackboard on at 11:30am Saturday, Sep 9, 2016

Part 2: Due ONLINE on blackboard **and** in class at 11:30am Monday, Sep 11, 2016

Part 1 guidelines:

- Work your solution **independently** and **neatly,** on **one side** only on college-rule / engineering paper.
- You may use any combination of mix of **black / blue / green** pens or pencils (but not red).
- Start every problem on a **new** page.
- All **diagrams** must be drawn **neatly** using a straight edge.
- All work should be presented in a **logical sequence**.
- **Scan & submit your homework online** on Blackboard as a **single pdf-file**.
- **Do not email** your homework to the instructor.
- Make sure that your **scan is good quality** and your pdf-file is **clearly readable.**
  Cell-phone / camera pictures of your homework will **not** be accepted / graded.
  Illegible or light scans will **not** be graded**.**
- All the scans must be in a **single pdf-file**. To edit, combine or create pdf-files you may use any of the following freely software programs:
  - *PDF Architect* and/or *PDF Creator* (`http://www.pdfforge.org/`)
  - *Primo-pdf* (`http://www.primopdf.com`)

  Try to make sure that your pdf-file size is not more than 5MB (Maximum 10MB).
- The **file name** of your scan must be in the format "`HW??-FirstLast-1.pdf`" where "`??`" is the HW number, "`First`" and "`Last`" are your first and last names, and the "`-1`" denotes Part 1. e.g. `HW01-ArunPrakash-1.pdf`.

Part 2 guidelines: (Work in red pen only)

- The solutions will be posted online at 5pm on Friday (on the due date for Part-1).
- Based on the posted solutions:
  - Correct any errors in your work and revise your solution. If you made any errors, comment why you think you made the error(s) and how you will avoid such error(s) in the future.
  - For each problem, list the most important concepts that you learned.
  - Briefly comment how you may be able to verify / cross-check your revised solution and the posted solution. Also comment, if you think that the posted solution is incorrect.
- You may add pages if necessary, but do **not** submit an entirely new homework file for Part 2.
- **Scan & submit your revised homework online** on Blackboard as a **single pdf-file**.
- The **file name** of your scan must be in the format "`HW??-FirstLast-2.pdf`"

Grading & Solutions:

- **Part 1**: 10 points = 3 problems x 3 points each + 1 presentation point
  - For Part-1, we will grade based only on your effort: You can get full 3 points for a problem, if you made an **honest independent effort** (even if your solution was incorrect!).
- **Part 2**: 5 points (for revisions and comments)
- Total: **15 points**

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HW Guidelines:

- Review multi-variate calculus: line, area and volume integrals, partial derivatives etc.
- Read Chapter 1 (very carefully!) from *Fundamentals of Structural Mechanics* by KD Hjelmstad.
- Work your solution neatly, starting all the problems on a new page.
- Be **very precise** with notation. You will lose ½ point for every notational error that you make. So, if you make 10 notational errors in 1 question, you will receive a *zero* score even though your solution may have the right idea.

Problem 1: (5 points)

(a) Using only indicial notation, solve Problem 6 from the textbook:

6. Show that the triple scalar product is skew-symmetric with respect to changing the order in which the vectors appear in the product. For example, show that

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = -(\mathbf{v} \times \mathbf{u}) \cdot \mathbf{w}$$

To generalize this notion, any cyclic permutation (e.g., **u, v, w** → **w, u, v**) of the order of the vectors leaves the algebraic sign of the product unchanged, while any acyclic permutation (e.g., **u, v, w** → **v, u, w**) of the order of the vectors changes the sign. How does this observation relate to swapping rows of a matrix in the computation of the determinant of that matrix?

(b) Solve Problem 14 from the textbook:

14. The components of tensors **T** and **S** and the components of vectors **u** and **v** are

$$\mathbf{T} \sim \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad \mathbf{S} \sim \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \quad \mathbf{v} \sim \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{u} \sim \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Compute the components of the vector **Su**. Find the cosine of the angle between **u** and **Su**. Compute the determinants of **T**, **S**, and **TS**. Compute $T_{ij}T_{ij}$ and $u_i T_{ik} S_{kj} v_j$.

Problem 2: (5 points)

Solve Problem 13 from the textbook:

13. Consider two Cartesian coordinate systems, one with basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ and the other with basis $\{\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3\}$. Let $Q_{ij} \equiv \mathbf{g}_i \cdot \mathbf{e}_j$ be the cosine of the angle between $\mathbf{g}_i$ and $\mathbf{e}_j$.

(a) Show that $\mathbf{g}_i = Q_{ij}\mathbf{e}_j$ and $\mathbf{e}_j = Q_{ij}\mathbf{g}_i$ relate the two sets of base vectors.

(b) We can define a rotation tensor $\mathbf{Q}$ such that $\mathbf{e}_i = \mathbf{Q}\mathbf{g}_i$. Show that this tensor can be expressed as $\mathbf{Q} \equiv Q_{ij}[\mathbf{g}_i \otimes \mathbf{g}_j]$, that is, Q_{ij} are the components of $\mathbf{Q}$ with respect to the basis $[\mathbf{g}_i \otimes \mathbf{g}_j]$. Show that the tensor can also be expressed in the form $\mathbf{Q} = [\mathbf{e}_i \otimes \mathbf{g}_i]$.

(c) We can define a rotation tensor $\mathbf{Q}^T$, such that $\mathbf{g}_i = \mathbf{Q}^T\mathbf{e}_i$ (the reverse rotation from part (b). Show that this tensor can be expressed as $\mathbf{Q}^T \equiv Q_{ij}[\mathbf{e}_j \otimes \mathbf{e}_i]$, that is, Q_{ij} are the components of $\mathbf{Q}^T$ with respect to the basis $[\mathbf{e}_j \otimes \mathbf{e}_i]$. Show that the tensor can also be expressed in the form $\mathbf{Q}^T = [\mathbf{g}_i \otimes \mathbf{e}_i]$.

(d) Show that $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$, which implies that the tensor $\mathbf{Q}$ is orthogonal.

Problem 3: (5 points)

Solve Problem 27 from the textbook:

27. A certain state of deformation at a point in a body is described by the tensor $\mathbf{T}$, having the components relative to a certain basis of

$$\mathbf{T} \sim 10^{-2}\begin{bmatrix} 14 & 2 & 14 \\ 2 & -1 & -16 \\ 14 & -16 & 5 \end{bmatrix}$$

Let the principal values and principal directions be designated as μ and $\mathbf{n}$. Show that $\mathbf{n}_1 = (-1, 2, 2)$ is a principal direction and find μ_1. The second principal value is $\mu_2 = 9 \times 10^{-2}$, find $\mathbf{n}_2$. Find μ_3 and $\mathbf{n}_3$ with as little computation as possible.