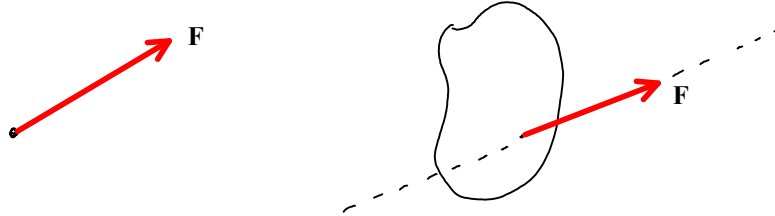


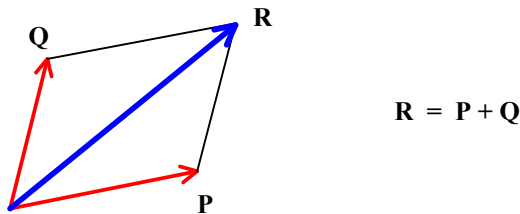
Chapter 2: Statics of Particles

2.1 - 2.3 Forces as Vectors & Resultants

- Forces are drawn as directed arrows. The length of the arrow represents the magnitude of the force and the arrow shows its direction. Forces on rigid bodies further have a line of action.



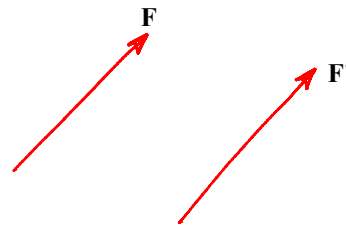
- Forces (and in general all vectors) follow the parallelogram law of vector addition. In fact, vectors are defined as quantities that follow the parallelogram law.



- Vector addition is represented by the same symbol +
The meaning of plus will be clear from the context it is used in.

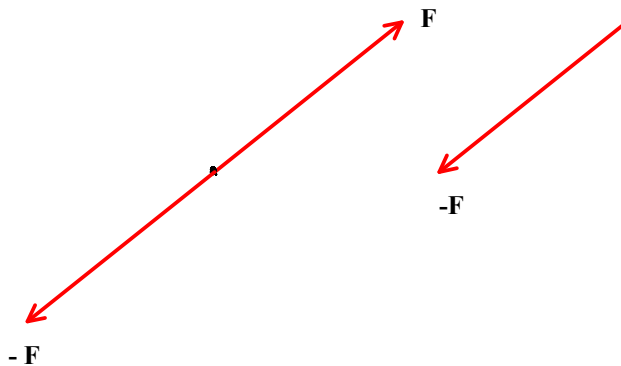
Note:

Vector addition is independent of any chosen coordinate system.
Two vectors are equal if they have the same magnitude and direction.



The negative of a vector **F** is simply **-F** denoted by arrow of the same size in the opposite direction.

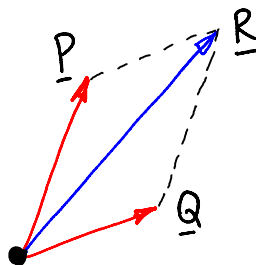
$$F + (-F) = 0$$



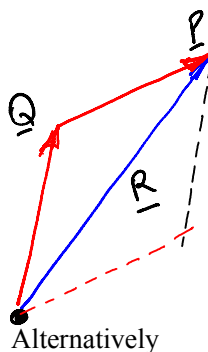
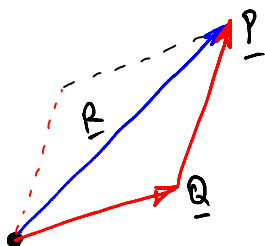
2.4-2.5 Addition of Vectors, Force resultants

- Vector Addition Parallelogram law:
 - Commutative property

$$\mathbf{P} + \mathbf{Q} = \mathbf{Q} + \mathbf{P} = \mathbf{R}$$

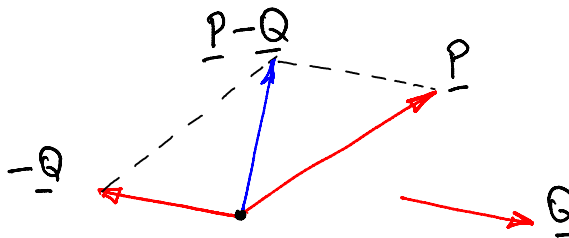


- Vector Addition Triangle law (tip-to-tail):
Derives from the parallelogram law.



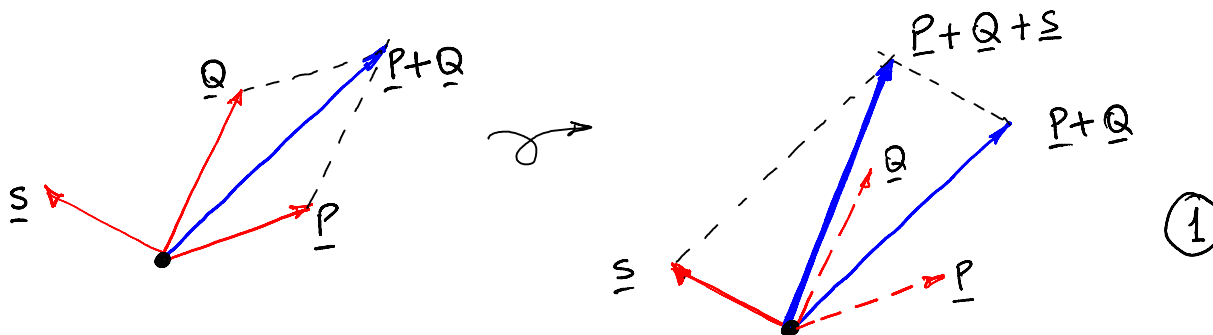
- Subtraction of a vector from another vector:
(Addition of the negative vector)

$$\mathbf{P} - \mathbf{Q} = \mathbf{P} + (-\mathbf{Q})$$

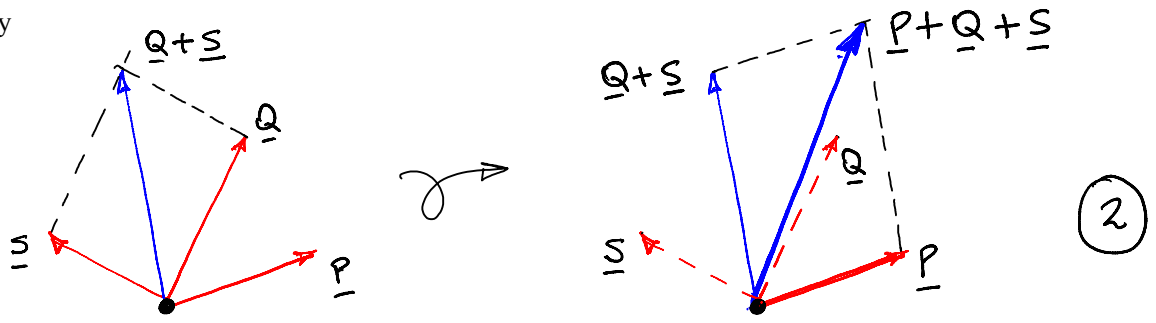


- Addition of multiple vectors
 - Associative property

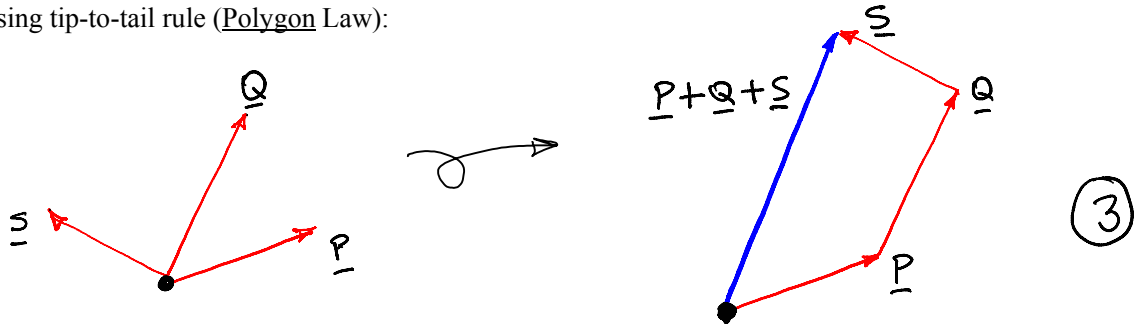
$$\mathbf{P} + \mathbf{Q} + \mathbf{S} = (\mathbf{P} + \mathbf{Q}) + \mathbf{S} = \mathbf{P} + (\mathbf{Q} + \mathbf{S}) = \mathbf{S} + \mathbf{Q} + \mathbf{P}$$



Alternatively

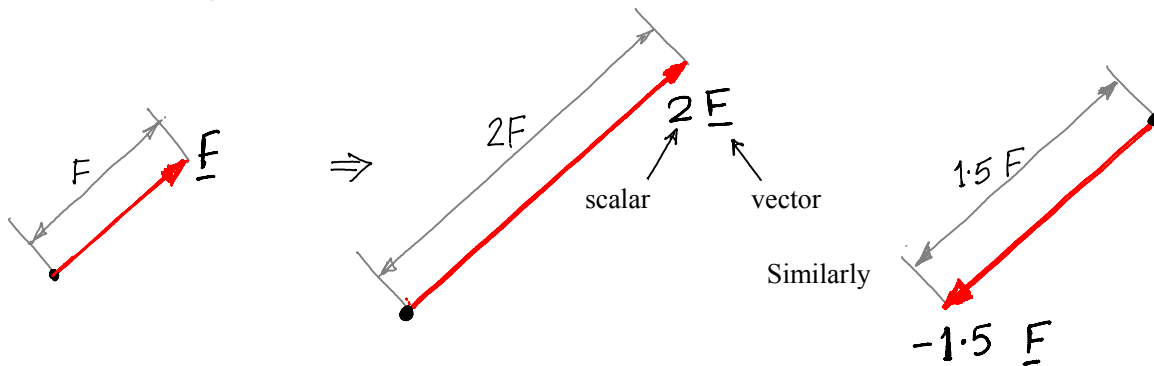


- Using tip-to-tail rule (Polygon Law):



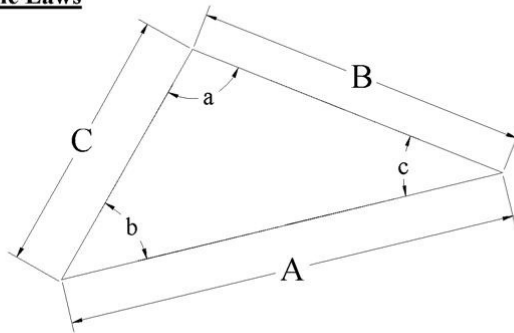
Compare 1, 2 and 3.

- Product of a scalar & a vector
"Scales" the length of the vector.



CE 297 Useful Trigonometric Relationships

Sine and Cosine Laws



Sine Law:

$$\frac{\sin(a)}{A} = \frac{\sin(b)}{B} = \frac{\sin(c)}{C}$$

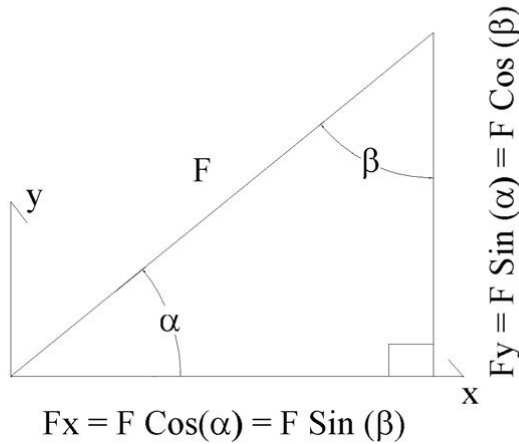
Cosine Law:

$$A^2 = B^2 + C^2 - 2 \times B \times C \times \cos(a)$$

$$B^2 = A^2 + C^2 - 2 \times A \times C \times \cos(b)$$

$$C^2 = A^2 + B^2 - 2 \times A \times B \times \cos(c)$$

Rectangular Components



$$\tan(\alpha) = F_y/F_x$$

$$\tan(\beta) = F_x/F_y$$

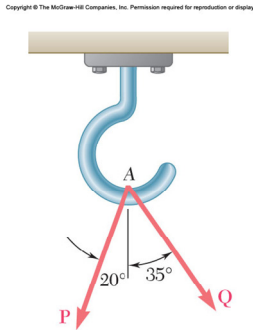
Sine and Cosine of a Sum of Angles

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \sin(B)\cos(A)$$

$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

Examples

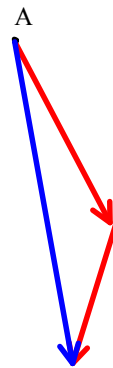
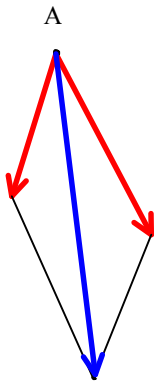
- Read Example 2.1 in book.
- Exercise 2.1



$$P = 75 \text{ N}$$

$$Q = 125 \text{ N}$$

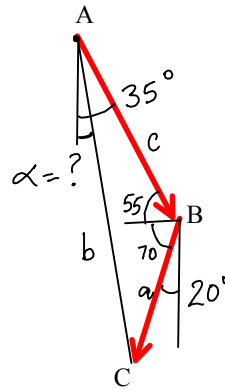
Determine the resultant using
 (1) Parallelogram Law,
 (2) Triangle Rule,
 (3) Trigonometry.



Using trigonometry

Length $AC = b$ (unknown)
 Angle α of AC with vertical
 (unknown)

Length $AB = c = 125 \text{ N}$
 Length $BC = a = 75 \text{ N}$



$$\angle B = 125^\circ$$

$$\angle A = 35^\circ - \alpha$$

$$\angle C = 20^\circ + \alpha$$

Using the cosine Law :-

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$= 75^2 + 125^2 - 2 \times 75 \times 125 \cos (125^\circ)$$

solving:

$$b = 178.8982 \text{ N} \approx \boxed{179 \text{ N}}$$

Now using Sine Law:

$$\frac{\sin B}{b} = \frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin(125)}{178.8982} = \frac{\sin(35-\alpha)}{75} = \frac{\sin(20+\alpha)}{125}$$

$$\Rightarrow \sin(20+\alpha) = \frac{125}{178.8982} \times \sin(125^\circ) = 0.5724$$

$$\Rightarrow 20+\alpha = \sin^{-1}(0.5724) = 34.9149$$

$$\Rightarrow \alpha = 14.9149 \approx \boxed{15^\circ}$$

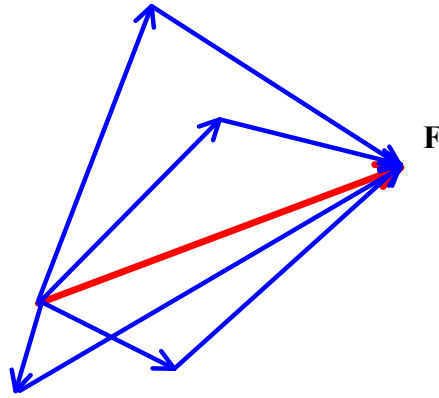
2.6 Resolving Forces into Components

- Reverse process of vector addition. Split a force into two (or more) components.

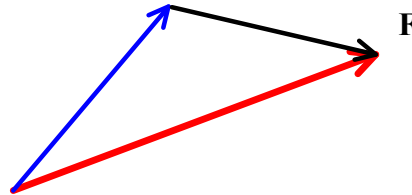
$$\mathbf{F} = \mathbf{P} + \mathbf{Q}$$

Given \mathbf{F} , Find \mathbf{P} and \mathbf{Q} ?

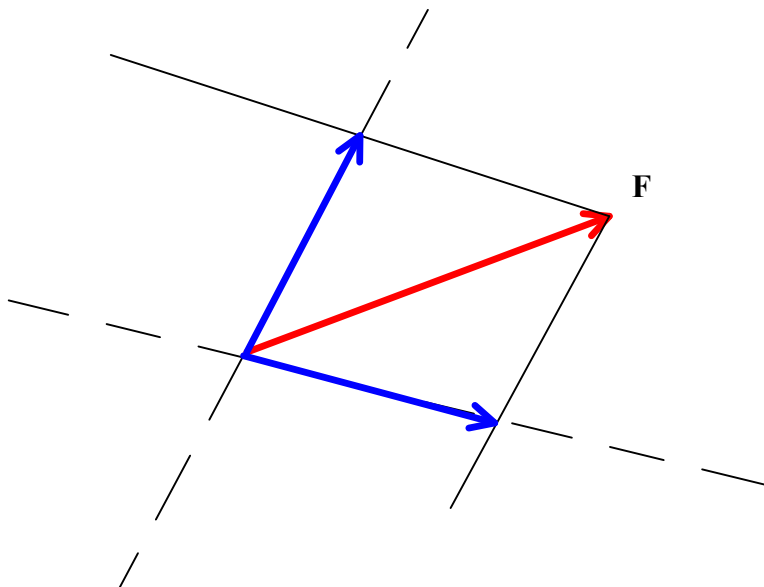
Many Possibilities



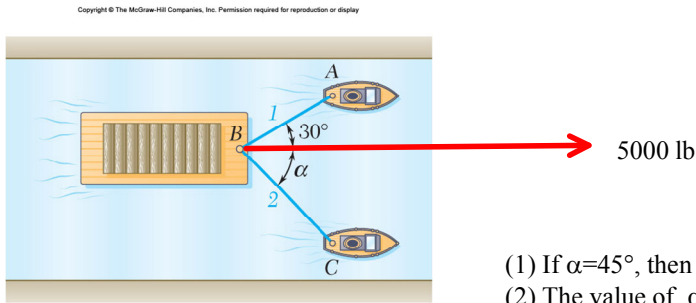
- Special cases:
 - One component (say \mathbf{P}) is known:



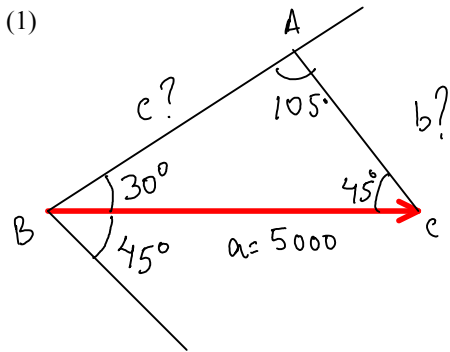
- The line of action of both components is known



Example 2.2 in book:

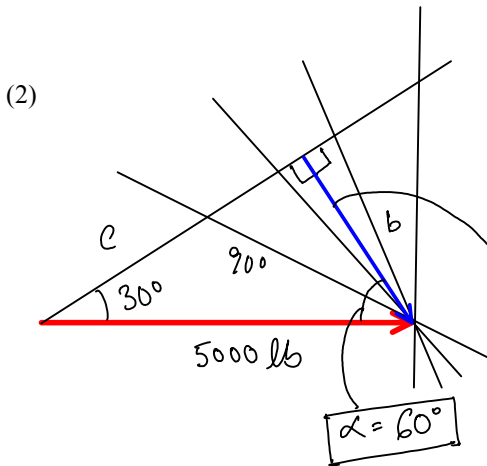


- (1) If $\alpha=45^\circ$, then find T_1 and T_2
- (2) The value of α for which T_2 is minimum.



$$T_2 = b = \frac{a}{\sin A} \cdot \sin B = 2590 \text{ lb.}$$

$$T_1 = c = \frac{a}{\sin A} \cdot \sin C = 3660 \text{ lb}$$

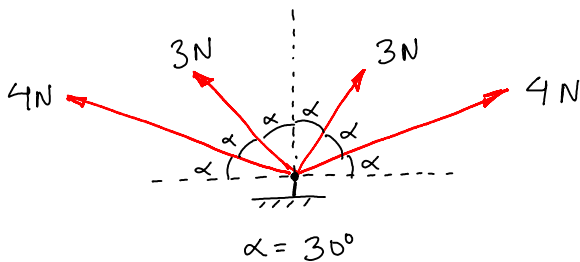


$$T_1 = c = 5000 \cos(30^\circ) = 4330 \text{ lb}$$

$$T_2 = b = 5000 \sin(30^\circ) = 2500 \text{ lb}$$

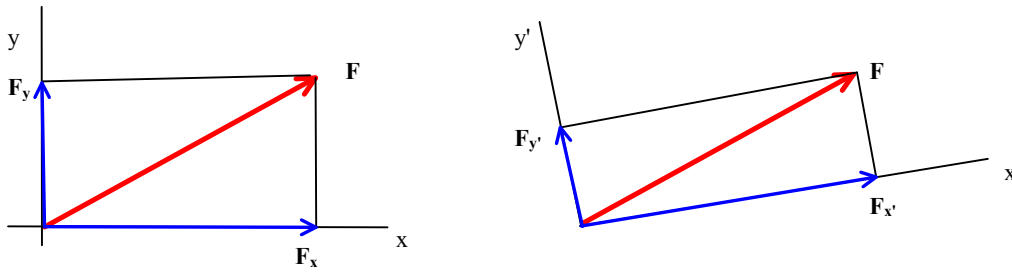
(minimum T_2)

In class exercise.



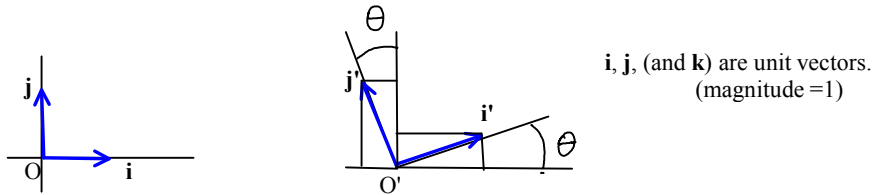
2.7 Rectangular Components of Vectors; Unit vectors

- For ease in mathematical manipulation, forces (and vectors) can be resolved into rectangular components along predefined x, y (and z) directions.



$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y = \mathbf{F}_{x'} + \mathbf{F}_{y'}$$

- One can **choose** any coordinate system $[O, \mathbf{i}, \mathbf{j}, \mathbf{k}]$ and resolve forces and vectors along these directions.



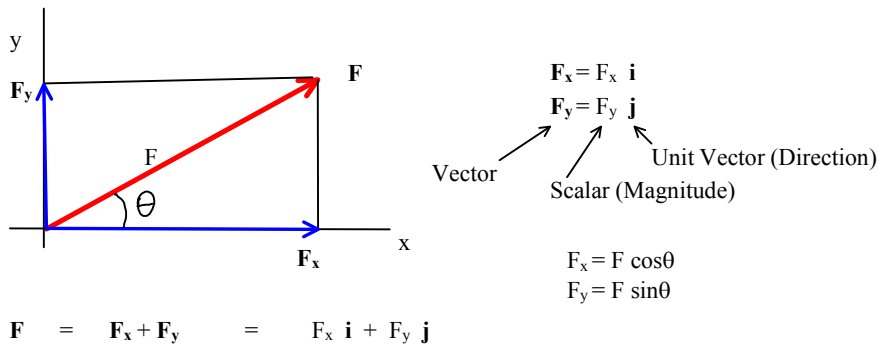
Note:

$$\begin{aligned} \mathbf{i}' &= \cos\theta \mathbf{i} + \sin\theta \mathbf{j} \\ \mathbf{j}' &= -\sin\theta \mathbf{i} + \cos\theta \mathbf{j} \end{aligned}$$

$$\begin{bmatrix} \mathbf{i}' \\ \mathbf{j}' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \mathbf{i} \\ \mathbf{j} \end{bmatrix}$$

Useful in converting one coordinate system to another.

- Using the unit vectors:



$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y = F_x \mathbf{i} + F_y \mathbf{j}$$

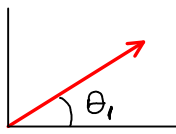
Examples

- Read examples 1, 2 & 3 in section 2.7 of the book.

• Exercise 2.21

Determine the x, y components of the 3 forces.

(i) $F = 800 \text{ N}$



Note: $\tan \theta_1 = \frac{600}{800} = \frac{3}{4}$

$\Rightarrow F_x = F \cos \theta_1$
 $= 800 \times \frac{4}{5} = \boxed{640 \text{ N}}$

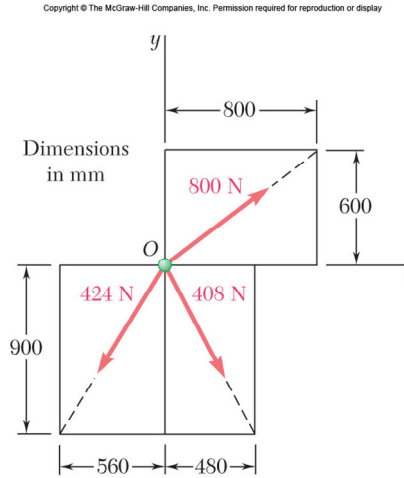
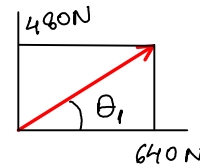
$\Rightarrow \underline{F}_x = 640 \underline{i} \text{ N}$

$F_y = F \sin \theta_1$
 $= 800 \times \frac{3}{5} = \boxed{480 \text{ N}}$

$\Rightarrow \underline{F}_y = 480 \underline{j} \text{ N}$

$\Rightarrow \underline{F} = \underline{F}_x + \underline{F}_y$

$\underline{F} = 640 \underline{i} + 480 \underline{j} \text{ N}$



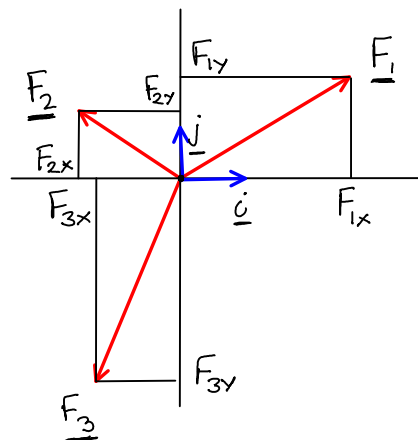
Similarly solve for the other two forces.

2.8 Addition of forces in x-y components

- The vector sum of two or more forces can be obtained by
 - Resolving each force into x-y components
 - Simply adding the individual components

$\underline{F} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3$

$\underline{F} = (F_{1x} + F_{2x} + F_{3x}) \underline{i}$
 $+ (F_{1y} + F_{2y} + F_{3y}) \underline{j}$

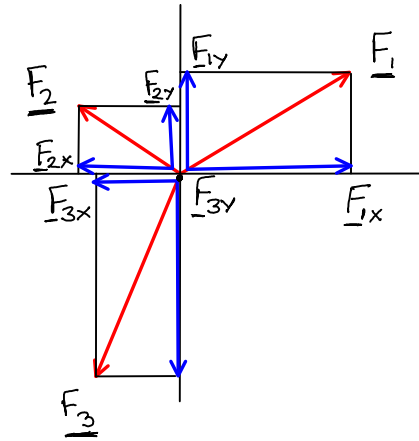


$$\underline{F} = F_x \underline{i} + F_y \underline{j}$$

where

$$F_x = F_{1x} + F_{2x} + F_{3x}$$

$$F_y = F_{1y} + F_{2y} + F_{3y}$$

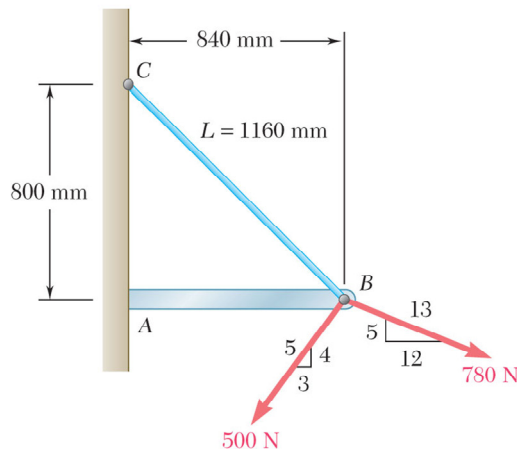
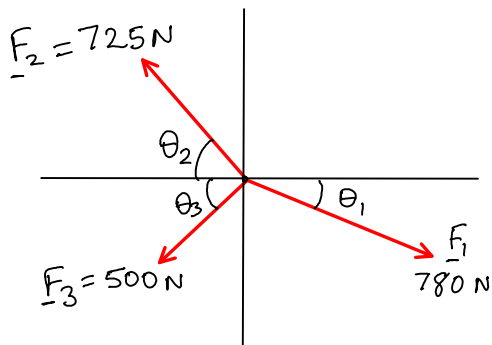


Example

- Read example 2.3 in book.
- Exercise 2.36

Tension in BC = 725 N.
Determine the resultant of the 3 forces at B.

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Note: $\sin \theta_1 = \frac{5}{13}$; $\cos \theta_1 = \frac{12}{13}$

$$\sin \theta_2 = \frac{800}{1160} ; \cos \theta_2 = \frac{840}{1160}$$

$$\sin \theta_3 = \frac{4}{5} ; \cos \theta_3 = \frac{3}{5}$$

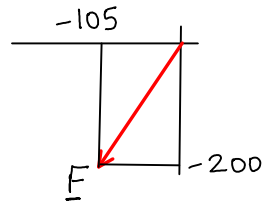
$$\left. \begin{aligned} F_{1x} &= F_1 \cos \theta_1 = 780 \times \frac{12}{13} = 720 \text{ N} \\ F_{1y} &= (-)F_1 \sin \theta_1 = -780 \times \frac{5}{13} = -300 \text{ N} \end{aligned} \right\}$$

$$\left. \begin{aligned} F_{2x} &= (+) F_2 \cos \theta_2 = -725 \times \frac{840}{1160} = -525 \text{ N} \\ F_{2y} &= F_2 \sin \theta_2 = 725 \times \frac{800}{1160} = 500 \text{ N} \\ F_{3x} &= (-) F_3 \cos \theta_3 = -500 \times \frac{3}{5} = -300 \text{ N} \\ F_{3y} &= (-) F_3 \sin \theta_3 = -500 \times \frac{4}{5} = -400 \text{ N} \end{aligned} \right\}$$

Thus the resultant is:

$$\underline{F} = (720 - 525 - 300) \underline{i} + (-300 + 500 - 400) \underline{j}$$

$$\underline{F} = -105 \underline{i} - 200 \underline{j} \text{ N}$$

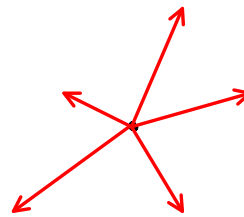


2.9-2.11 Equilibrium of a Particle: Free Body Diagrams

- If the resultant of all the forces acting on a particle is zero, the particle is said to be in **equilibrium**.

$$\mathbf{R} = \Sigma \mathbf{F} = \mathbf{0}$$

$$\Sigma F_x = 0 ; \quad \Sigma F_y = 0$$

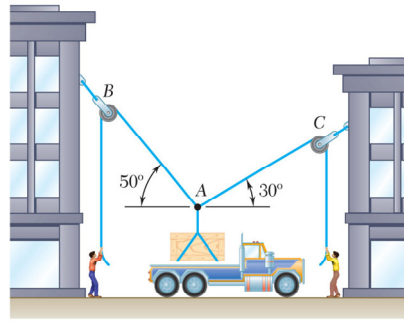
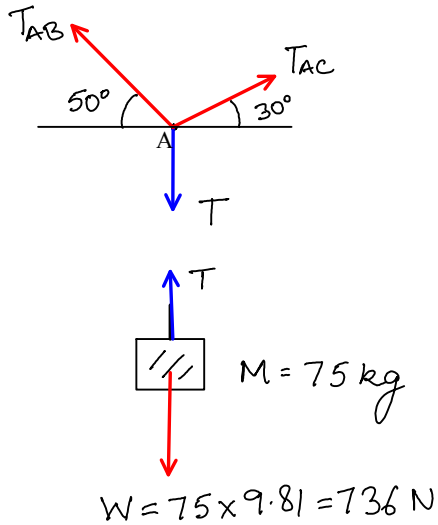


- Recall, Newton's 1st Law.
- Choose the particle judiciously.
- **FREE BODY DIAGRAMS**
 - A diagram showing all the forces acting on a particle or an object.

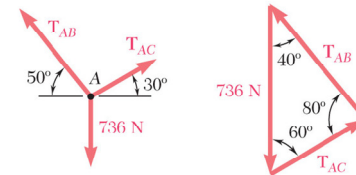
In 2 dimensions, we have two equations of equilibrium, thus the maximum number of unknowns we can solve for is two.

Examples:

- Unloading a truck.



(a) Space diagram



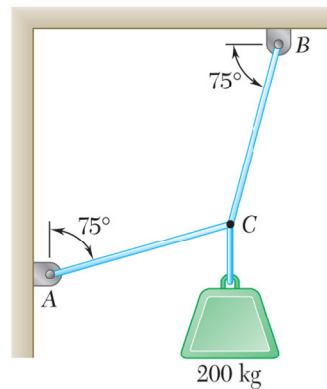
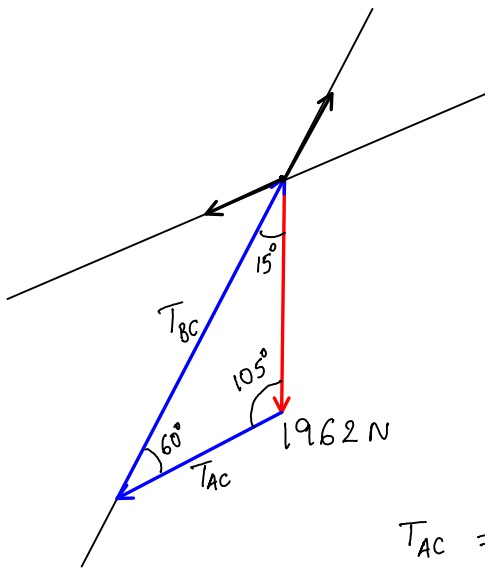
(b) Free-body diagram

(c) Force triangle

$$T_{AC} = \frac{736}{\sin(80^\circ)} \times \sin(40^\circ) = \boxed{480 \text{ N}}$$

$$T_{AB} = \frac{736}{\sin(80^\circ)} \times \sin(60^\circ) = \boxed{647 \text{ N}}$$

- Read Examples 2.4, 2.5 and 2.6 in book.
- Exercise 2.46:
Determine T_{AC} and T_{BC}



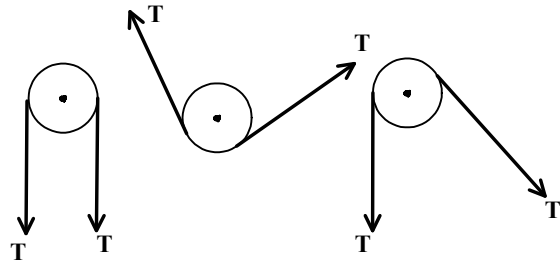
$W = 200 \times 9.81 = 1962 \text{ N}$

$$T_{AC} = \frac{1962}{\sin(60^\circ)} \cdot \sin(15^\circ) = 586.3604 \approx \boxed{586 \text{ N}}$$

$$T_{BC} = \frac{1962}{\sin(60^\circ)} \cdot \sin(105^\circ) = 2188.3 \approx \boxed{2188 \text{ N}}$$

Pulleys

- The tension on both sides of a frictionless pulley is the same (under Equilibrium).



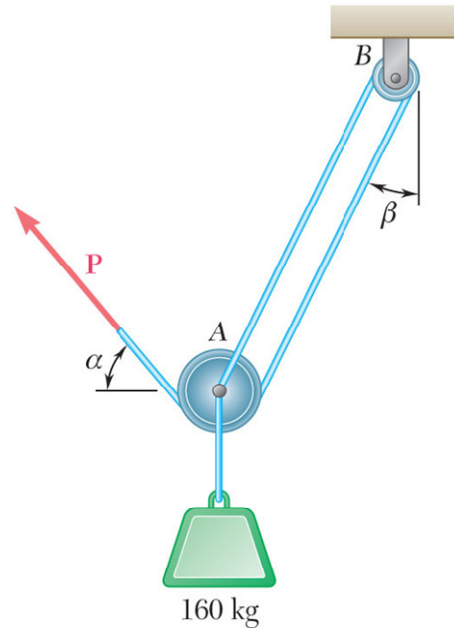
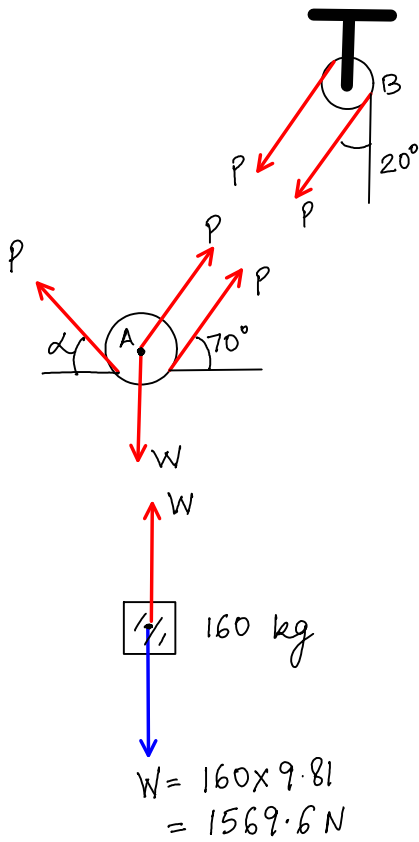
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- Exercise 2.65

$$\beta = 20^\circ$$

Find **P** for equilibrium.

We have to consider the free body diagrams of all the "particles" in the system here.



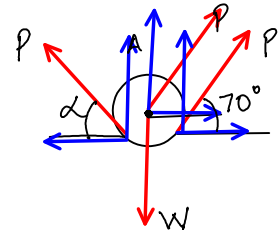
Equilibrium of the pulley A:

$$\sum F_x = 0$$

$$\Rightarrow (-)P \cos \alpha + 2P \cos(70^\circ) = 0$$

$$\Rightarrow \cos \alpha = 2 \cos(70^\circ)$$

$$\Rightarrow \alpha = 46.84^\circ$$



$$\sum F_y = 0$$

$$\Rightarrow -W + P \sin \alpha + 2P \sin(70^\circ) = 0$$

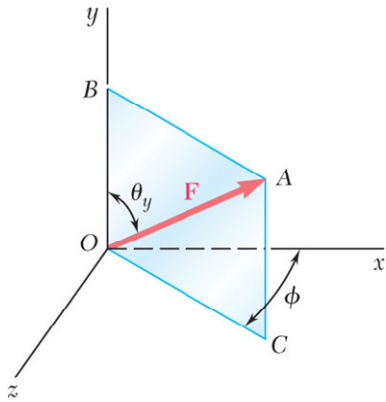
$$\Rightarrow P = \frac{W}{(\sin \alpha + 2 \sin(70^\circ))}$$

$$\Rightarrow P = 601.6486 \approx$$

$$P = 601.65 \text{ N}$$

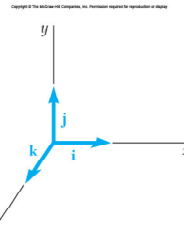
2.12 - 2.14 Forces (and Vectors) in 3-Dimensional Space

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(a)

A force in 3-dimensional space can be represented with the angles ϕ and θ that it makes with the x-axis and the vertical axis respectively.



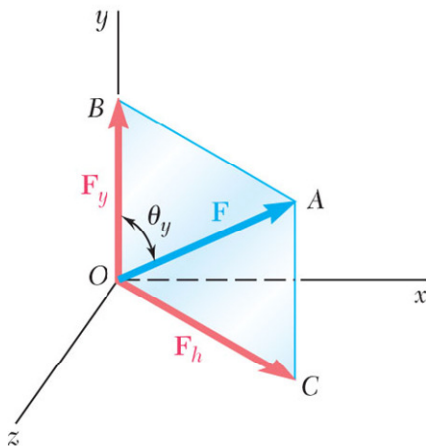
Unit vectors in space.

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k}$$

$$\underline{F} = \underbrace{F_x}_{\underline{F}_x} \underbrace{\mathbf{i}}_{\text{Unit vector}} + \underbrace{F_y}_{\underline{F}_y} \underbrace{\mathbf{j}}_{\text{Unit vector}} + \underbrace{F_z}_{\underline{F}_z} \underbrace{\mathbf{k}}_{\text{Unit vector}}$$

Vector

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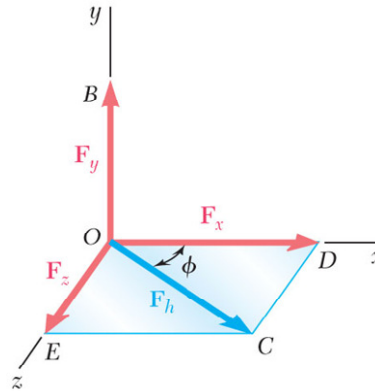
(b)

- Resolve \vec{F} into horizontal and vertical components.

$$F_y = F \cos \theta_y$$

$$F_h = F \sin \theta_y$$

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(c)

- Resolve F_h into rectangular components

$$F_x = F_h \cos \phi$$

$$= F \sin \theta_y \cos \phi$$

$$F_z = F_h \sin \phi$$

$$= F \sin \theta_y \sin \phi$$

Note:

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

Example:

Consider the force shown:

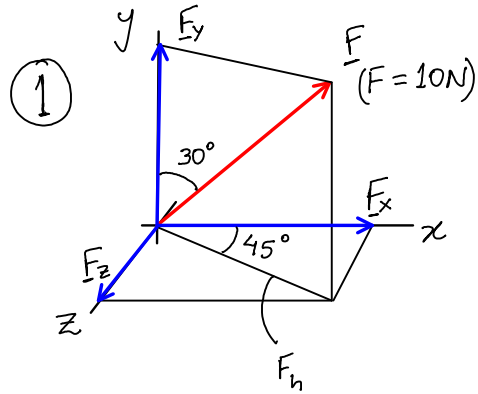
$$F_y = F \cos 30^\circ = 10 \times \frac{\sqrt{3}}{2} \text{ N}$$

$$\Rightarrow \underline{F_y} = 8.66 \underline{j} \text{ N}$$

$$F_h = F \sin 30^\circ = 5 \text{ N}$$

$$\text{Thus } F_x = F_h \cos 45^\circ = 5 \times \frac{1}{\sqrt{2}} = 3.54 \text{ N}$$

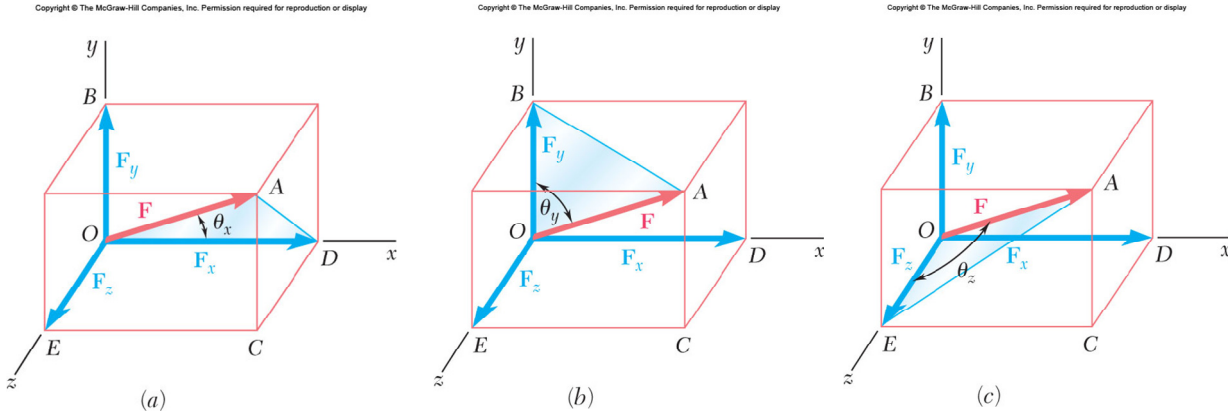
$$F_z = F_h \sin 45^\circ = 5 \times \frac{1}{\sqrt{2}} = 3.54 \text{ N}$$



$$\Rightarrow \underline{F_x} = 3.54 \underline{i} \text{ N}$$

$$\underline{F_z} = 3.54 \underline{k} \text{ N}$$

Direction Cosines



$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

$$F_x = F \cos \theta_x$$

$$F_y = F \cos \theta_y$$

$$F_z = F \cos \theta_z$$

Example:

For the example shown above in figure 1:

$$\cos \theta_x = \frac{F_x}{F} = \frac{3.54}{10} \Rightarrow \theta_x = 69.3^\circ$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{8.66}{10} \Rightarrow \theta_y = 30^\circ$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{3.54}{10} \Rightarrow \theta_z = 69.3^\circ$$

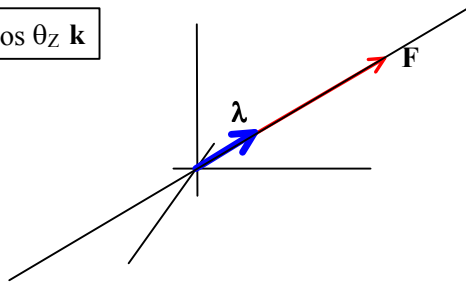
General 3-dimensional Unit Vector

A general 3-D unit vector can be used to represent the line of action of a 3-D force.

$$\lambda = \cos \theta_x \mathbf{i} + \cos \theta_y \mathbf{j} + \cos \theta_z \mathbf{k}$$

$$\mathbf{F} = F \lambda$$

$$\underline{\mathbf{F}} = F \underline{\lambda}$$



Example

For the example in figure 1:

$$\lambda = 0.354 \mathbf{i} + 0.866 \mathbf{j} + 0.354 \mathbf{k}$$

$$\mathbf{F} = 10 \lambda \text{ N}$$

Addition of forces (vectors) in 3-D space

Simply add the x, y, and z components.

$$\mathbf{R} = \Sigma \mathbf{F} \quad \text{or} \quad \begin{aligned} R_x &= \Sigma F_x \\ R_y &= \Sigma F_y \\ R_z &= \Sigma F_z \end{aligned}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

2.15 Equilibrium of Particles in 3D Space

$$\mathbf{R} = \Sigma \mathbf{F} = \mathbf{0}$$

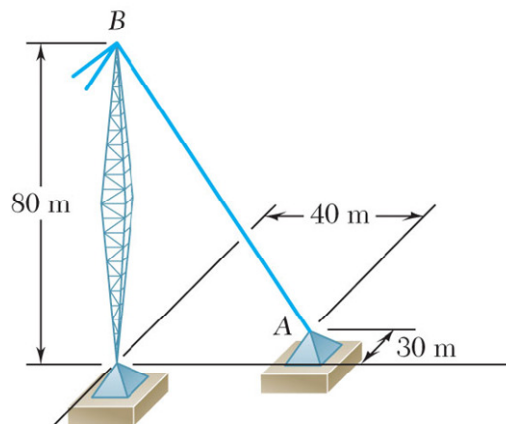
$$\Sigma F_x = 0 ; \quad \Sigma F_y = 0 ; \quad \Sigma F_z = 0$$

Note:

3 independent equations. Thus 3 unknowns can be solved for.

• Example 2.7 in book

Tension in wire = 2500 N
 Find
 F_x, F_y, F_z at A
 and $\theta_x, \theta_y, \theta_z$.



Approach:

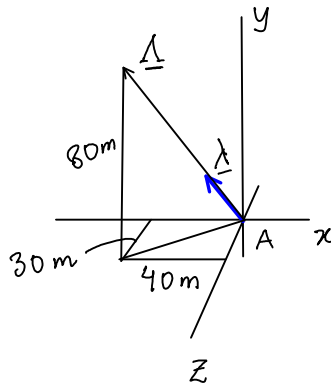
If we know the "direction" of \underline{F} , we can find F_x, F_y, F_z .

Direction \leftrightarrow Unit Vector $\underline{\lambda}$

$$\underline{\Delta} = -40 \underline{i} + 80 \underline{j} + 30 \underline{k} \text{ m}$$

$$|\underline{\Delta}| = \Delta = \sqrt{(-40)^2 + (80)^2 + (30)^2}$$

$$= 94.34 \text{ m}$$



\Rightarrow Unit Vector

$$\underline{\lambda} = \frac{\underline{\Delta}}{\Delta} = \frac{-40}{94.34} \underline{i} + \frac{80}{94.34} \underline{j} + \frac{30}{94.34} \underline{k}$$

$$\Rightarrow \underline{\lambda} = -0.424 \underline{i} + 0.848 \underline{j} + 0.318 \underline{k}$$

Recall:

$$\underline{F} = F \underline{\lambda} = 2500 (\underline{\lambda}) \text{ N}$$

$$\underline{F} = \underbrace{-1060}_{F_x} \underline{i} + \underbrace{2120}_{F_y} \underline{j} + \underbrace{795}_{F_z} \underline{k} \text{ N}$$

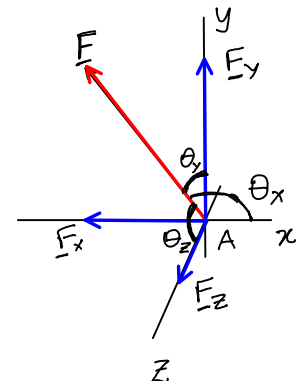
Also recall:

$$\Rightarrow \underline{\lambda} = \underbrace{-0.424}_{\cos \theta_x} \underline{i} + \underbrace{0.848}_{\cos \theta_y} \underline{j} + \underbrace{0.318}_{\cos \theta_z} \underline{k}$$

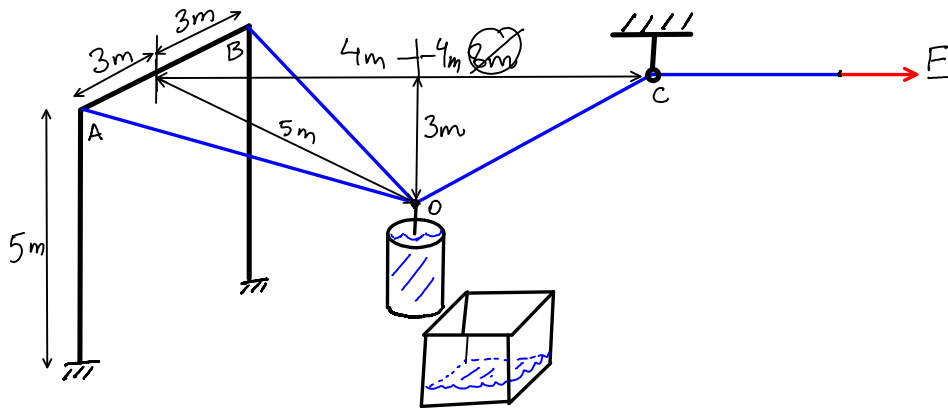
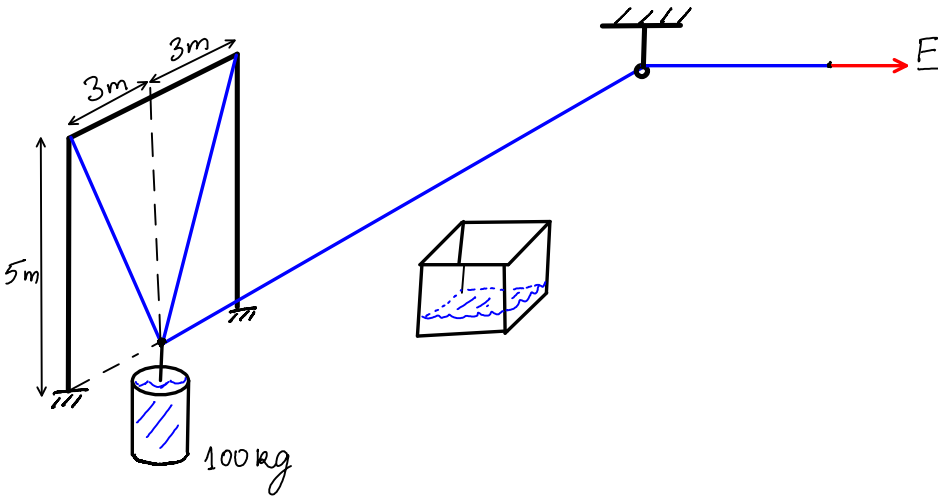
$$\Rightarrow \theta_x = \cos^{-1}(-0.424) = 115.1^\circ$$

$$\theta_y = \cos^{-1}(0.848) = 32^\circ$$

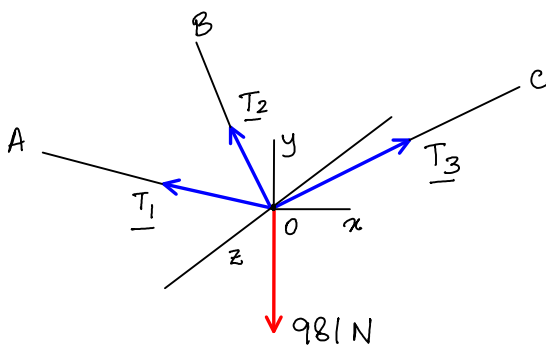
$$\theta_z = \cos^{-1}(0.318) = 71.5^\circ$$



Example



Free body diagram of point O:



Note:

$$\underline{T}_1 = T_1 \underline{\lambda}_1$$

$$\underline{T}_2 = T_2 \underline{\lambda}_2$$

$$\underline{T}_3 = T_3 \underline{\lambda}_3$$

$$\vec{OA} = -4 \underline{i} + 3 \underline{j} + 3 \underline{k} \text{ m}$$

$$\Rightarrow |OA| = \sqrt{4^2 + 3^2 + 3^2} = 5.83 \text{ m}$$

$$\Rightarrow \text{Unit vector } \underline{\lambda}_1 = \frac{\vec{OA}}{|OA|} = \frac{-4}{5.83} \underline{i} + \frac{3}{5.83} \underline{j} + \frac{3}{5.83} \underline{k}$$

Similarly

$$\vec{OB} = -4\hat{i} + 3\hat{j} - 3\hat{k} \text{ m}$$

$$\Rightarrow |OB| = \sqrt{4^2 + 3^2 + 3^2} = 5.83 \text{ m}$$

$$\Rightarrow \lambda_2 = \frac{\vec{OB}}{|OB|} = \frac{-4}{5.83}\hat{i} + \frac{3}{5.83}\hat{j} - \frac{3}{5.83}\hat{k}$$

and

$$\vec{OC} = 4\hat{i} + 3\hat{j} \text{ m}$$

$$\Rightarrow |OC| = 5 \text{ m}$$

$$\Rightarrow \lambda_3 = 0.8\hat{i} + 0.6\hat{j}$$

Finally enforcing equilibrium of point O:-

$$\boxed{\sum F_x = 0}$$

$$\Rightarrow T_1 \left(\frac{-4}{5.83} \right) + T_2 \left(\frac{-4}{5.83} \right) + T_3 (0.8) = 0 \quad \text{--- (1)}$$

$$\boxed{\sum F_y = 0}$$

$$\Rightarrow T_1 \left(\frac{3}{5.83} \right) + T_2 \left(\frac{3}{5.83} \right) + T_3 (0.6) - 981 = 0 \quad \text{--- (2)}$$

$$\boxed{\sum F_z = 0}$$

$$\Rightarrow T_1 \left(\frac{3}{5.83} \right) + T_2 \left(\frac{-3}{5.83} \right) = 0 \quad \Rightarrow \boxed{T_1 = T_2}$$

From (1):-

$$T_3 = 2T_1 \left(\frac{5}{5.83} \right) \Rightarrow \boxed{T_1 = 0.583 T_3}$$

Substituting into (2):-

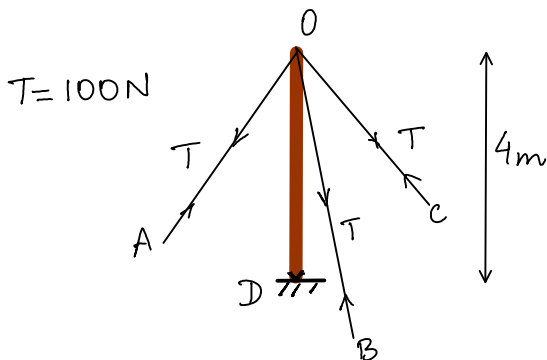
$$T_3 \left(\frac{0.583}{5.83} \times 3 + \frac{0.583}{5.83} \times 3 + 0.6 \right) = 981 \Rightarrow \boxed{T_3 = 817.5 \text{ N}}$$

Thus the force

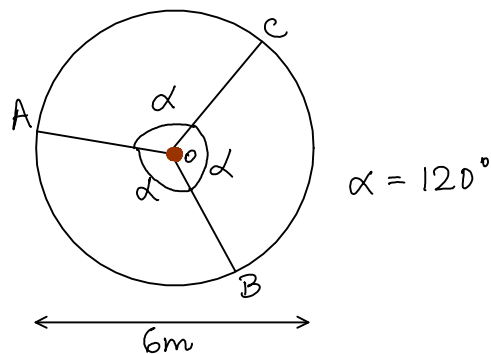
$$\boxed{\underline{F} = 817.5 \hat{i} \text{ N}}$$

In Class Assignment 2:

Side View:



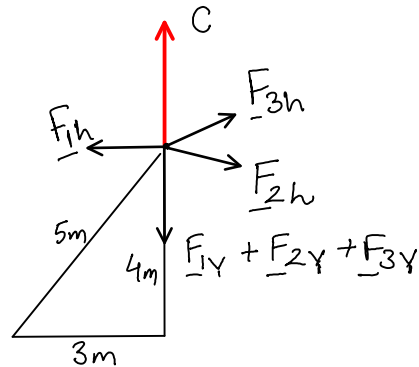
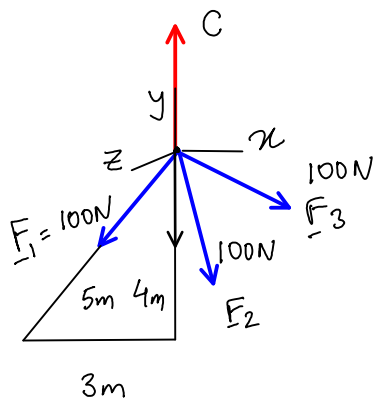
Top View:



Find:

- (i) Is the pole OD in "tension" or "compression".
- (ii) Magnitude of the force in the pole OD.

Consider point O:



Choose x, y, z such that "y" is vertical; and F_{1h} is in the (-) "x" direction.

Note:

This is an arbitrary choice. You may choose a different x, y, z , and the result will be the same.

$$\begin{aligned} \underline{F}_{1y} &= \underline{F}_{2y} = \underline{F}_{3y} \\ &= -80 \underline{j} \text{ N} \\ \underline{F}_{1h} &= \underline{F}_{2h} = \underline{F}_{3h} \\ &= 60 \text{ N (magnitude)} \end{aligned}$$

Resolving the 3 forces into "vertical" and "horizontal" components, rather than x, y, z .

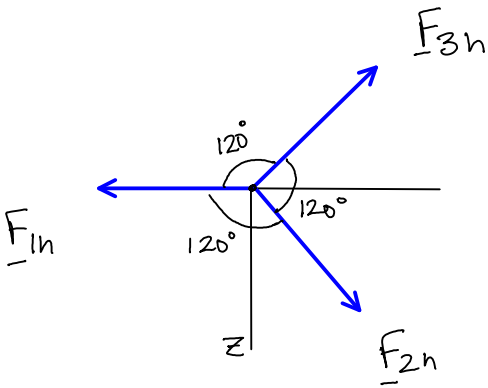
Each of the 3 triangles will be "similar".

$$\boxed{\sum F_y = 0}$$

$$\Rightarrow F_{1y} + F_{2y} + F_{3y} + C = 0$$

$$\Rightarrow \boxed{C = 240 \underline{j} \text{ N}}$$

On the x-z plane:



$$\sum F_x = 0$$

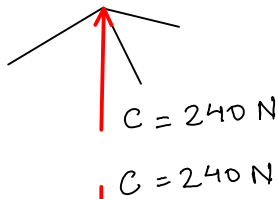
$$\Rightarrow -60 + 60 \cos(60^\circ) + 60 \cos(60^\circ) = 0$$

satisfied!

$$\sum F_z = 0$$

$$\Rightarrow 60 \sin(60^\circ) - 60 \sin(60^\circ) = 0$$

satisfied!



Free body diagram of the pole:

\Rightarrow Compression