Plane Stress - 2D Behavior

Ref: Ch-9-10 Hibbeler, 10th Ed.

Numerous real-life applications lead to a state of plane stress where only those stress components that are in a particular 2D plane are non-zero. All out-of-plane stress components are zero.



Examples: Thin plates used in walls, roofs, sheet-metal bodies of automobiles, aircrafts, ships etc.

State of stress is represented as:

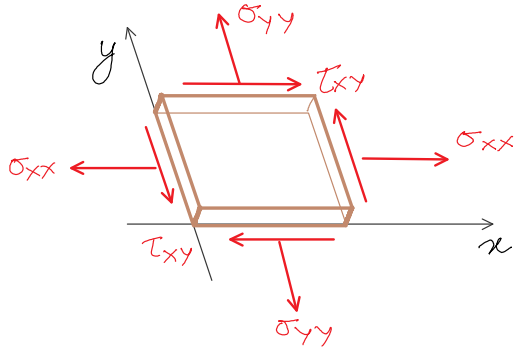
In-plane

$$\begin{bmatrix} \sigma_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix}$$

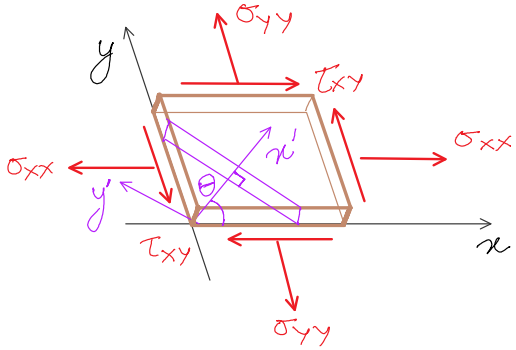
Note:

$$\sigma_{zz} = \tau_{zx} = \tau_{zy} = 0$$

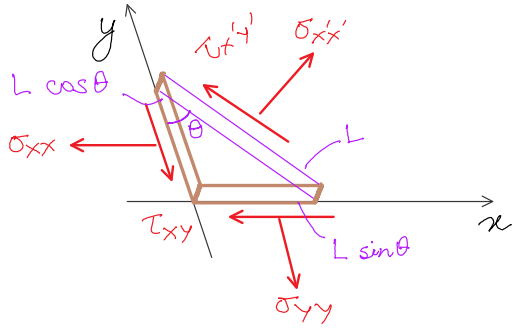
Out-of-plane



Often, there is a need to find normal and shear stress components on faces that are not normal to x or y axes. This is done by **Stress Transformation**:



FBD-1



From FBD-1:

$$\Sigma F_{x'} = 0 \Rightarrow \sigma_{x'x'}(\cancel{Lt}) - \sigma_{xx}(\cancel{Lt}\cos\theta)\cos\theta - \sigma_{yy}(\cancel{Lt}\sin\theta)\sin\theta - \tau_{xy}(\cancel{Lt}\sin\theta)\cos\theta - \tau_{xy}(\cancel{Lt}\cos\theta)\sin\theta = 0$$

$$\Rightarrow \sigma_{x'x'} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta - \tau_{xy}(2\sin\theta\cos\theta)$$

Note:

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$
$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\Rightarrow \boxed{\sigma_{x'x'} = \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) + \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta}$$

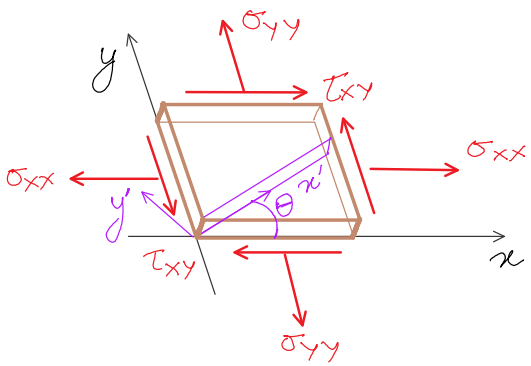
Similarly

$$\sum F_{y'} = 0 \Rightarrow \tau_{x'y'}(Lt) + \sigma_{xx}(Lt\cos\theta)\sin\theta - \tau_{xy}(Lt\cos\theta)\cos\theta - \sigma_{yy}(Lt\sin\theta)\cos\theta + \tau_{xy}(Lt\sin\theta)\sin\theta = 0$$

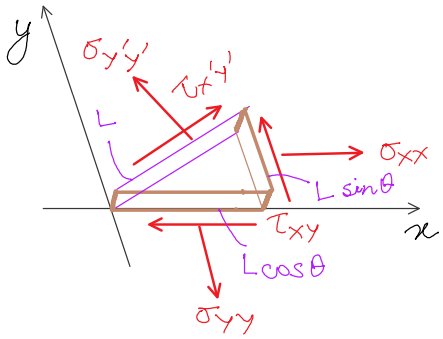
$$\Rightarrow \tau_{x'y'} = -(\sigma_{xx}-\sigma_{yy})\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\Rightarrow \boxed{\tau_{x'y'} = -\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta}$$

Note also:



FBD-2:



From FBD-2:

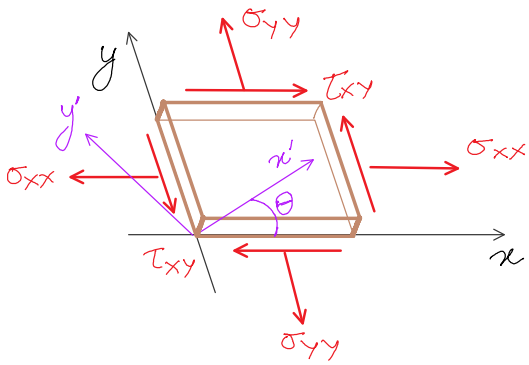
$$\sum F_{y'} = 0 \Rightarrow \sigma_{y'y'}(Lt) - \sigma_{yy}(Lt\cos\theta)\cos\theta - \sigma_{xx}(Lt\sin\theta)\sin\theta + \tau_{xy}(Lt\sin\theta)\cos\theta + \tau_{xy}(Lt\cos\theta)\sin\theta = 0$$

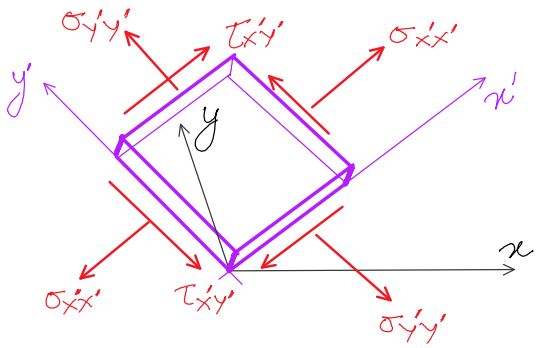
$$\Rightarrow \sigma_{y'y'} = \sigma_{yy}\cos^2\theta + \sigma_{xx}\sin^2\theta + \tau_{xy}(2\sin\theta\cos\theta)$$

$$\Rightarrow \boxed{\sigma_{y'y'} = \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) - \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta}$$

Note that this expression could have also been obtained by substituting θ+90° for θ in $\sigma_{x'x'}$.

Combining:

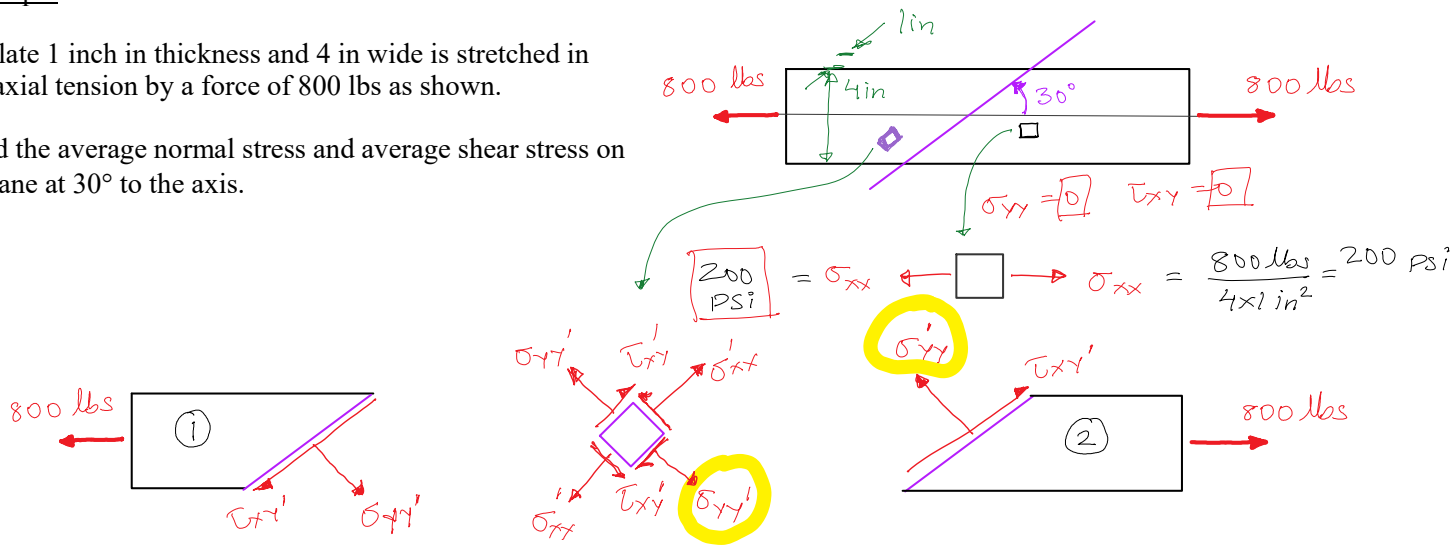




Example

A plate 1 inch in thickness and 4 in wide is stretched in uniaxial tension by a force of 800 lbs as shown.

Find the average normal stress and average shear stress on a plane at 30° to the axis.

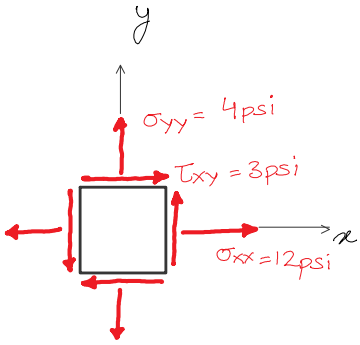


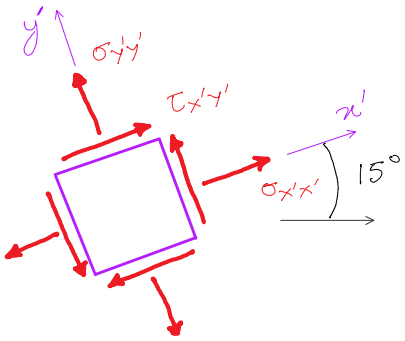
$$\sigma_{y'y'} = \left(\frac{200 + 0}{2}\right) - \left(\frac{200 - 0}{2}\right)\cos 120^\circ - 0 \sin 120^\circ = 50\ psi$$

$$\tau_{x'y'} = -\left(\frac{200 - 0}{2}\right)\sin 120^\circ + 0\cos 120^\circ = -86.67\ psi$$

Example

Given state of stress in *xy* axes
Find state of stress in *x'y'* axes





$$\underset{\sim}{\sigma} \sim \begin{bmatrix} 12 & 3 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{(x,y)}$$

$$\sim \begin{bmatrix} \sigma_{x'x'} & \tau_{x'y'} & 0 \\ \tau_{x'y'} & \sigma_{y'y'} & 0 \\ 0 & 0 & 0 \end{bmatrix}_{(x'y')}$$

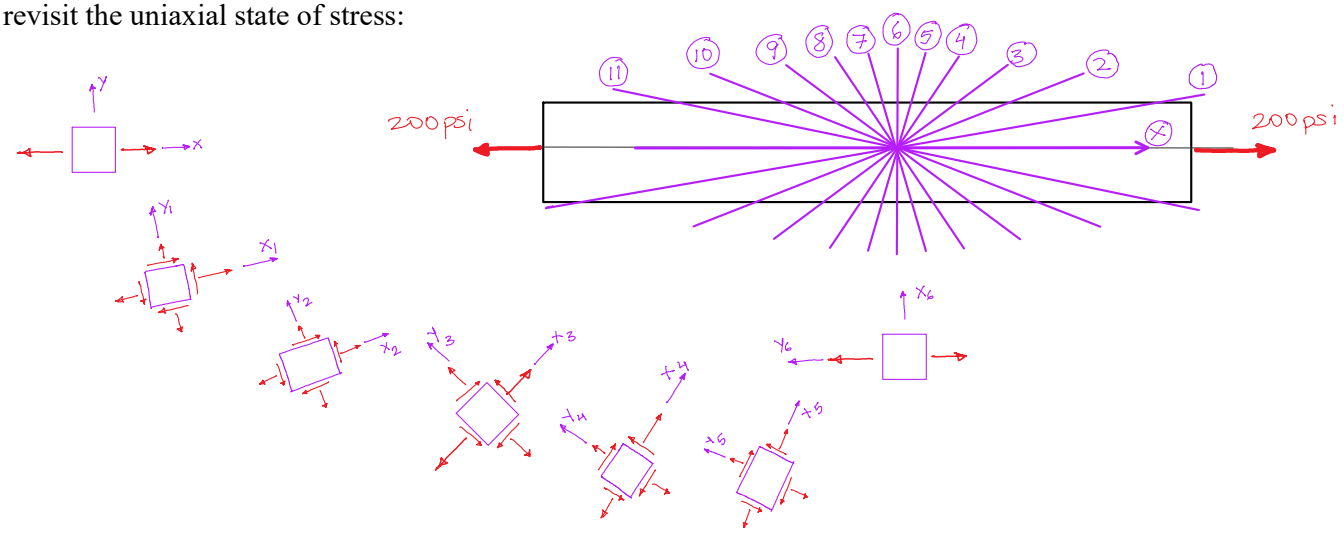
$$\sigma_{x'x'} = \left(\frac{12 + 4}{2}\right) + \left(\frac{12 - 4}{2}\right)\cos 60^\circ + 3 \sin 60^\circ \Rightarrow \sigma_{x'x'} = 12.96\ psi$$

$$\sigma_{y'y'} = \left(\frac{12 + 4}{2}\right) - \left(\frac{12 - 4}{2}\right)\cos 60^\circ - 3 \sin 60^\circ \Rightarrow \sigma_{y'y'} = 3.04\ psi$$

$$\tau_{x'y'} = -\left(\frac{12 - 4}{2}\right)\sin 60^\circ + 3\cos 60^\circ \Rightarrow \tau_{x'y'} = 0.6\ psi$$

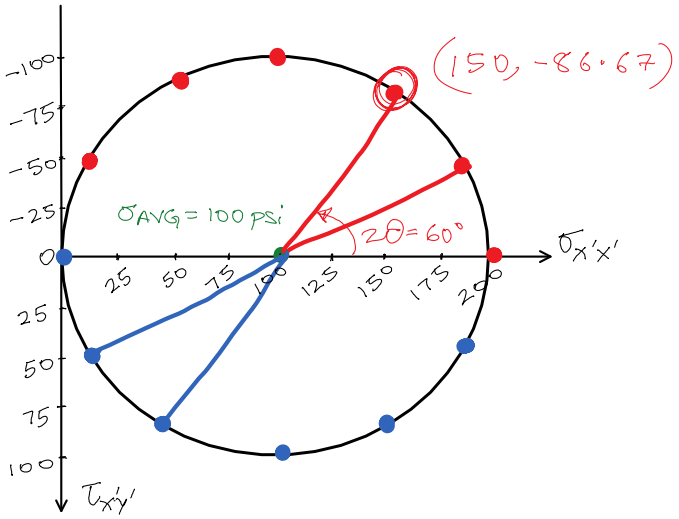
Graphical Representation of Stress: **Mohr's Circle**

Let's revisit the uniaxial state of stress:



Using the transformation relations, we can compute the normal and shear stress components on all these planes:

θ	$\sigma_{x'x'}$	$\tau_{x'y'}$	$\sigma_{y'y'}$	$-\tau_{x'y'}$
0	200.00	0.0	0.00	-0.00
15	186.60	-50.0	13.39	50.00
30	150.00	-86.6	50.00	86.60
45	100.00	-100.0	100.00	100.00
60	50.00	-86.6	150.00	86.60
75	13.39	-50.0	186.60	50.00
90	0	-0.0	200.00	0.00
105	13.39	50.0	186.60	-50.00
120	50.00	86.6	150.00	-86.60
135	100.00	100.0	100.00	-100.00
150	150.00	86.6	50.00	-86.60
165	186.60	50.0	13.39	-50.00
180	200.00	0.0	0.00	-0.00



Thus, the normal stress ($\sigma_{x'x'}$) and shear stress ($\tau_{x'y'}$) on an arbitrary plane with normal to the x' direction can be interpreted as points on a circle:

Proof:

$$\sigma_{x'x'} - \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) = \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Square both sides and add the above equations:

$$\underbrace{\left[\sigma_{x'x'} - \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right)\right]^2}_{\sigma_{AVG}} + \left[\tau_{x'y'}\right]^2 = \underbrace{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \left(\tau_{xy}\right)^2}_{R^2}$$

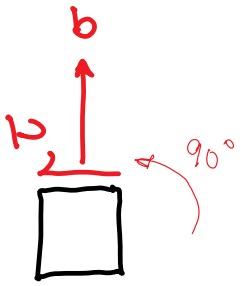
This is nothing but an equation of a circle with center at (σ_{AVG},0) and radius R.

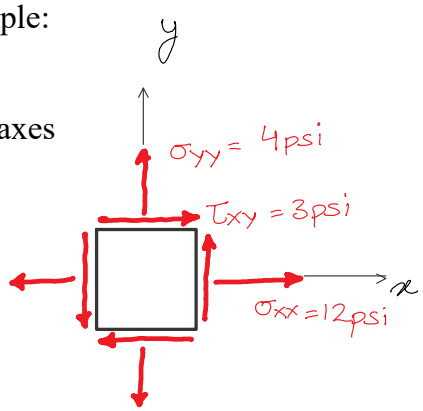
Example

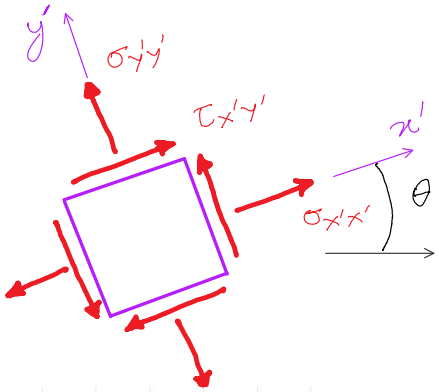
Let's revisit another previous example:

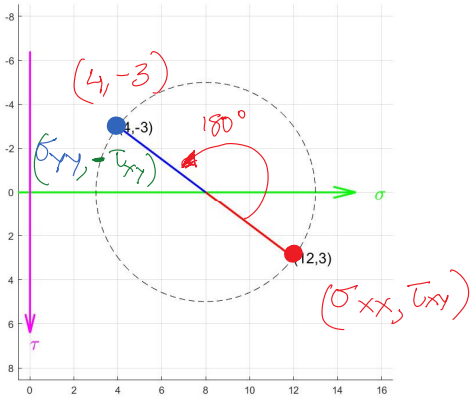
Given state of stress in *xy* axes

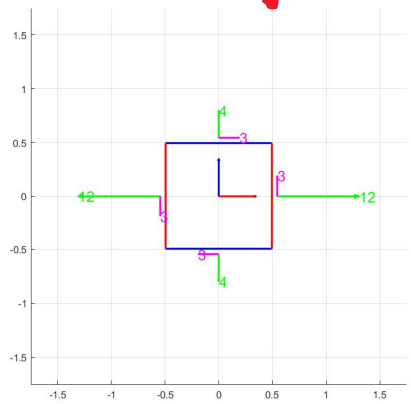
Find state of stress for various *x'y'* axes









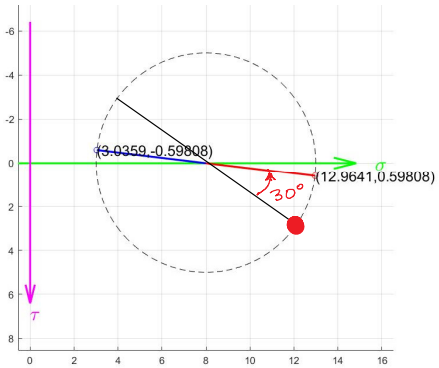


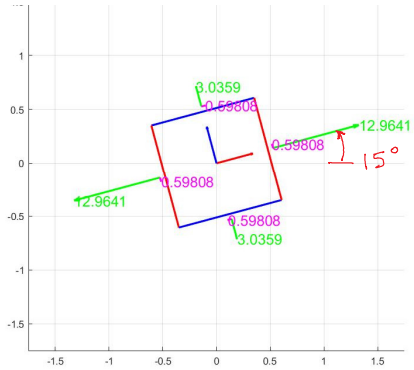
Recall, for θ=15°:

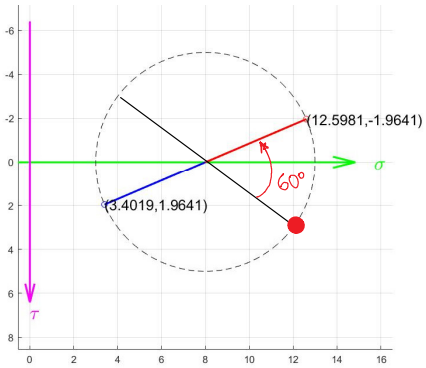
$\sigma_{x'x'} = 12.96$ psi

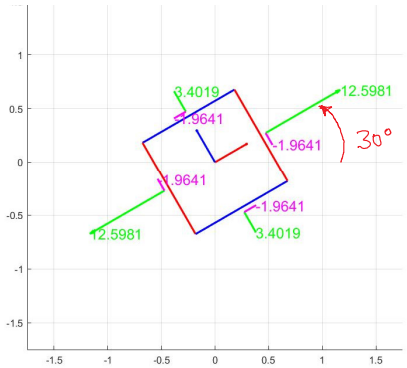
$\tau_{x'y'} = 0.6$ psi

$\sigma_{y'y'} = 3.04$ psi

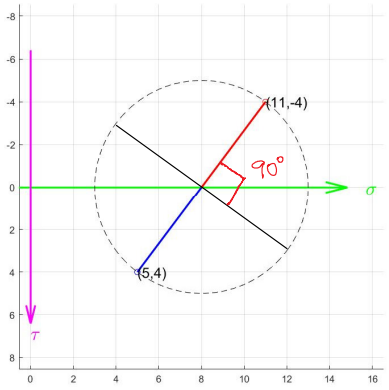


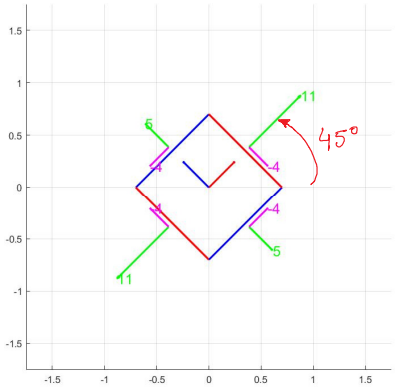


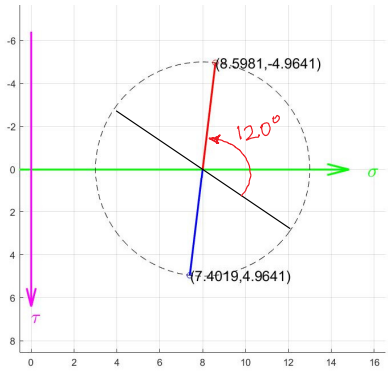


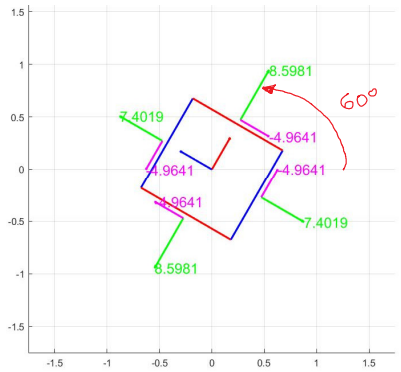


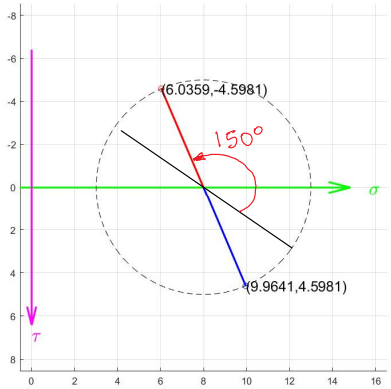
Example continued:

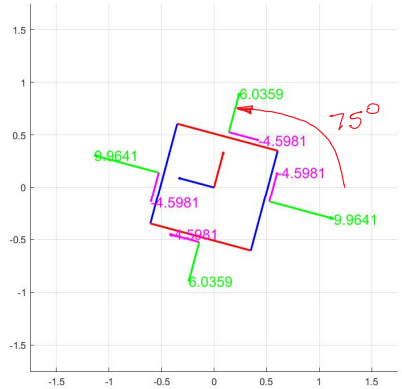


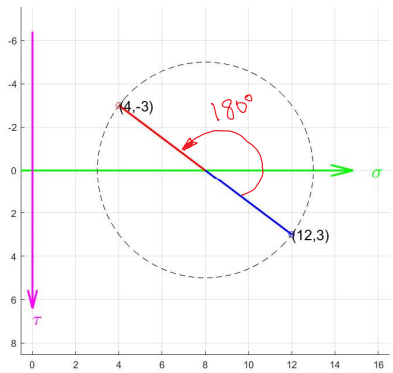


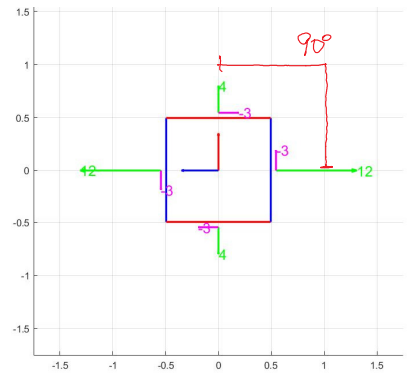








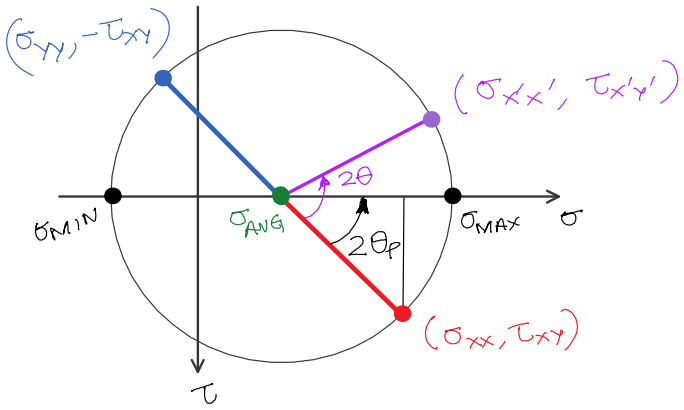


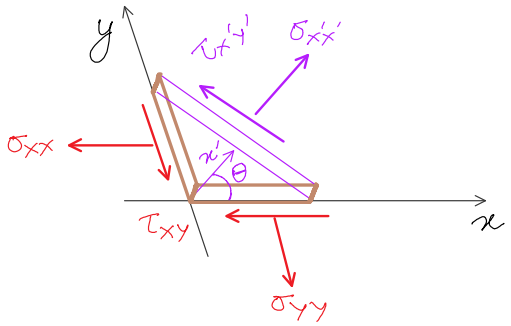


Planes of Maximum and Minimum Normal Stress: **Principal Stresses**

Since the state of stress at arbitrary angles of θ can be represented as points on the $\sigma-\tau$ plot, then there must be certain values of θ_P at which the normal stress $\sigma_{x'x'}$ is maximum or minimum:







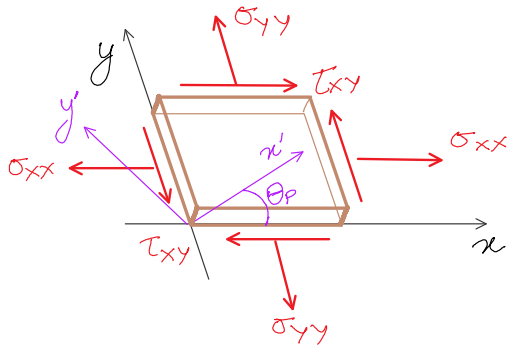
The value of θ_P can be found using geometry of the triangle shown: $\tan(2\theta_p) = \left(\frac{\tau_{xy}}{\sigma_{xx} - \sigma_{AVG}}\right)$

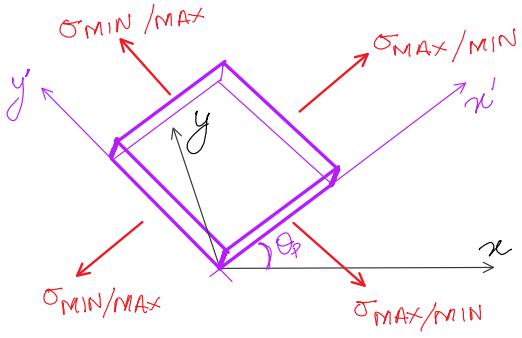
Since $\sigma_{AVG} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \Rightarrow$ $\boxed{\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{\tau_{xy}}{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)}\right)}$

This plane (at angle θ_P to the x-axis) is called a Principal plane and the normal stress on this plane is called the Principal stress at that point.

Notes:

1. Shear stress on the Principal Plane is zero.
2. There are two Principal planes θ_{P1} and θ_{P2}: one for σ_{max} and one for σ_{min} where $|\theta_{P1} - \theta_{P2}| = 90°$

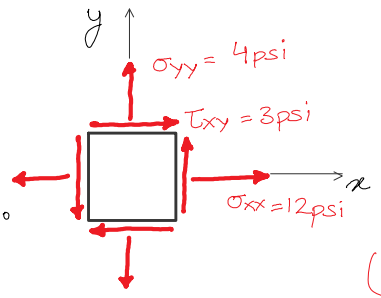


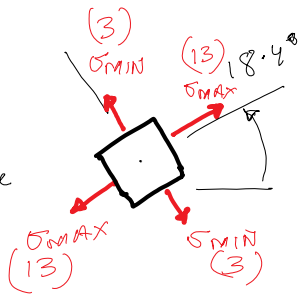


Example:
The state of stress at a point in a body is as shown.

Find the Principal directions and the Principal Stresses.

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{\tau_{xy}}{\frac{\sigma_{xx}-\sigma_{yy}}{2}}\right) \Rightarrow \theta_{p_1} = 18.4^\circ, \quad \theta_{p_2} = 108.4^\circ$$



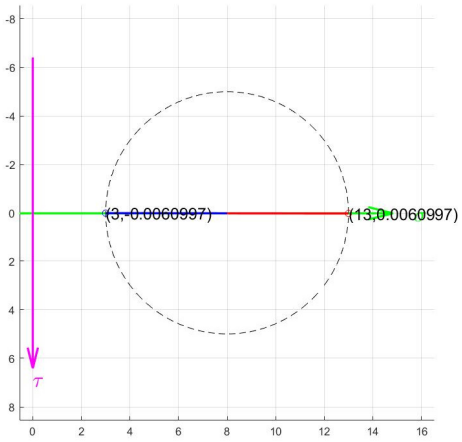


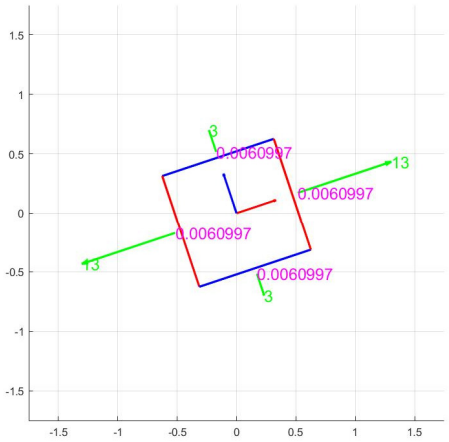
Normal stresses on these two planes:

On θ_{p_1}: $\sigma_{x'x'} = \left(\frac{12+4}{2}\right) + \left(\frac{12-4}{2}\right)\cos(2\theta_{p_1}) + 3\sin(2\theta_{p_1}) = 13\ psi$

On θ_{p_2}: $\sigma_{x'x'} = \left(\frac{12+4}{2}\right) + \left(\frac{12-4}{2}\right)\cos(2\theta_{p_2}) + 3\sin(2\theta_{p_2}) = 3\ psi$

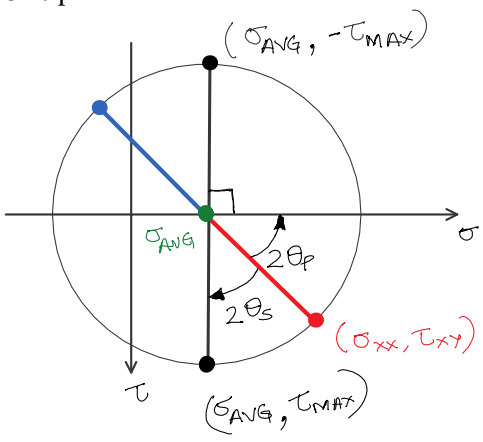
On both θ_{p_1} and θ_{p_2}: $\tau_{x'y'} = 0$

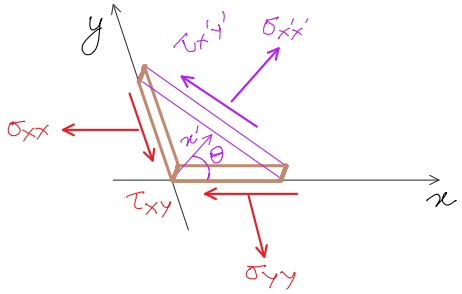




Planes of Maximum Shear Stress:

Similar Principal planes (on which Normal stress is maximum/minimum), there are planes on which the Shear stress achieves a maximum value. The angles θ_S for these planes can be represented as points on the $\sigma-\tau$ plot:





Notes:

1. $2\theta_P - 2\theta_S = 90^\circ \Rightarrow \boxed{\theta_P - \theta_S = 45^\circ}$

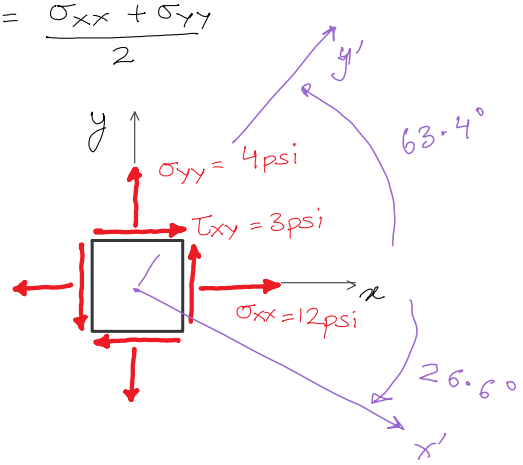
2. θ_{S_1} and θ_{S_2} have same magnitude of τ_{MAX} but opposite sign

3. $\tau_{MAX} = R = \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$

4. On θ_{S_1} and θ_{S_2}: Normal Stress $= \sigma_{AVG} = \frac{\sigma_{xx}+\sigma_{yy}}{2}$

Example:

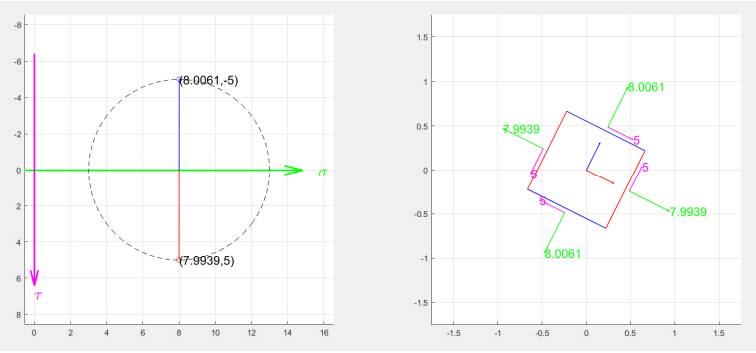
The state of stress at a point in a body is as shown.
Find the plane of maximum Shear stress.

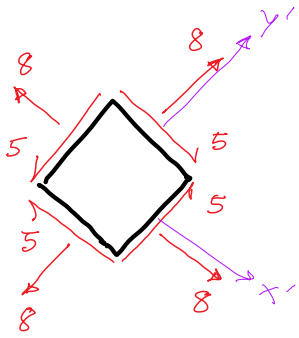


Recall: $\theta_P = \frac{1}{2}\tan^{-1}\left(\frac{\tau_{xy}}{\frac{\sigma_{xx}-\sigma_{yy}}{2}}\right)$ (3, 12, 4) $\Rightarrow \theta_{P_1} = 18.4^\circ$

$\theta_{P_2} = 108.4^\circ$

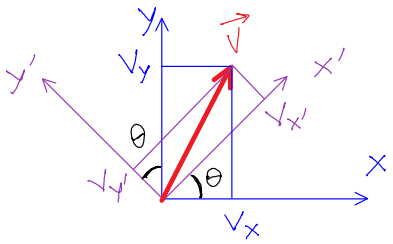
$\Rightarrow \theta_{S_{1/2}} = \theta_{P_1} \pm 45^\circ = -26.6^\circ \;/\; 63.4^\circ$





Aside: Stress transformation as a change of coordinate system:

$$\vec{V} = \{V_x, V_y\}_{x,y} = \{V_{x'}, V_{y'}\}_{x'y'}$$



For a vector:

$$\left.\begin{aligned} V_{x'} &= V_x \cos\theta + V_y \sin\theta \\ V_{y'} &= -V_x \sin\theta + V_y \cos\theta \end{aligned}\right\} \Rightarrow \begin{Bmatrix} V_{x'} \\ V_{y'} \end{Bmatrix} = \underbrace{\begin{bmatrix} c & s \\ -s & c \end{bmatrix}}_{\underset{\sim}{T}} \begin{Bmatrix} V_x \\ V_y \end{Bmatrix}$$

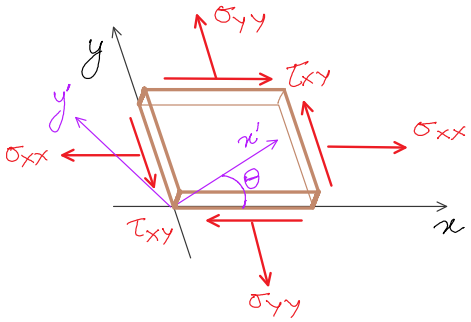
$\underset{\sim}{T}$ → orthogonal

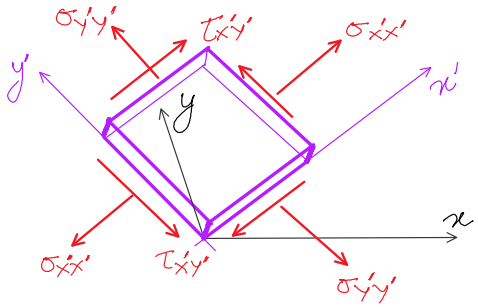
Reverse:

$$\left.\begin{aligned} V_x &= V_{x'} \cos\theta - V_{y'} \sin\theta \\ V_y &= V_{x'} \sin\theta + V_{y'} \cos\theta \end{aligned}\right\} \Rightarrow \begin{Bmatrix} V_x \\ V_y \end{Bmatrix} = \underbrace{\begin{bmatrix} c & -s \\ s & c \end{bmatrix}}_{\underset{\sim}{T}^{-1} = \underset{\sim}{T}^{T}} \begin{Bmatrix} V_{x'} \\ V_{y'} \end{Bmatrix}$$

Similarly, transformation of the stress tensor:

$$\underset{\sim}{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}_{(x,y,z)} \qquad \underset{\sim}{\sigma} = \begin{bmatrix} \sigma_{x'x'} & \sigma_{x'y'} & 0 \\ \sigma_{x'y'} & \sigma_{y'y'} & 0 \\ 0 & 0 & 0 \end{bmatrix}_{(x',y',z')}$$





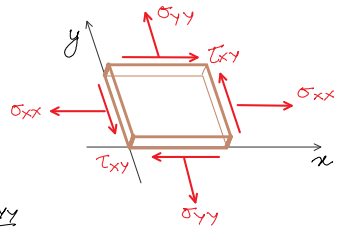
$$\underbrace{\begin{bmatrix} c & s \\ -s & c \end{bmatrix}}_{\underset{\sim}{T}} \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} \underbrace{\begin{bmatrix} c & -s \\ s & c \end{bmatrix}}_{\underset{\sim}{T}^{T}} = \begin{bmatrix} \sigma_{x'x'} & \sigma_{x'y'} \\ \sigma_{x'y'} & \sigma_{y'y'} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} = \underbrace{\begin{bmatrix} c & -s \\ s & c \end{bmatrix}}_{\underset{\sim}{T}^{T}} \begin{bmatrix} \sigma_{x'x'} & \sigma_{x'y'} \\ \sigma_{x'y'} & \sigma_{y'y'} \end{bmatrix} \underbrace{\begin{bmatrix} c & s \\ -s & c \end{bmatrix}}_{\underset{\sim}{T}}$$

State of strain in Plane stress

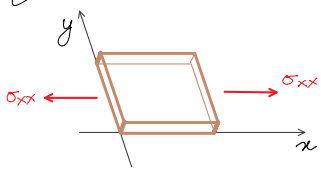
Stress:

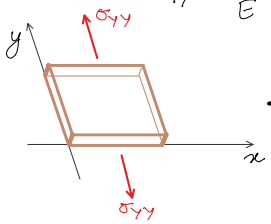
$$\begin{bmatrix} \sigma_{xx} & \tau_{yx} & 0 \\ \tau_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

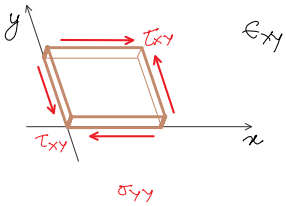


$$\frac{\sigma_{xx}}{E} = \epsilon_{xx} \qquad \epsilon_{yy} = \frac{\sigma_{yy}}{E} \qquad \epsilon_{xy} = \frac{\gamma_{xy}}{2} = \frac{\tau_{xy}}{2G}$$

$$= \quad + \quad + \quad$$







Strain:

$$= \begin{bmatrix} \frac{\sigma_{xx}}{E} & 0 & 0 \\ 0 & -\frac{\nu\sigma_{xx}}{E} & 0 \\ 0 & 0 & -\frac{\nu\sigma_{xx}}{E} \end{bmatrix} + \begin{bmatrix} -\frac{\nu\sigma_{yy}}{E} & 0 & 0 \\ 0 & \frac{\sigma_{yy}}{E} & 0 \\ 0 & 0 & -\frac{\nu\sigma_{yy}}{E} \end{bmatrix} + \begin{bmatrix} 0 & \frac{\tau_{xy}}{2G} & 0 \\ \frac{\tau_{xy}}{2G} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{yx} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix}$$

Thus the state of strain is given by:

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu\sigma_{yy}}{E}$$

$$\epsilon_{yy} = -\frac{\nu\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E}$$

$$\epsilon_{zz} = -\frac{\nu\sigma_{xx}}{E} - \frac{\nu\sigma_{yy}}{E}$$

$$\gamma_{xy} = 2\epsilon_{xy} = \frac{\tau_{xy}}{G}$$

$$\epsilon_{xz} = \epsilon_{yz} = 0$$

Example

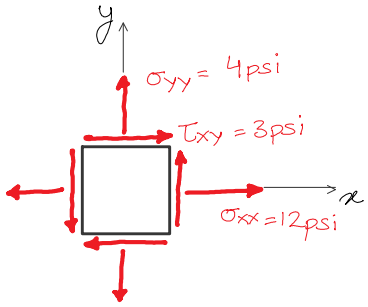
Given state of stress in *xy* axes.

Let $E = 10{,}000$ psi and $\nu = 0.25$

Find state of strain.

Stress:

$$\begin{bmatrix} 12 & 3 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



Note: $G = \frac{E}{2(1+\nu)} = 4000$ psi

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu\sigma_{yy}}{E} = \frac{12\ \text{psi}}{10000\ \text{psi}} - 0.25 \times \frac{4\ \text{psi}}{10000\ \text{psi}}$$

$$\epsilon_{yy} = -\frac{\nu\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} = 0.04 \times 10^{-3}$$

$$\epsilon_{zz} = -\frac{\nu\sigma_{xx}}{E} - \frac{\nu\sigma_{yy}}{E} = -0.48 \times 10^{-3}$$

Strain:

$$= 10^{-3} \begin{bmatrix} 1.08 & 0.375 & 0 \\ 0.375 & 0.04 & 0 \\ 0 & 0 & -0.48 \end{bmatrix}$$

$$\gamma_{xy} = 2\epsilon_{xy} = \frac{\tau_{xy}}{G} = \frac{3\ \text{psi}}{4000\ \text{psi}} = 0.75 \times 10^{-3} \Rightarrow \epsilon_{xy} = \frac{\gamma_{xy}}{2} = 0.375 \times 10^{-3}$$

$$\epsilon_{xz} = \epsilon_{yz} = 0$$

Plane Strain Problems

In problems where all deformation in a particular direction (say z) is zero (i.e. all deformation occurs only in the x-y plane) leads to a state of plane strain.



In this case:

Out-of-plane

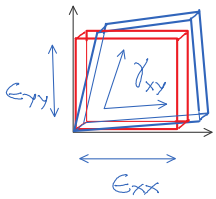
$\epsilon_{zz} = \epsilon_{zx} = \epsilon_{yz} = 0$

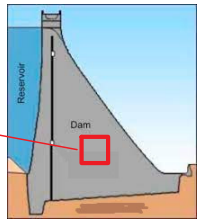
$(\epsilon_{xz}) \quad (\epsilon_{zy})$

In-plane

$$\begin{bmatrix} \epsilon_{xx} & \epsilon_{yx} & \epsilon_{zx} \\ \epsilon_{xy} & \epsilon_{yy} & \epsilon_{zy} \\ \epsilon_{xz} & \epsilon_{yz} & \epsilon_{zz} \end{bmatrix}$$

Cross-section:

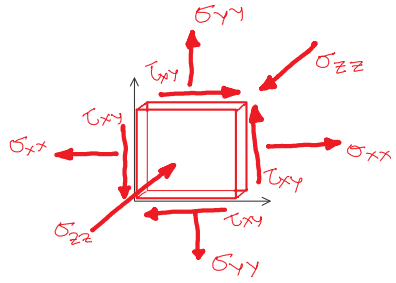




Note, however, that stress $\sigma_{zz} \neq 0$

Stress-strain relationship for Plane Strain:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & 0 \\ \nu & (1-\nu) & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2\epsilon_{xy} \end{Bmatrix}$$

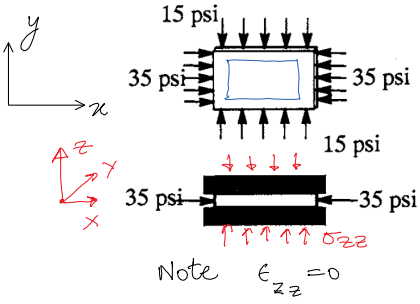


$$\sigma_{zz} = \frac{\nu E}{(1+\nu)(1-2\nu)} (\epsilon_{xx} + \epsilon_{yy})$$

Example: Plane Strain

Given $E = 1000$ ksi, $\nu = 0.2$, initial thickness = 0.3 inch

Find state of stress and state of strain.



plane strain

Stress:

$$\begin{bmatrix} -35 & 0 & 0 \\ 0 & -15 & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix}$$

Strain:

$$\begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sigma_{zz} = \frac{0.2 \times 1000\ \text{ksi}}{(1.2)(0.6)} (\epsilon_{xx} + \epsilon_{yy}) \quad \text{— (1)}$$

$$\Rightarrow \sigma_{zz} = -10\ \text{psi}$$

$$\epsilon_{xx} = \frac{-35\ \text{psi}}{1000\ \text{ksi}} - \frac{0.2(-15)\ \text{psi}}{1000\ \text{ksi}} - \frac{0.2(\sigma_{zz})}{1000\ \text{ksi}} \quad \text{— (2)}$$

$$\epsilon_{yy} = -\frac{0.2(-35)\ \text{psi}}{1000\ \text{ksi}} + \frac{(-15)\ \text{psi}}{1000\ \text{ksi}} - \frac{0.2(\sigma_{zz})}{1000\ \text{ksi}} \quad \text{— (3)}$$

$$\epsilon_{xx} = -30 \times 10^{-6}$$

$$\epsilon_{yy} = -6 \times 10^{-6}$$

OR

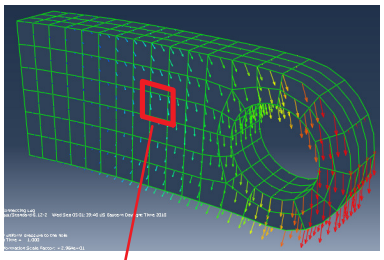
$$0 = \epsilon_{zz} = \frac{\sigma_{zz}}{E} - \nu \frac{-35}{E} - \nu \frac{-15}{E} \Rightarrow \sigma_{zz} = -10\ \text{psi}$$

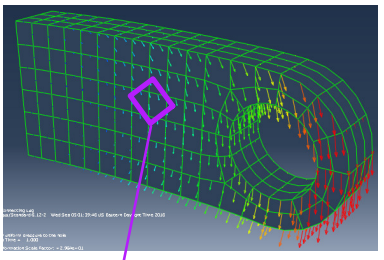
Transformation on Strain: **Mohr's circle** of strain

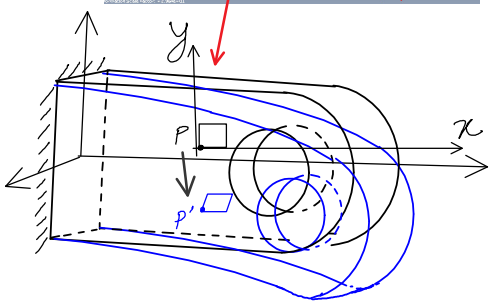
The state of Strain in either plane stress or plane strain will be:

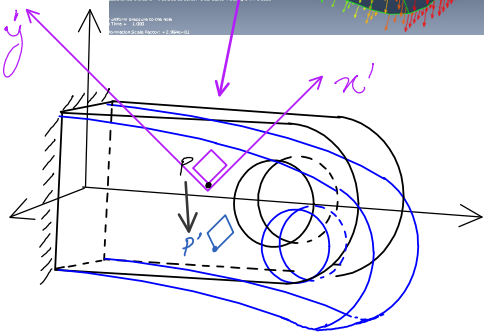
$$\begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{yx} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix}$$

where $\epsilon_{zz} = 0$ (for plane strain)



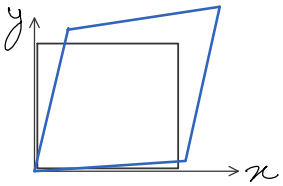


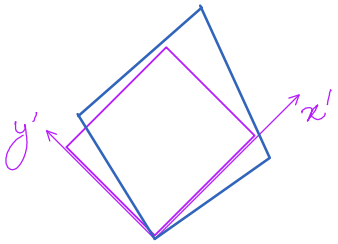




To transform:

$$\begin{bmatrix} \epsilon_{x'x'} & \epsilon_{x'y'} & 0 \\ \epsilon_{y'x'} & \epsilon_{y'y'} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix}$$





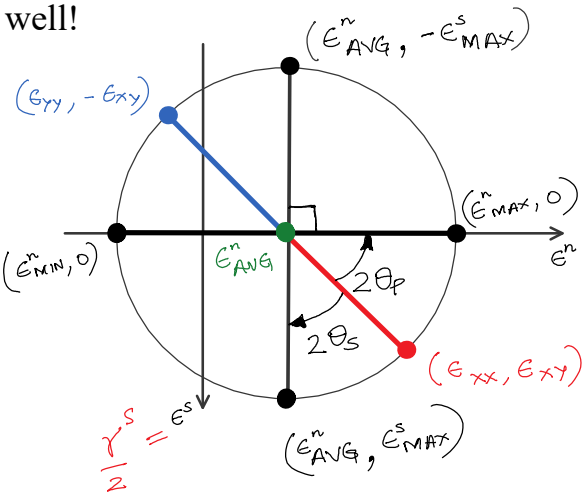
Similar equations of transformation hold for strains too!

$$\epsilon_{x'x'} = \left(\frac{\epsilon_{xx} + \epsilon_{yy}}{2}\right) + \left(\frac{\epsilon_{xx} - \epsilon_{yy}}{2}\right)\cos 2\theta + \epsilon_{xy}\sin 2\theta$$

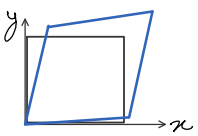
$$\frac{\gamma_{x'y'}}{2} = \epsilon_{x'y'} = -\left(\frac{\epsilon_{xx} - \epsilon_{yy}}{2}\right)\sin 2\theta + \epsilon_{xy}\cos 2\theta$$

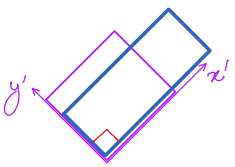
$$\epsilon_{y'y'} = \left(\frac{\epsilon_{xx} + \epsilon_{yy}}{2}\right) - \left(\frac{\epsilon_{xx} - \epsilon_{yy}}{2}\right)\cos 2\theta - \epsilon_{xy}\sin 2\theta$$

Thus, there is a Mohr's circle representation of Strain as well!
And similar ideas as Stress, hold for Strain too:



- Principal planes: on which normal strain is maximum (θ_p) and shear strain is *zero*!





- Planes of maximum shear strain (θ_s)

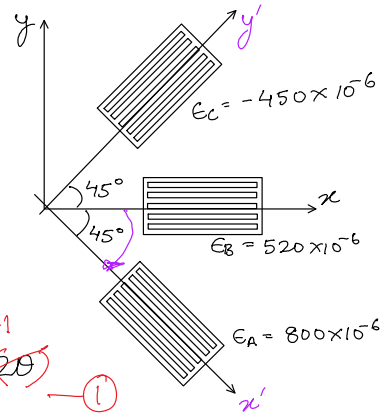
$$2\theta_s + 2\theta_p = 90^\circ \quad \Rightarrow \quad \theta_s = 45^\circ - \theta_p$$

Example: Strain Rosettes:

Given ε_A, ε_B, ε_C

Find ε_{xx}, ε_{yy}, ε_{xy}

Plot Mohr's circle of Strain



$\epsilon_{xx} = \epsilon_B = 520\times10^{-6}$

$(\theta = -45^\circ)$

$800\times10^{-6} = \epsilon_{x'x'} = \left(\frac{\epsilon_{xx}+\epsilon_{yy}}{2}\right) + \left(\frac{\epsilon_{xx}-\epsilon_{yy}}{2}\right)\cos(2\theta) + \epsilon_{xy}\sin(2\theta)$ — (1)

$\frac{\gamma_{x'y'}}{2} = \epsilon_{x'y'} = -\left(\frac{\epsilon_{xx}-\epsilon_{yy}}{2}\right)\sin 2\theta + \epsilon_{xy}\cos(2\theta)$ — (3)

$-450\times10^{-6} = \epsilon_{y'y'} = \left(\frac{\epsilon_{xx}+\epsilon_{yy}}{2}\right) - \left(\frac{\epsilon_{xx}-\epsilon_{yy}}{2}\right)\cos(2\theta) - \epsilon_{xy}\sin(2\theta)$ — (2)

(1) + (2) $\Rightarrow$ $35\times10^{-6} = \left(\frac{520\times10^{-6}+\epsilon_{yy}}{2}\right)\times 2 \Rightarrow \epsilon_{yy} = -170\times10^{-6}$

(3) $\Rightarrow$ $\epsilon_{x'y'} = -\left(\frac{520\times10^{-6} - (-170\times10^{-6})}{2}\right) \Rightarrow \epsilon_{x'y'} = 345\times10^{-6}$

(1) − (2) $\Rightarrow$ $1250\times10^{-6} = -2\epsilon_{xy} \Rightarrow \epsilon_{xy} = -625\times10^{-6}$

$\Rightarrow \gamma_{xy} = 2\epsilon_{xy} = -1250\times10^{-6}$

Mohr's circle:

