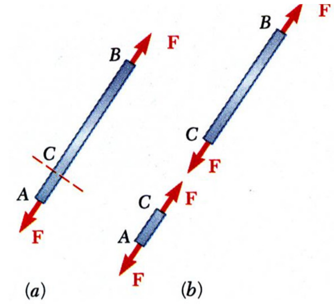
Chapter 7: Internal forces in Frames and Beams

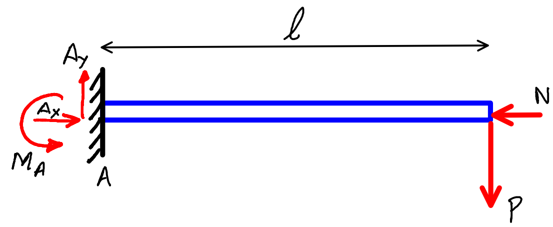


In Chapter 6, we considered internal forces in trusses. We saw that all the members are 2-force members that carry only tension or compression.

In this chapter, we will consider internal forces in Frames and Beams. Recall that these structures have atleast one multi-force member.

Multi-force members can carry additional types of **internal forces** such as **shear** and **bending moment** in addition to **tension/compression**.

For example, consider the cantilever beam shown with an end load. We can find the external forces using the FBD of the entire beam.



External:

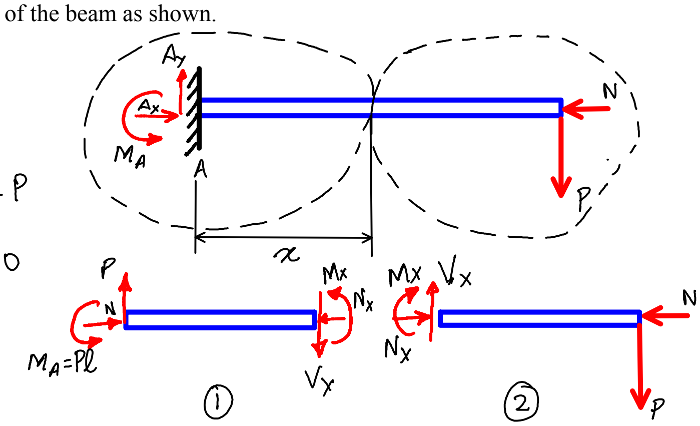
$\sum F_x = 0 \Rightarrow A_x = N$

$\sum F_y = 0 \Rightarrow A_y = P$

$\sum M_{Az} = 0 \Rightarrow M_A = P\ell$

However we may also want to find out the internal forces (and moments) at different points of the beam. This will help us decide if the beam can support the applied load or not.

To do this, we imagine two (or more) sub-parts of the beam as shown.



FBD ①

$\sum F_x = 0 \Rightarrow N_x = N$

$\sum F_y = 0 \Rightarrow +P - V_x = 0 \Rightarrow V_x = P$

$\sum M_A = 0 \Rightarrow +P\ell + M_x - V_x x = 0$

$\Rightarrow M_x = -P\ell + Px$

$M_x = P(x-\ell)$

FBD ②

$\sum F_x = 0 \Rightarrow N_x = N$

$\sum F_y = 0 \Rightarrow V_x = P$

$\sum M_x = 0 \Rightarrow -M_x - P(\ell - x) = 0 \Rightarrow M_x = P(x-\ell)$

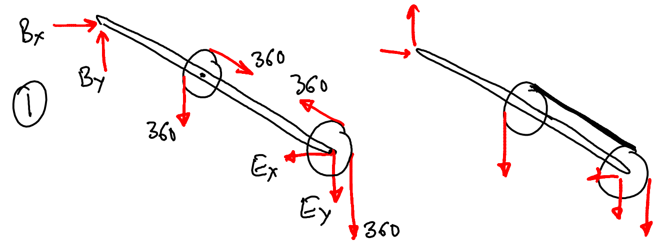
N_x :	Axial force (Tension/Compression)
V_x :	SHEAR Force
M_x :	BENDING Moment

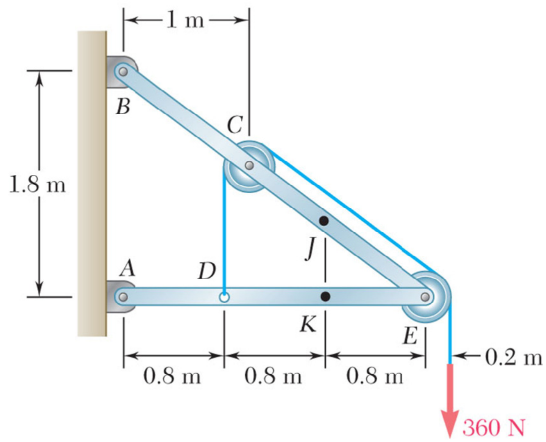
Read example 7.1

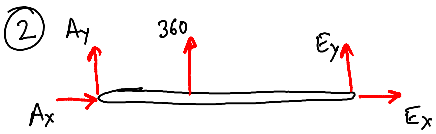
Exercise 7.17 & 7.18

Radius of pulleys = 200 mm

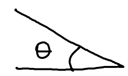
Find the internal forces (& moments) at J & K.







$36.9°$



PART-A External

FBD (2)

$\sum F_x = 0 \Rightarrow A_x + E_x = 0$

$\sum F_y = 0 \Rightarrow A_y + 360 + E_y = 0 \Rightarrow \boxed{E_y = -120\,N}$

$\sum M_E = 0 \Rightarrow -A_y \times 2.4 - 360 \times 1.6 = 0 \Rightarrow A_y = -\frac{\cancel{360}^{120} \times \cancel{1.6}^{2}}{\cancel{2.4}\,\cancel{3}} = \boxed{-240\,N}$

FBD (1)

$\sum F_x = 0 \Rightarrow B_x - E_x = 0 \Rightarrow B_x = E_x$

$\sum F_y = 0 \Rightarrow B_y - 360 - E_y - 360 = 0 \Rightarrow \boxed{B_y = 600\,N}$

$\sum M_E = 0 \Rightarrow -B_x \times 1.8 - B_y \times 2.4 + 360 \times 1.6 - 360 \times 0.2 = 0$

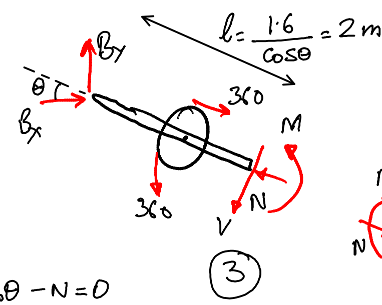
$\Rightarrow B_x = \frac{360 \times 1.4 - 600 \times 2.4}{1.8} = \boxed{-520\,N} \Rightarrow \boxed{E_x = -520\,N}$

$\Rightarrow \boxed{A_x = 520\,N}$

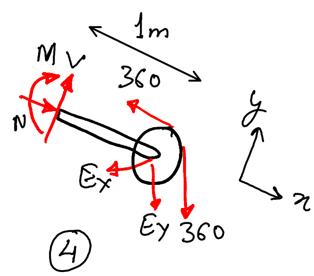
PART-B Internal forces & moments

Take the cut at point J perpendicular to BCE.

$l = \frac{1.6}{\cos\theta} = 2\,m$



To find M, V, N :-



FBD (3) : (Rotated co-ordinate axis)

$\sum F_x = 0 \Rightarrow B_x \cos\theta - B_y \sin\theta + 360 + 360 \sin\theta - N = 0$

$\Rightarrow \boxed{N = -200\,N \quad (\text{TENSILE})}$

$\sum F_y = 0 \Rightarrow$

$B_x \sin\theta + B_y \cos\theta - 360 \cos\theta - V = 0$

$\Rightarrow \boxed{V = -120\,N}$

$\sum M_B = 0$

$\Rightarrow M - V \times 2 - 360 \times 0.2 - 360 \times 0.8 = 0$

$\Rightarrow \boxed{M = +120\,Nm}$

FBD (4) (To check)

$\sum F_x = N - E_x \cos\theta + E_y \sin\theta + 360 \sin\theta - 360 = 0$ ✓

$\sum F_y = V - E_x \sin\theta - E_y \cos\theta - 360 \cos\theta = 0$ ✓

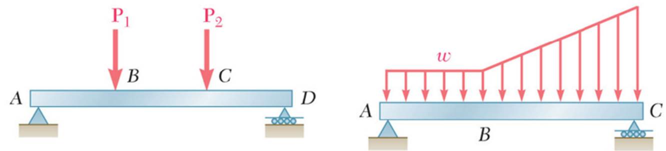
$\sum M_E = -M - V \times 1 + 360 \times 0.2 - 360 \times 0.2 = 0$ ✓

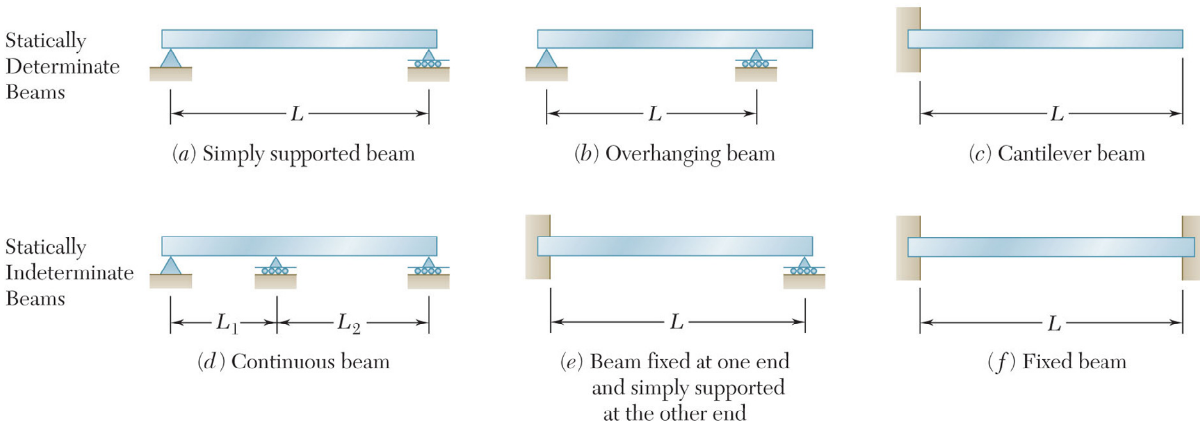
Tuesday, November 17, 2009
11:01 PM

7.3 - 7.4 Internal Forces in Beams

Beams can point or distributed loads acting on them.

Beams may also be externally determinate or indeterminate depending upon the type of support.

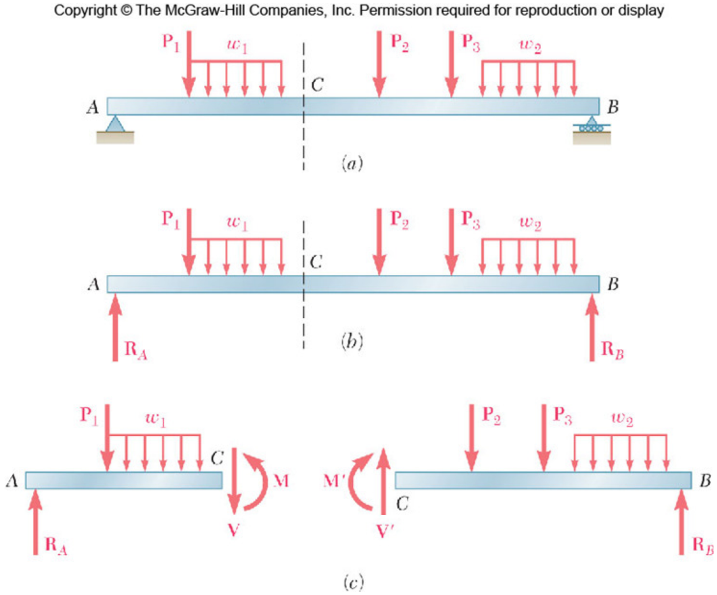


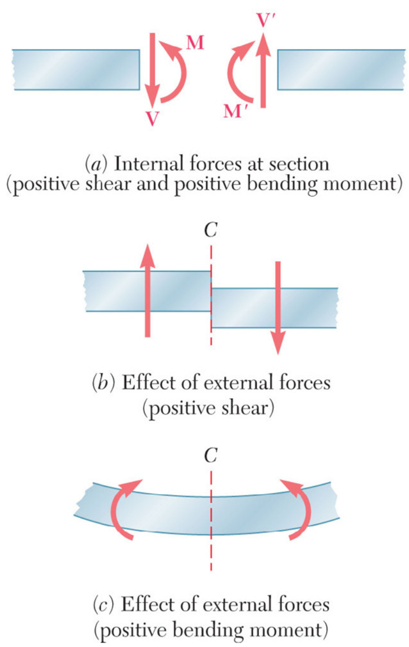


Shear and Bending Moment in Beams

Consider the Beam shown carrying some loads. We can find out the reactions $\mathbf{R}_A$ and $\mathbf{R}_B$ for external equilibrium. To find the internal forces, consider the cut shown.

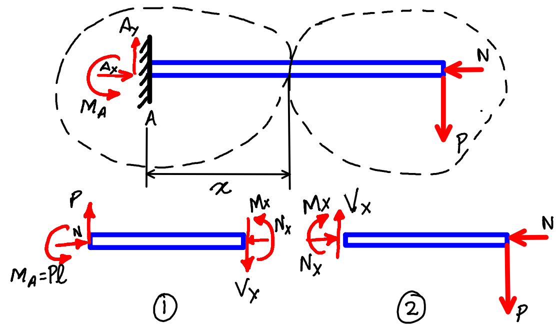
The following convention is adopted for the positive shear and bending moments in beams.





7.5 Shear and Bending Moment Diagrams

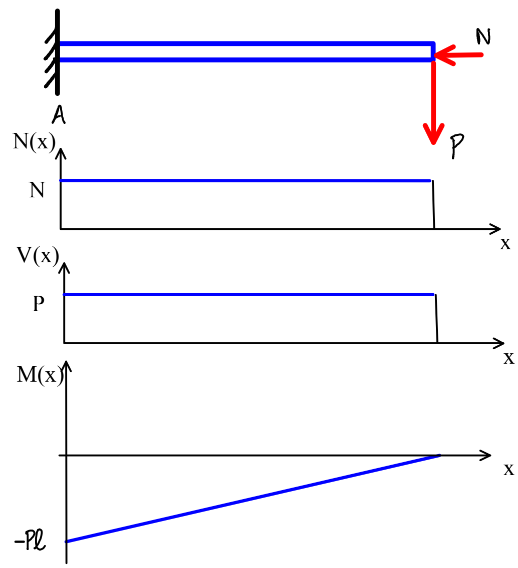
Recall the cantilever beam from the previous section.



N_x : Axial force (Tension/Compression)
V_x : SHEAR Force
M_x : BENDING Moment

Using the FBD of individual parts of the beam we found:

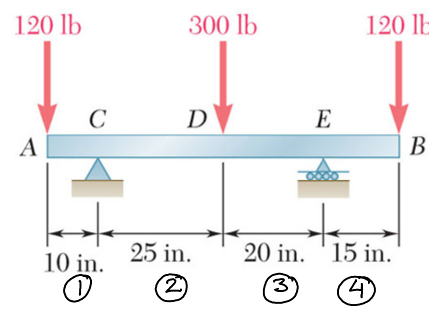
$$N_x = N$$
$$V_x = P$$
$$M_x = P(x - l)$$

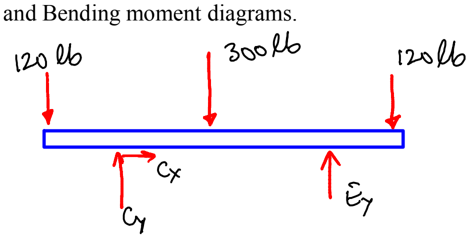


If we plot these INTERNAL forces and moments along the length of the beam, the resulting diagrams are called
- Axial force diagram N(x)
- Shear force diagram V(x)
- Bending moment diagram M(x)

Exercise 7.38

Plot the Shear force and Bending moment diagrams.





$\sum f_x = 0 \Rightarrow C_x = 0$

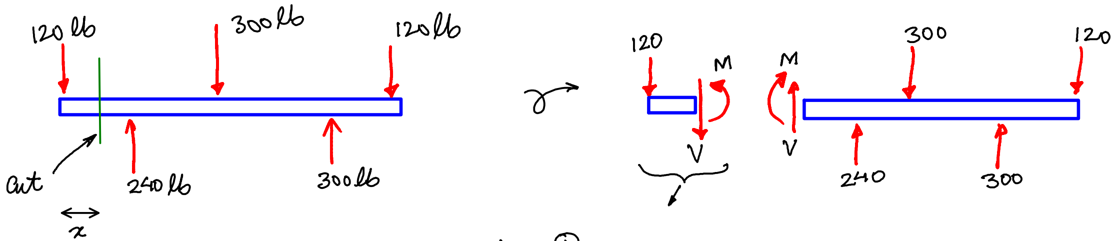
$\sum M_C = 0 \Rightarrow +120 \times 10 - 300 \times 25 + E_y \, 45 - 120 \times 60 = 0 \Rightarrow \boxed{E_y = 300 \text{ lb}}$

$\sum M_E = 0 \Rightarrow 120 \times 55 + 300 \times 20 - C_y \times 45 - 120 \times 15 = 0 \Rightarrow \boxed{C_y = 240 \text{ lb}}$

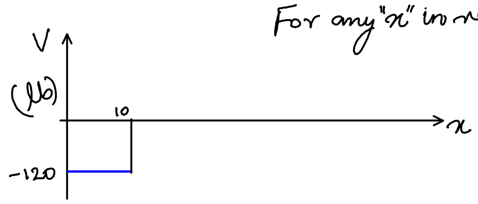
To draw the shear force & Bending Moment diagrams:-
Consider the following "regions" of x along the beam:

1. $0 < x < 10$
2. $10 < x < 35$
3. $35 < x < 55$
4. $55 < x < 70$

For region 1 FBDs:



For any "x" in region 1:



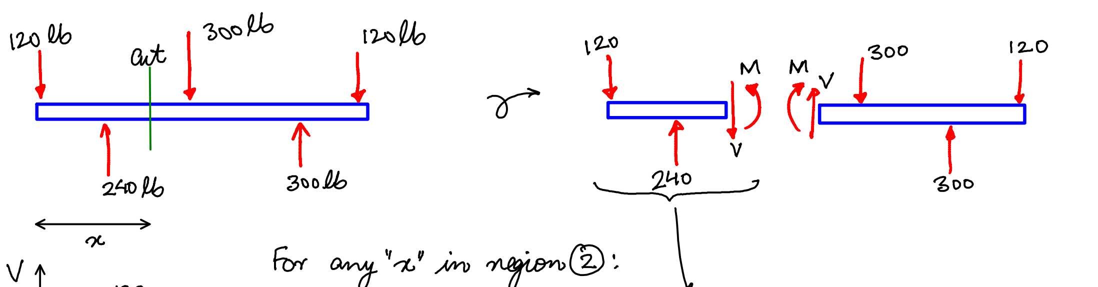
$\sum F_y = 0 \Rightarrow -120 - V = 0$

$\Rightarrow \boxed{V = -120}$

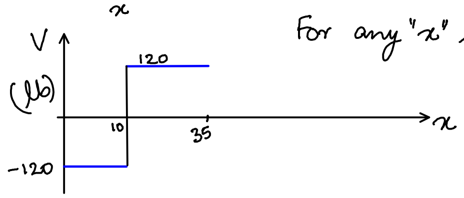
$\sum M_x = 0 \Rightarrow +120x + M = 0$

$\Rightarrow \boxed{M = -120x}$

For region 2 FBDs:

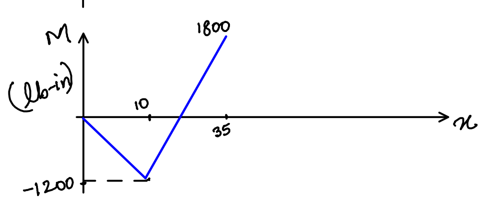


For any "x" in region 2:



$\sum F_y = 0 \Rightarrow -120 + 240 - V = 0$

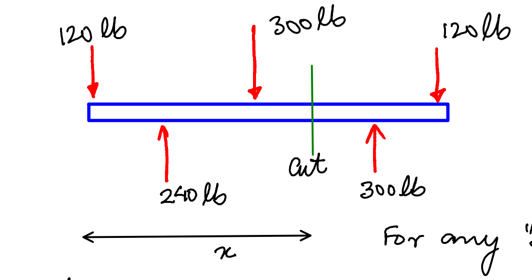
$\Rightarrow \boxed{V = 120}$

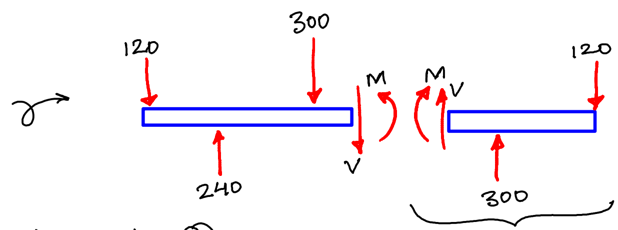


$\sum M_x = 0 \Rightarrow 120x - 240(x-10) + M = 0$

$\Rightarrow \boxed{M = -2400 + 120x}$

For region (3) FBDs:





For any "x" in region (3):

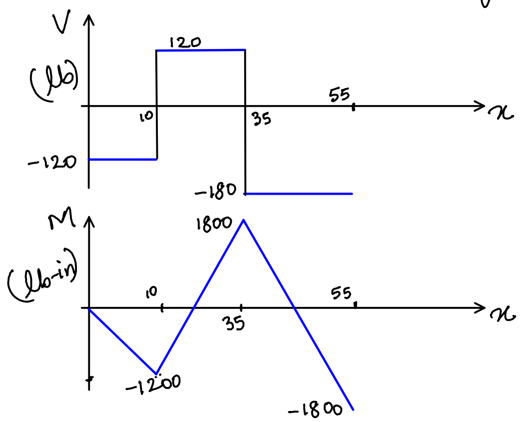
$\sum F_y = 0 \Rightarrow +V + 300 - 120 = 0$

$\Rightarrow \boxed{V = -180}$

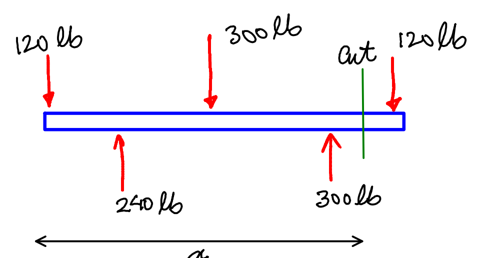
$\sum M_x = 0$

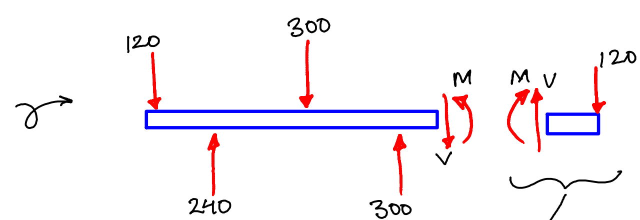
$\Rightarrow -M + 300(55-x) - 120(70-x) = 0$

$\Rightarrow M = \boxed{8100 - 180x}$



For region (4) FBDs:



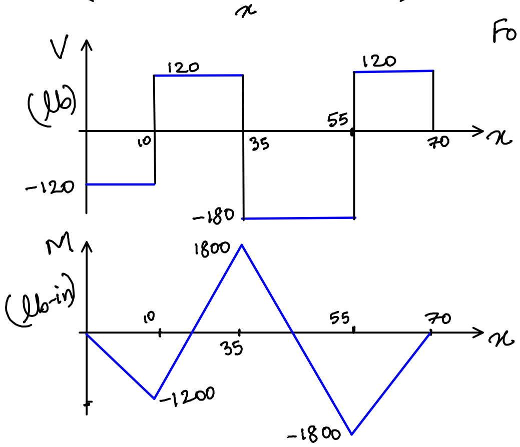


For any "x" in region (4):

$\sum F_y = 0 \Rightarrow \boxed{V = 120}$

$\sum M_x = 0 \Rightarrow -M - 120(70-x) = 0$

$\Rightarrow \boxed{M = -8400 + 120x}$



Maximum Absolute Value of

Shear : 180 lb

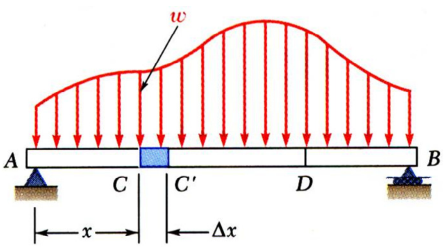
Moment : 1800 lb-in

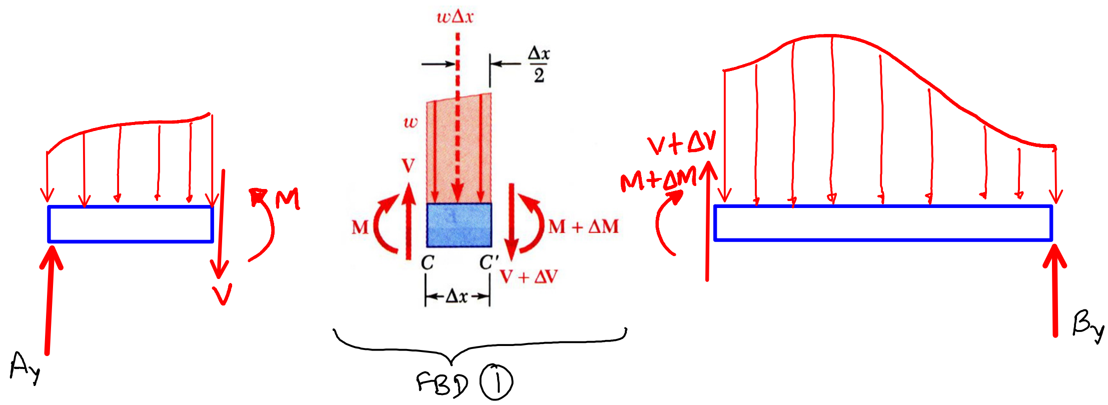
7.6 Load vs. Shear vs. Bending moment

Drawing Shear force and Bending moment diagrams for a beam can be simplified by using relationships between Load vs. Shear and Shear vs. Bending Moment.

These relationships can be derived simply from statics as follows.

Consider a small Δx length of **any beam** carrying a distributed load.





FBD (1):

$$\Sigma F_y = 0 \Rightarrow \cancel{V} - w\Delta x - (\cancel{V} + \Delta V) = 0$$

$$\Rightarrow w = \frac{-\Delta V}{\Delta x} \qquad \text{Take } \lim_{\Delta x \to 0} \frac{-\Delta V}{\Delta x} \Rightarrow \boxed{w = -\frac{dV}{dx}}$$

$$\Sigma M_{C'} = 0 \Rightarrow -M - V\Delta x + (M + \Delta M) + (w\Delta x)\frac{\Delta x}{2} = 0$$

$$\Rightarrow V = \frac{\Delta M}{\Delta x} + \frac{w\Delta x}{2} = 0 \Rightarrow \boxed{V = \frac{dM}{dx}}$$

Integrating

$$\boxed{V_D - V_C = -\int_{x_C}^{x_D} w\,dx = -(\text{area under load curve})}$$

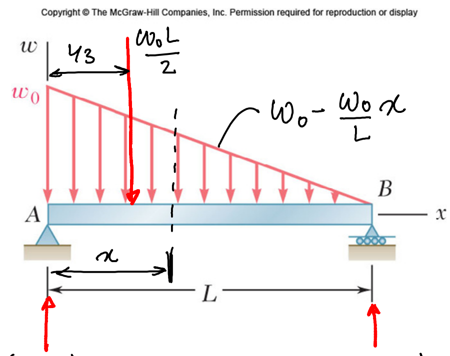
$$\boxed{M_D - M_C = \int_{x_C}^{x_D} V\,dx = (\text{area under shear curve})}$$

Read examples 7.4, 7.5, 7.6 and 7.7.

Exercise 7.85

Write the expressions of Shear and Bending Moments. Draw the diagrams. Verify the relationships between Load vs. Shear and Shear vs. Bending Moment.

Find the location of the maximum Bending Moment.

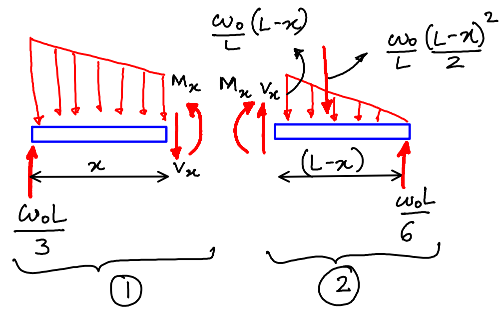


$$w = -\frac{dV}{dx}$$

$$\Rightarrow V_x - V_A = -\int_0^x \left(w_0 - \frac{w_0}{L}x\right) dx$$

Note: $V_A = A_y = \left(\frac{w_0 L}{2}\right) \times \frac{2}{3}$ $\qquad B_y = \left(\frac{w_0 L}{2}\right)\frac{1}{3}$

$$\Rightarrow \boxed{V_x = \frac{w_0 L}{3} - w_0 x + \frac{w_0}{L}\frac{x^2}{2}} \quad (*)$$



Alternatively: FBD (2):

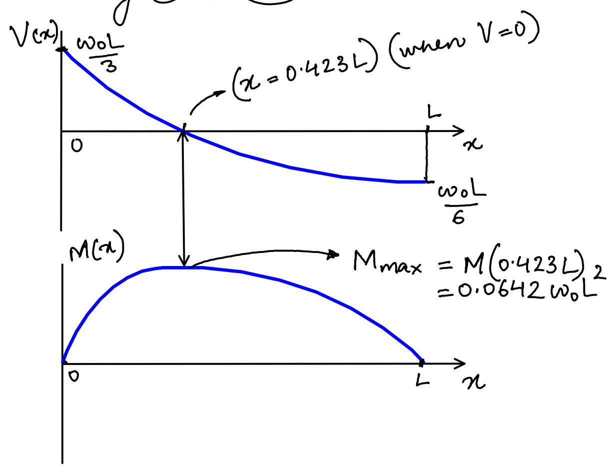
$\sum F_y = 0$

$$\Rightarrow V + \frac{w_0 L}{6} - \frac{w_0}{L}\frac{(L-x)^2}{2} = 0$$

$$\Rightarrow V = -\frac{w_0 L}{6} + \frac{w_0}{2L}\left(L^2 - 2Lx + x^2\right)$$

$$\Rightarrow \boxed{V = \frac{w_0 L}{3} - w_0 x + \frac{w_0}{L}\frac{x^2}{2}}$$

Plotting (*) & (**)



Similarly:

$$V = \frac{dM}{dx}$$

Note: $M_A = 0$

$$\Rightarrow M_x - M_A = \int_0^x \left(\frac{w_0 L}{3} - w_0 x + \frac{w_0}{L}\frac{x^2}{2}\right) dx$$

$$\Rightarrow \boxed{M_x = \frac{w_0 L}{3}x - \frac{w_0 x^2}{2} + \frac{w_0}{L}\frac{x^3}{6}} \quad (**)$$

Alternatively from FBD (2):

$$\sum M_B = 0 \Rightarrow -M_x - V_x(L-x) + \frac{w_0}{L}\frac{(L-x)^2}{2} \times \frac{2}{3}(L-x) = 0$$

$$\Rightarrow M_x = -\left(\frac{w_0 L}{3} - w_0 x + \frac{w_0}{L}\frac{x^2}{2}\right)(L-x) + \frac{w_0}{L}\frac{(L-x)^3}{3}$$

$$= (L-x)\left(-\cancel{\frac{w_0 L}{3}} + w_0 x - \frac{w_0}{L}\frac{x^2}{2} + \frac{w_0}{3L}\left(\cancel{L^2} - 2Lx + x^2\right)\right)$$

$$= (L-x)\left(\frac{w_0 x}{3} - \frac{w_0 x^2}{6L}\right) \quad \Rightarrow \boxed{M_x = \frac{w_0 L x}{3} - \frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L}}$$