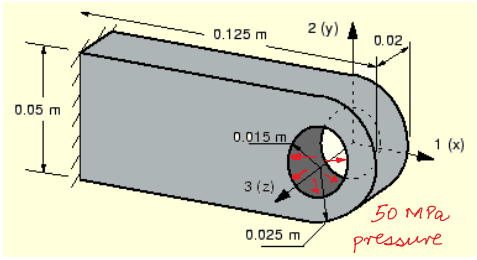
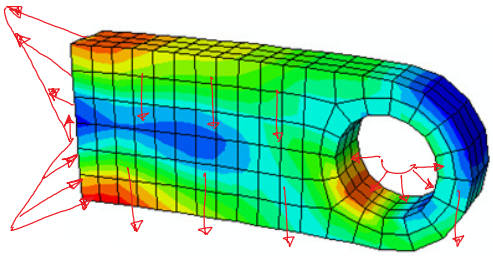
Stresses in Materials

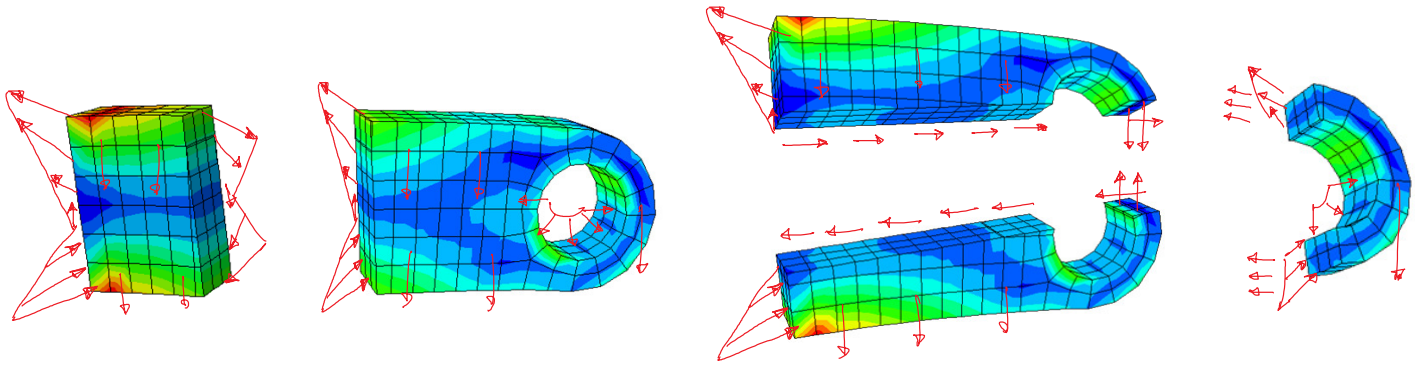
When structures / deformable bodies are acted upon by loads, they build up distributed internal forces (stresses) within them to be able to carry those loads (equilibrium) without breaking apart / failing. These internal forces are usually electromagnetic forces between atoms and molecules of the constituent material.



Assumption: We will assume that the body is a continuum and these forces are *distributed* uniformly over surfaces / volumes. Further, the material is assumed to be cohesive (i.e. connected together without cracks / voids / breaks).



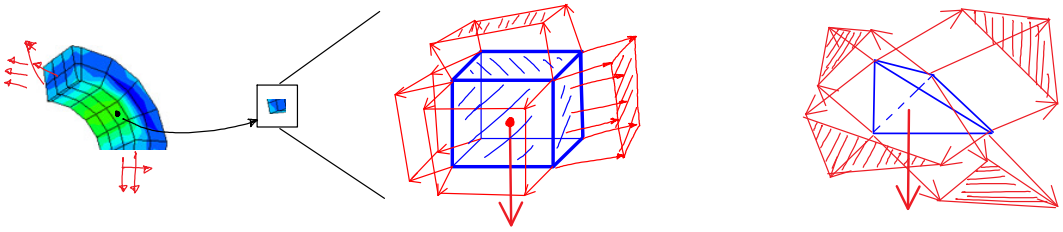
Free body diagrams:
FBDs are one of the most important tools to determine if a structure / body is in equilibrium or not.
In FBDs we draw a specific body (or a specific part of a body) and mark all the (distributed) *external* forces that are acting on it.



Newton's Laws:

1. If sum of all the external forces acting on a body (or a specific part of a body) is **0**, then that body (or that specific part of the body) is at rest or constant velocity (in an inertial frame of reference).
2. If sum of all the external forces acting on a body (or a specific part of a body) is NOT **0**, then the instantaneous acceleration of the body (or a specific part of a body) is given by: $\boldsymbol{F} = m\,\boldsymbol{a}$
3. All forces in the universe occur as pairs of equal and opposite forces between two interacting bodies.

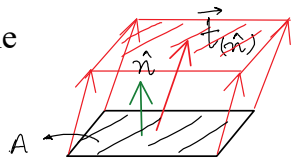
=> In order for a structure / deformable body to be in equilibrium, each and every part of the body (no matter how small) must be in equilibrium *i.e.* all points within a body must be in equilibrium:



NOTE: As we reduce the size of the material "element," the distributed forces on the surfaces of that element become more uniform.

Traction

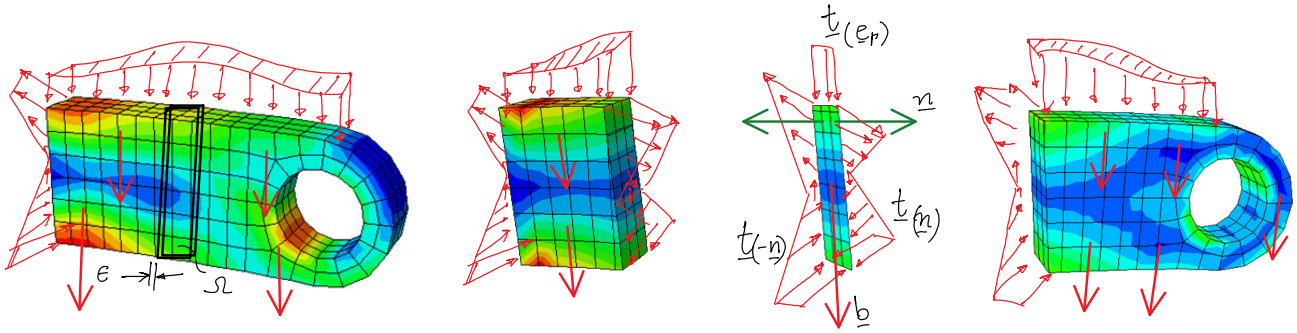
Traction is the distributed force per unit area acting at a point on a surface passing through that point either on an outside (boundary) surface or any surface within the body.

Example 

Total force (on A) $\quad \vec{F} = \int_A \vec{t}_{(\hat{n})}\, dA$

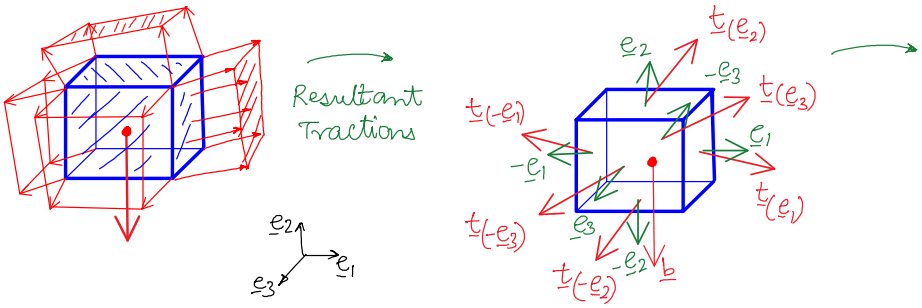
or $\quad$ Traction : $\vec{t}_{(\hat{n})} = \lim_{\Delta A \to 0} \frac{\Delta \vec{F}}{\Delta A}$

(Your book calls this stress)

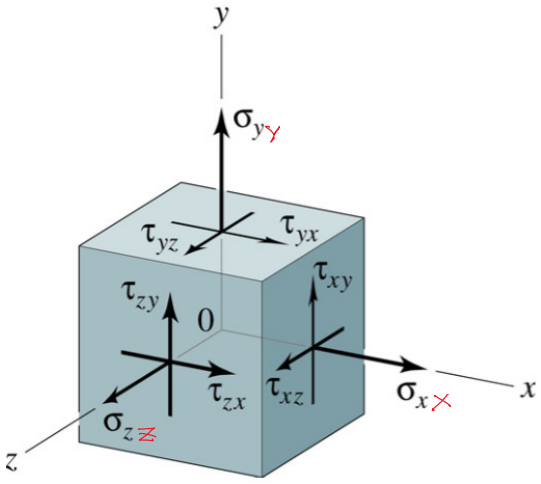


General State of Stress at a point: (Stress tensor)

Tractions at a point is related to internal forces that develop within a body to maintain equilibrium. These internal forces within a body are best represented with a stress tensor field within the body.



In the limit, as the size of the material element tends to zero, (i.e. **at a point**) the state of stress (at that point) converges to 9 unique values which collectively represent the Stress Tensor in a matrix:



Stress Components

$$\begin{bmatrix} \sigma_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix}$$

SHEAR STRESS

NORMAL STRESSES

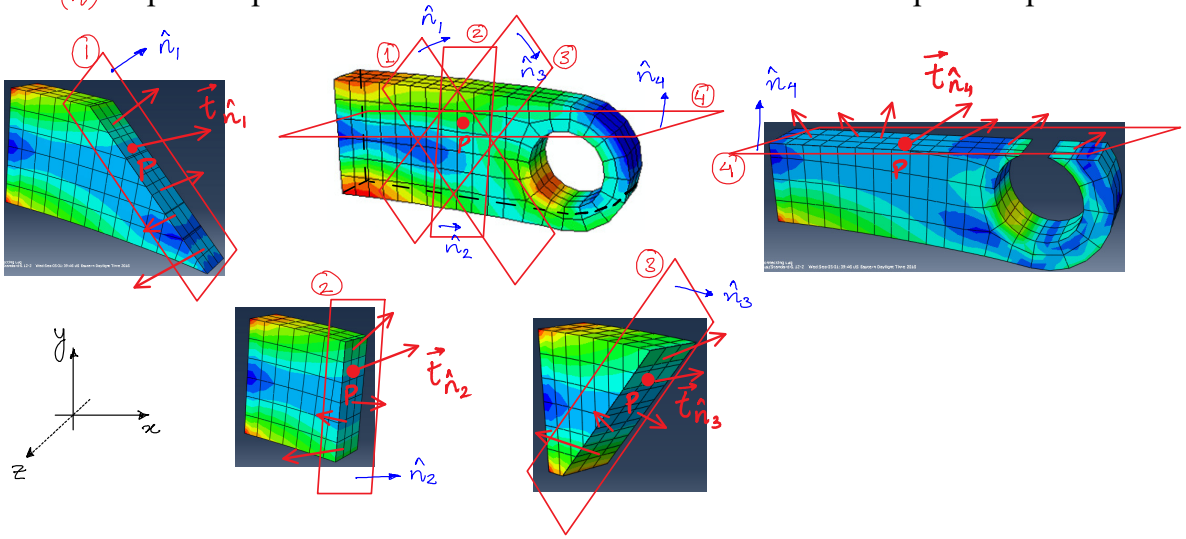
Stress tensor is symmetric => 6 independent values.

Example : $\tau_{zx} = \tau_{xz}$

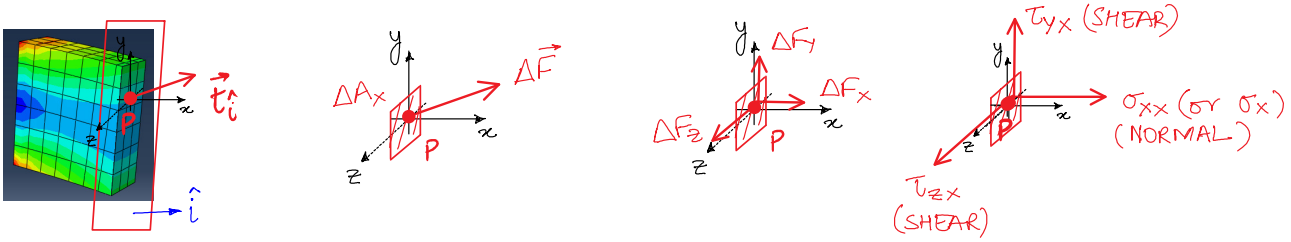
face, Direction, Face, Direction

Resolving into Components Normal and Shear stresses (on a surface):

Traction $\vec{t}_{(\hat{n})}$ depends upon the surface normal $\hat{n}$ and also varies from point to point on a surface.



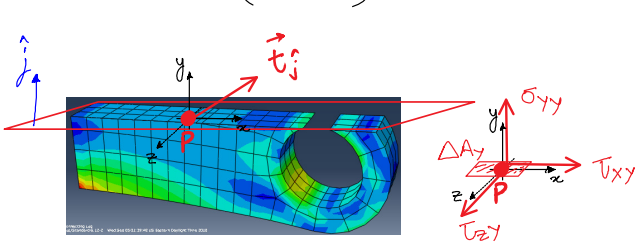
If we take cuts along coordinate axes, then we obtain Normal and Shear components of stress:

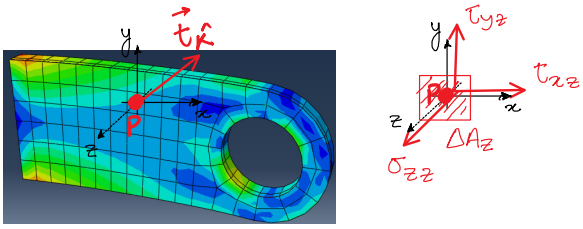


Normal Components of Stress:

$$\sigma_{xx}\ (\text{or } \sigma_x) = \lim_{\Delta A_x \to 0} \frac{\Delta F_x}{\Delta A_x}$$

Similarly $\sigma_{yy}\ (\text{or } \sigma_y) = \lim_{\Delta A_y \to 0} \frac{\Delta F_y}{\Delta A_y}$; $\sigma_{zz}\ (\text{or } \sigma_z) = \lim_{\Delta A_z \to 0} \frac{\Delta F_z}{\Delta A_z}$





Note: Tension positive
(i.e. when stress component points in the same direction as the outward normal to the body)

Shear Components of Stress

$$\tau_{yx} = \lim_{\Delta A_x \to 0} \frac{\Delta F_y}{\Delta A_x} \quad = \quad \tau_{xy} = \lim_{\Delta A_y \to 0} \frac{\Delta F_x}{\Delta A_y}$$

$$\tau_{zx} = \lim_{\Delta A_x \to 0} \frac{\Delta F_z}{\Delta A_x} \quad = \quad \tau_{xz} = \lim_{\Delta A_z \to 0} \frac{\Delta F_x}{\Delta A_z}$$

$$\tau_{zy} = \lim_{\Delta A_y \to 0} \frac{\Delta F_z}{\Delta A_y} \quad = \quad \tau_{yz} = \lim_{\Delta A_z \to 0} \frac{\Delta F_y}{\Delta A_z}$$

Units of Stress (and traction):

$\frac{\Delta F}{\Delta A} \rightarrow \frac{\text{Force}}{\text{Area}} \rightarrow \frac{lb}{ft^2}$ (psf) ; $\frac{lb}{in^2}$ (psi) ; $\frac{kips}{in^2}$ (ksi) ; $\frac{N}{m^2}$ (Pa) ; $\frac{N}{mm^2} = \frac{MN}{m^2}$ (MPa)

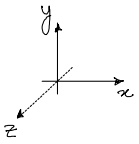
US units | SI units

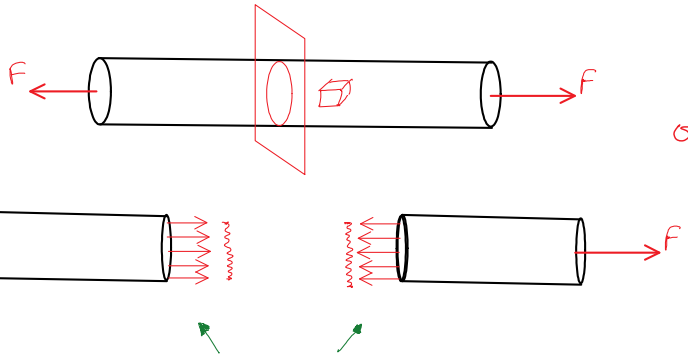
Average Normal Stress (over the cross-section in Axial members)

Equilibrium :

$\sum f_x = 0$

$\Rightarrow F = \int_A \sigma_{xx}\, dA$







Note: $\underset{\sim}{\sigma} = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (state of Stress)

This internal distributed force (traction) is NOT uniform.

$\underset{\sim}{\sigma} \rightarrow \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Assume:
- Homogeneous bar; Isotropic (uniform deformation)
- Remains straight (line of load passes through centroid)
- Section is far from applied loads

Then $F = \int_A \sigma_{xx}\, dA$

$F = \boxed{\bar{\sigma}_{AVG}\ \text{Area}}$

$\bar{\sigma}_{AVG}$: Average Normal Stress (over the entire cross section)

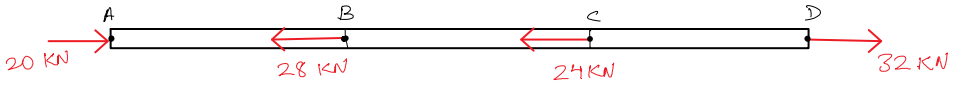
$\Rightarrow \bar{\sigma}_{AVG} = \frac{\int \sigma_{xx}\, dA}{\int dA} = \frac{\text{Force}}{\text{Area}}$

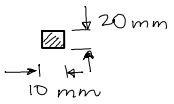
Note: very restricted special case of stress

The same convention still holds: Tension positive.

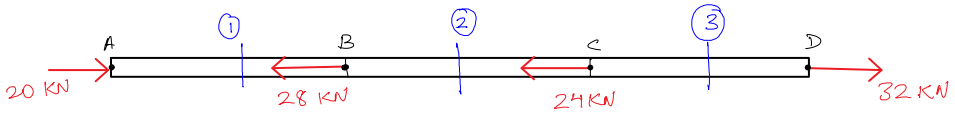
i.e. $\bar{\sigma}_{AVG} > 0$ (tension)
$\bar{\sigma}_{AVG} < 0$ (compression)

Example:

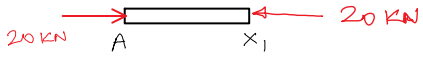




Find average normal stress everywhere in the bar.

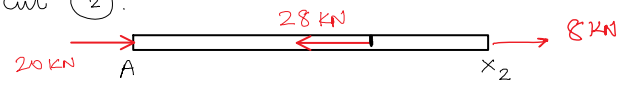


Cut (1):



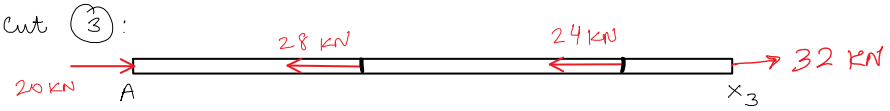
σ_{xx} (AB) $= \frac{\ominus 20\,KN}{10\,mm \times 20\,mm} = \ominus 0.1\,\frac{KN}{mm^2} = \ominus 100\,MPa$ (C)

Cut (2):



σ_{xx} (BC) $= \frac{\oplus 8\,KN}{10\,mm \times 20\,mm} = \frac{8000\,N}{200\,mm^2} = 40\,MPa$ (T)

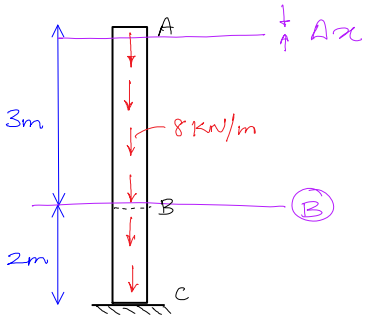
Cut (3):



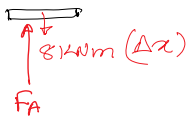
σ_{xx} (CD) $= \frac{32\,KN}{200\,mm^2} = 160\,MPa$ (T)

Example:

Find internal forces and average normal stress @ A, B, and C



At A:

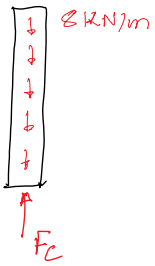


$F_A = (8\,KN/m)\Delta x$

As $\Delta x \to 0$ $F_A \to 0$

$\bar{\sigma}_A = \frac{F_A}{Area} = 0\,MPa$

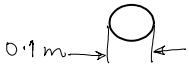
At C:



$F_C = 40\,KN$

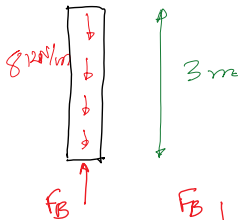
$\bar{\sigma}_C = \frac{\ominus 40\,KN}{0.0079\,m^2}$

$\bar{\sigma}_C = -5.1\,MPa$ (C)

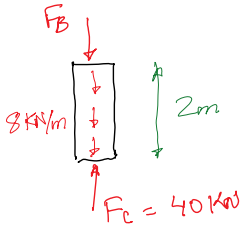


$Area = \frac{\pi d^2}{4} = \pi \left(\frac{d}{2}\right)^2 = 0.0079\,m^2$

At B:



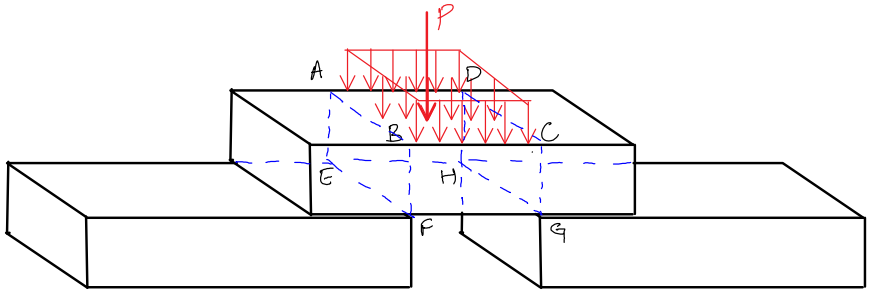
$F_B = 8\,KN/m \times 3m = 24\,KN$



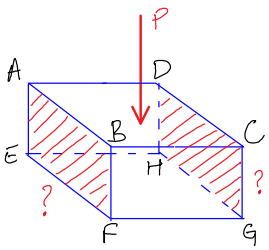
$\bar{\sigma}_B = \frac{\ominus 24\,KN}{0.0079\,m^2}$ (C) $= \frac{24000}{0.0079}\,\frac{N}{m^2}$ Pa

$= \frac{24000}{0.0079}\,\frac{MPa}{10^6} = -3.06\,MPa$

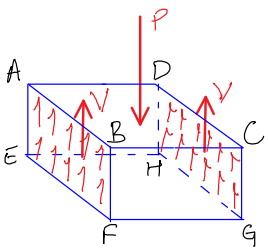
Average Shear Stress (over a cross-section)



FBD of the part "ABCDEFGH":



Assume Uniform Distribution



$P = 2V$

$\bar{\tau}_{AVG}$ (ABFE) $= \dfrac{V}{\text{Area}(ABFE)}$

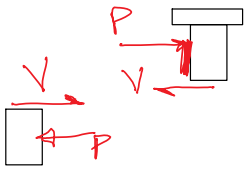
$\bar{\tau}_{AVG}$ (DCGH) $= \dfrac{V}{\text{Area}(DCGH)}$

Note $V = \dfrac{P}{2}$ for this case.

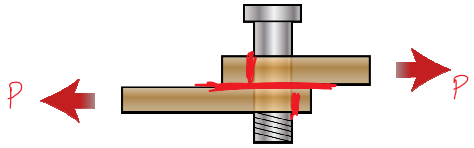
Single shear

FBD of Bolt:

$\bar{\tau}_{AVG} = \dfrac{V}{\text{Area(Bolt)}} = \boxed{\dfrac{P}{A}}$



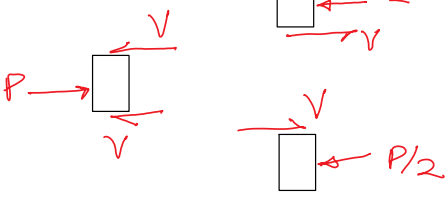
$\boxed{V = P}$

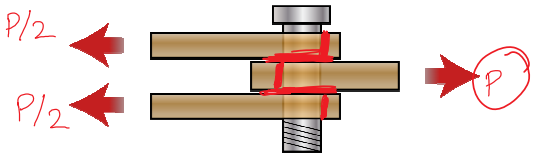


Double shear

FBD of bolt:

$\bar{\tau}_{AVG} = \dfrac{V}{\text{Area(Bolt)}} = \boxed{\dfrac{P}{2A}}$



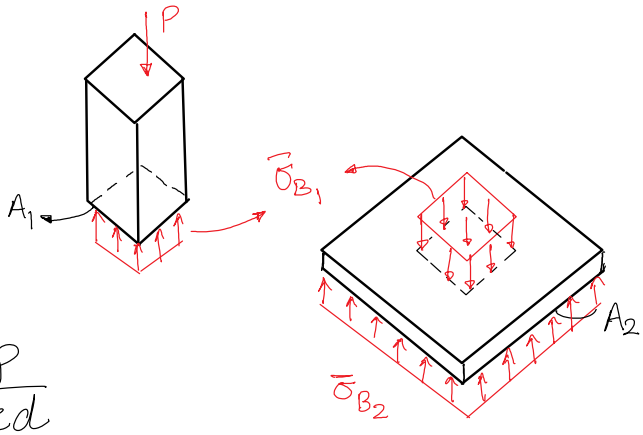


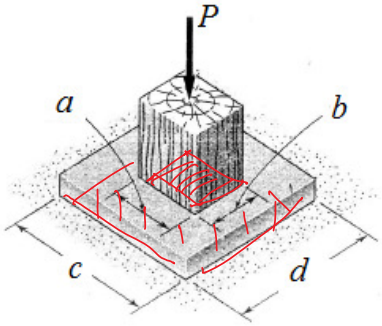
$\boxed{V = P/2}$

Average Bearing Stress

$\bar{\sigma}_{B_1} = \dfrac{P}{A_1} = \dfrac{P}{ab}$

$\bar{\sigma}_{B_2} = \dfrac{P}{A_2} = \dfrac{P}{cd}$

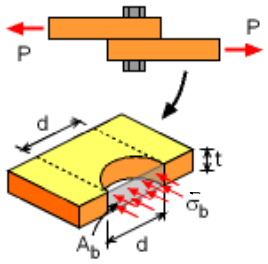




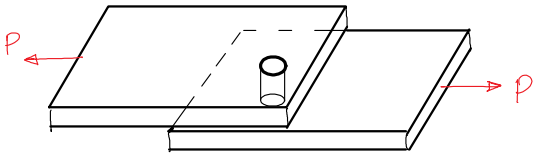
Average Bearing Stress on bolts / plates

$$\bar{\sigma}_B = \frac{\text{Force}}{\text{Projected Area}}$$

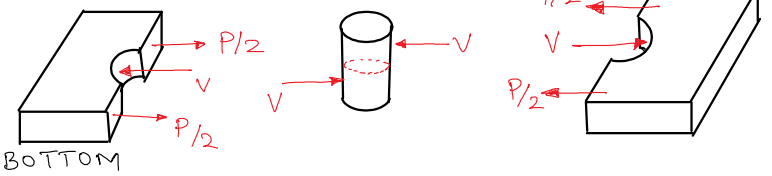
$$\bar{\sigma}_B = \frac{V}{dt}$$



FBD of plates and bolt :



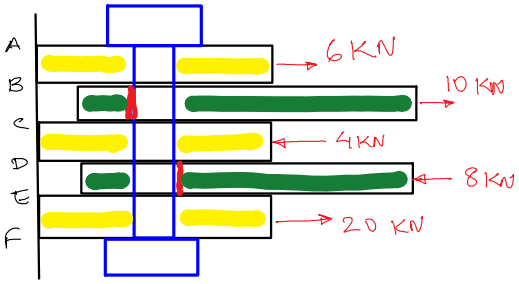
$V = P$



Example

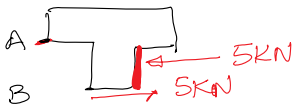
Bolt diameter: 20 mm
Plate Thickness: 10 mm
Find

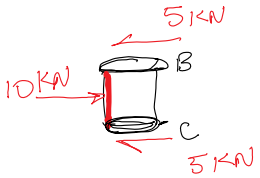
- Average Shear stresses in bolt @ A, B, C, D, E, F
- Average bearing stresses on faces AB, BC, CD, DE, EF



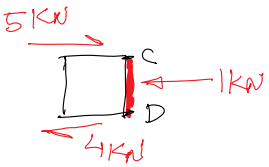
$\bar{\tau}_A = 0$

$\bar{\tau}_B = \frac{5KN}{314mm^2}$

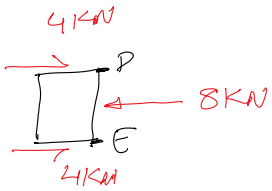




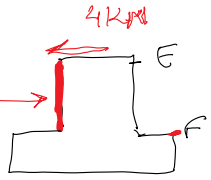
$\bar{\tau}_C = \frac{5KN}{314\,mm^2}$



$\bar{\tau}_D = \frac{4KN}{314\,mm^2}$

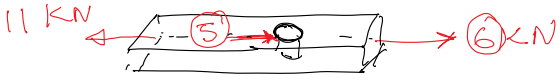


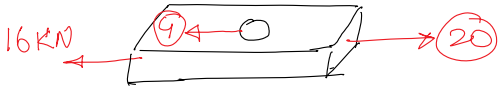
$\bar{\tau}_E = \frac{4KN}{314\,mm^2}$



$\bar{\tau}_F = 0$

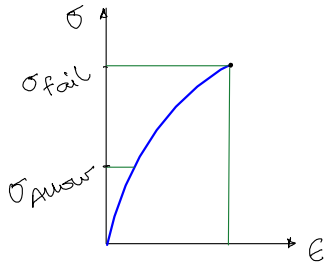
Area Bolt
$= \frac{\pi(20)^2}{4}$
$= 314\ mm^2$





$$\bar{\sigma}^{B}_{AB} = \frac{5KN}{(20\times10)mm^2} \qquad \bar{\sigma}^{B}_{BC} = \frac{10KN}{(20\times10)\,mm^2} \quad - - - - -$$

Allowable Stress Design:



Every material has a certain capacity for withstanding stresses.
Different capacities in different modes: normal / shear / bearing etc.
If internal stresses (caused by external loads) reach the capacity of the material, then the material fails.
We denote this capacity as : σ_{fail}.

In engineering design, we must ensure that working stresses are far below σ_{fail}.
Because:

- Uncertainty in loads / dynamic loads
- Imperfect construction / manufacturing
- Material defects / variability
- Deterioration with age

Thus we choose a factor of safety (FS) in designing components such that there is a maximum allowable stress : $\sigma_{allow} = \frac{\sigma_{fail}}{FS}$

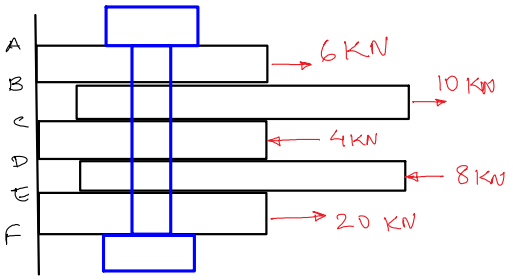
Under operating conditions stresses in components must remain less than this allowable stress.

Example

Assume $\sigma_{Fail} = 250\ MPa$

$\tau_{fail} = 125\ MPa$

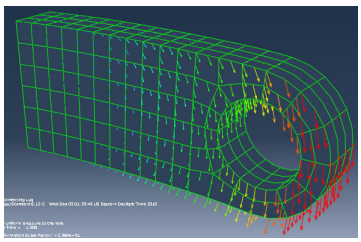
Determine if the components will fail.
If not, then find the factor of safety.



$$\sigma^{B}_{MAX_{BC}} = \frac{10\ KN}{(20\times10)\ mm^2} = \frac{10\times1000\ N}{200\ mm^2} = 50\ MPa \Rightarrow FOS = \frac{250}{50} = \boxed{5}$$

$$\tau_{MAX_{(B,C)}} = \frac{51\ KN}{314\ mm^2} = \Rightarrow FOS. = \boxed{7.8}$$

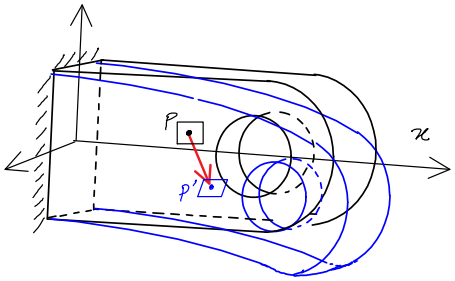
Deformation and Strain



Displacement and Deformation field:

Every point in the body undergoes some displacement in 3D. Given undeformed position of a point *P*, we can write the displacement of that point as a 3D vector:

$$\vec{u}(\vec{x}) = \begin{Bmatrix} u_x(x,y,z) \\ u_y(x,y,z) \\ u_z(x,y,z) \end{Bmatrix}$$



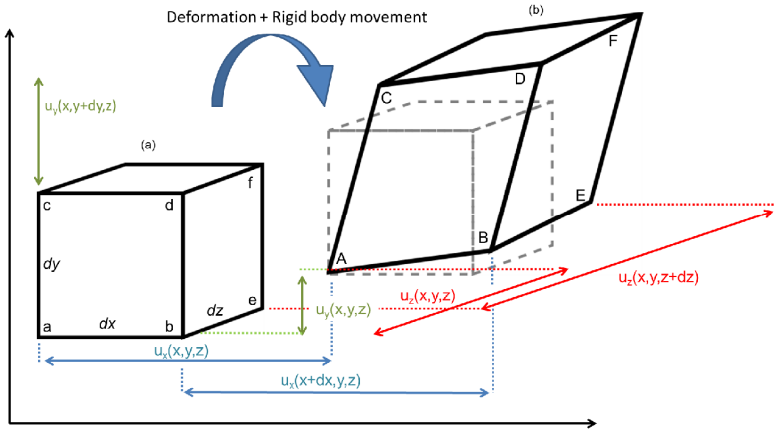
Strain:

Strain is defined as the rate of change of displacement in 3D.
Yes, strain (state of strain in 3D) is also a tensor!
Strain tensor is represented by a 3x3 matrix:

$$\begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}$$

SHEAR STRAINS

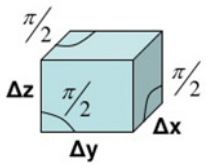
NORMAL STRAINS



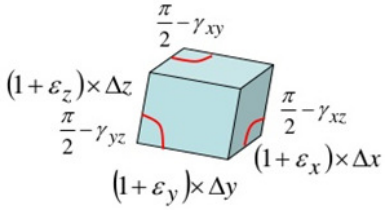
Normal Strains $\epsilon_{xx} \equiv \frac{\partial u_x}{\partial x}$

Similarly $\epsilon_{yy} \equiv \frac{\partial u_y}{\partial y}$

$\epsilon_{zz} = \frac{\partial u_z}{\partial z}$



Undeformed element



Deformed element

Shear strains

$$\epsilon_{xy} = \epsilon_{yx} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

$$\epsilon_{xz} = \epsilon_{zx} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right)$$

$$\epsilon_{yz} = \epsilon_{zy} = \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right)$$

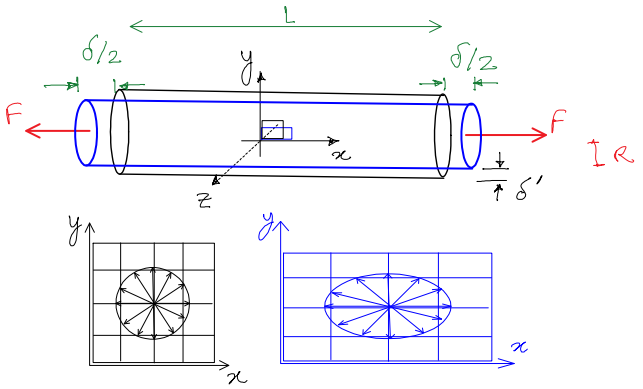
Engineering Shear Strains:

$$\gamma_{xy} (\gamma_{yx}) = 2\epsilon_{xy} = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

$$\gamma_{xz} (\gamma_{zx}) = 2\epsilon_{xz} = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

$$\gamma_{yz} (\gamma_{zy}) = 2\epsilon_{yz} = \left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right)$$

Average Normal Strain (in axial members)



Assume:
- Homogeneous bar; Isotropic (uniform deformation)
- Remains straight (line of load passes through centroid)
- Point under consideration is far from applied loads

$$\vec{u}(\vec{x}) = \begin{Bmatrix} \delta/L\, x \\ -\nu\, \delta/L\, y \\ -\nu\, \delta/L\, z \end{Bmatrix}$$

Average Normal Strain (in Longitudinal direction)

$$\epsilon_{xx} = \frac{\partial u_x}{\partial x} = \frac{\partial (\delta/L) x}{\partial x} = \boxed{\frac{\delta}{L} = \bar{\epsilon}_{LONG}}$$

$$\epsilon_{yy} = \epsilon_{zz} = \frac{\Theta \delta'}{R} = \frac{-\nu \delta}{L}$$

Average Normal Strain (in Lateral direction)

$$\epsilon_{yy} = \frac{\partial u_y}{\partial y} = \frac{\partial \left(-\nu\, \delta/L\right) y}{\partial y} = \boxed{\frac{-\nu \delta}{L} = \bar{\epsilon}_{LAT}}$$

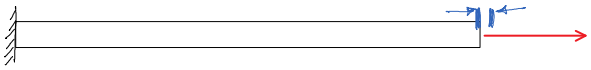
Note: $\nu = \dfrac{-\bar{\epsilon}_{LAT}}{\bar{\epsilon}_{LONG}}$ (Poisson's Ratio)

Also $\epsilon_{zz} = \bar{\epsilon}_{LAT} = -\nu \dfrac{\delta}{L}$

Note: Normal strains measure changes in dimension (length, area, and volume) of body at a point.

Note: Units of Strain: Dimensionless

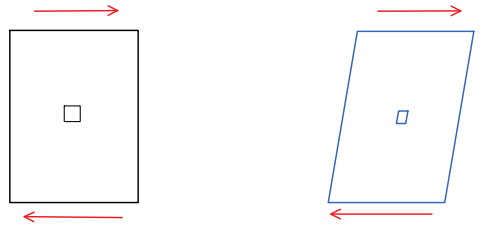
for example: $L = 0.5\ m$ $\qquad \delta = 100 \times 10^{-6}\ m = 100\ \mu m$ (microns)



SI: $\bar{\epsilon} = \dfrac{\delta}{L} = \dfrac{100 \times 10^{-6}\ m}{0.5\ m} = 200 \times 10^{-6} = 200\ (10^{-6})\ \mu$

US: $\bar{\epsilon} = \dfrac{\delta}{L} = \dfrac{100 \times 10^{-6}\ m \cdot 39.37\ in/m}{0.5\ m \cdot 39.37\ in/in} = 200\ \mu$

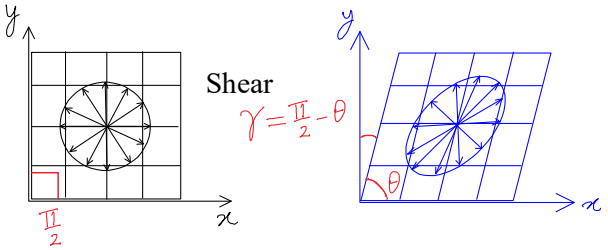
Average Shear Strain



Again assuming uniform deformation,

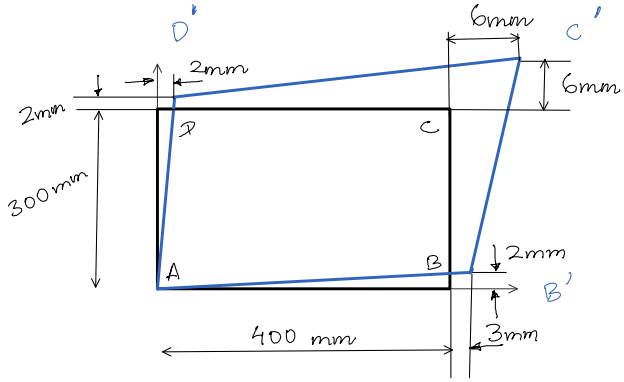
Average Shear Strain (in radians):

$$\gamma_{(xy)} = \frac{\pi}{2} - \theta$$



Note: Shear strains measure changes in shape of a body at a point.

Example: Find $\bar{\epsilon}_{BD}$, $\bar{\gamma}_{B\,(xy)}$



Av. Normal Strain $\bar{\epsilon}_{BD}$

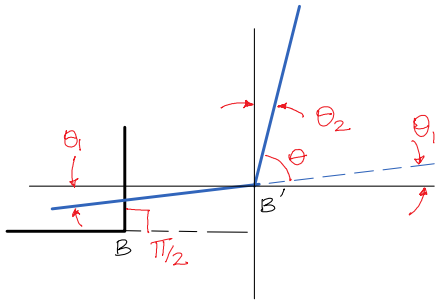
$$\bar{\epsilon}_{BD} = \frac{B'D' - BD}{BD} = \frac{\sqrt{(401)^2 + (300)^2} - \sqrt{(400)^2 + (300)^2}}{\sqrt{(400)^2 + (300)^2}}$$

(500 under the second square root; 500 under the denominator)

$$\bar{\epsilon}_{BD} = \frac{500.8 - 500}{500} = \boxed{0.0016}$$

Av. Shear Strain @ B

$$\bar{\gamma}_{B\,(xy)} = \frac{\pi}{2} - \theta = (\theta_1 + \theta_2) = \tan^{-1}\left(\frac{2}{403}\right) + \tan^{-1}\left(\frac{6-3}{6-2+300}\right)$$

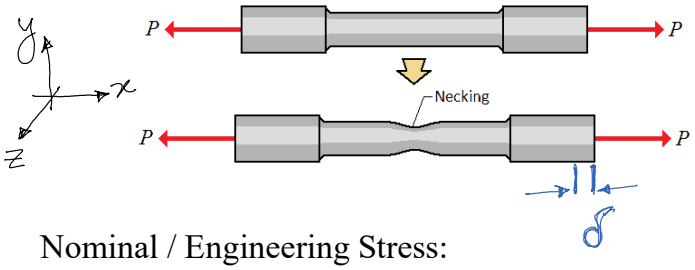


$$\Rightarrow \bar{\gamma}_{B\,(xy)} = 0.01483$$

$$\approx \boxed{0.015}$$

Material Behavior: (Uniaxial Stress-strain relationship):

Uni-axial tension test



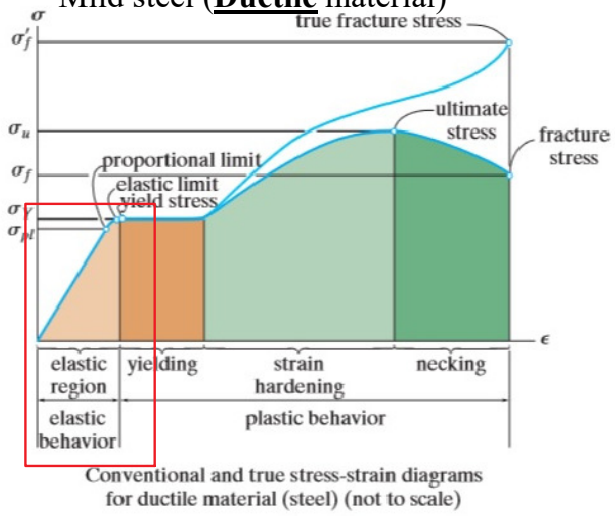
Nominal / Engineering Stress:

$$\begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \bar{\sigma}_{(xx)} = \frac{P}{A_0}$$

Nominal / Engineering Strain:

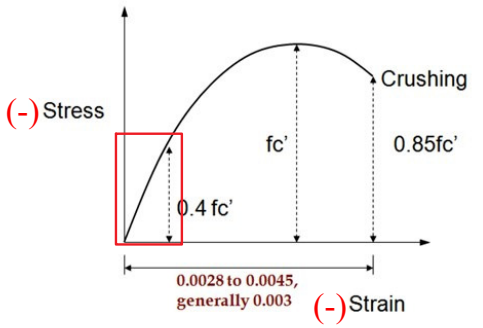
$$\begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \qquad \bar{\epsilon}_{(xx)} = \frac{\delta}{L_0}$$

Mild steel (**Ductile** material)



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

Stress Strain Curve of Concrete (**Brittle** material)



- The first portion of curve, to about 40% of the ultimate strength f_c', can be considered linear.
- The lower the strength of concrete the greater will be the failure strain

In the linear-elastic range, we can relate stress and strain using the Hooke's Model:

$$\bar{\sigma} = E\,\bar{\epsilon}$$

E: Young's modulus

For example, mild steel has a proportional limit of around 35 ksi and corresponding strain of 0.0012 in/in.

$$\text{Thus } E_{steel} = \frac{35}{0.0012} \approx 29000 \text{ ksi}$$

In contrast, mild steel yielding extends to a strain of 0.03 in/in at a stress of 36 ksi.
Further, it reaches its ultimate stress σ_u of 63 ksi before finally fracturing at about 47 ksi at a strain of about 0.38 in/in (i.e. locally - in the necked region - strain is 38%)!

This makes mild steel a **ductile** material.
Measures of ductility:

Percent Elongation (at failure): $\frac{L_f - L_0}{L_0}(\%)$ ~ 38%

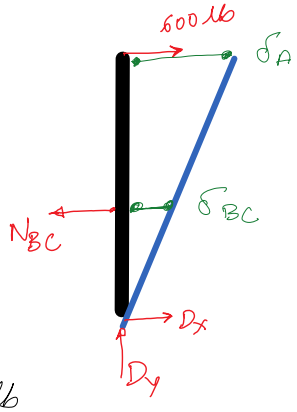
Percent reduction of area (at failure): $\frac{A_0 - A_f}{A_0}(\%)$ ~ 60%

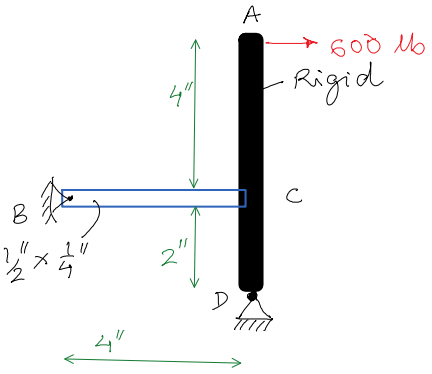
Mild Steel

Example

Find: N_{BC} , δ_A

Given: $E = 29000\ ksi$





$\sum M_D = 0$

$-600 \times 6'' + N_{BC} \times 2'' = 0$

$\Rightarrow N_{BC} = \frac{600 \times 6}{2} = 1800\ lb$

$\delta_{BC} = \frac{N_{BC} L_{BC}}{A_{BE} E}$

$\frac{\delta_A}{6''} = \frac{\delta_{BC}}{2''} \Rightarrow \delta_A = 3\delta_{BC}$

$\bar{\epsilon}_{BC} = \frac{\delta_{BC}}{L_{BC}} \Rightarrow \delta_{BC} = \bar{\epsilon}_{BC} L_{BC} = \left(\frac{\bar{\sigma}_{BC}}{E}\right) L_{BC} = \frac{N_{BC}}{A_{BC}} \cdot \frac{L_{BC}}{E} = \frac{1800\ lb}{(\frac{1}{2} \times \frac{1}{4})\ in^2} \cdot \frac{in^2 \cdot 4\ in}{29000\ kips}$ (1000 lbs)

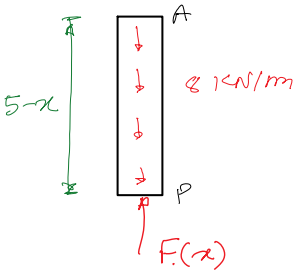
$\Rightarrow \delta_{BC} = 0.02\ in \Rightarrow \boxed{\delta_A = 3\delta_{BC} = 0.06\ in \rightarrow}$

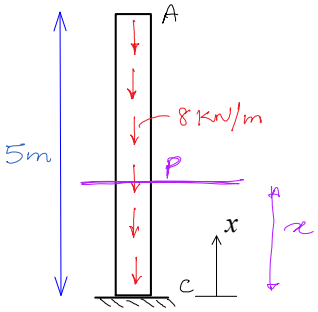
Example:

Concrete bar under distributed load. Young's modulus $E = 30$ GPa

Find:

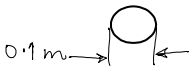
- Stress $\sigma(x)$
- Strain $\varepsilon(x)$
- Vertical displacement of point A





$\sum F_x = 0 \Rightarrow -8 \cdot (5-x) + F(x) = 0$

$\Rightarrow F(x) = 8(5-x)\ (C)$



$\sigma(x) = \frac{F(x)}{A} = \frac{-8(5-x)\ KN}{0.008\ m^2}$

$\text{Area} = \frac{\pi d^2}{4} = 0.007854\ m^2$

$\varepsilon(x) = \frac{\sigma(x)}{E} = \frac{-8(5-x)\ kN}{30 \times 10^9\ N/m^2} \cdot \frac{1000\ N}{1\ kN} \cdot \frac{m^2}{0.008} \cdot \frac{1}{8} = \frac{-(5-x)}{30000}$

$\epsilon = \frac{du}{dx} \Rightarrow \int_{u(0)}^{u(5)} du = \int_0^5 \epsilon\, dx = \int_0^5 \frac{-(5-x)}{30000}\, dx = \frac{-5 \times 5}{30000} + \left[\frac{5^2}{2}\right] \frac{1}{30000} = -4 \times 10^{-4}\ m$

$= -0.4\ mm$

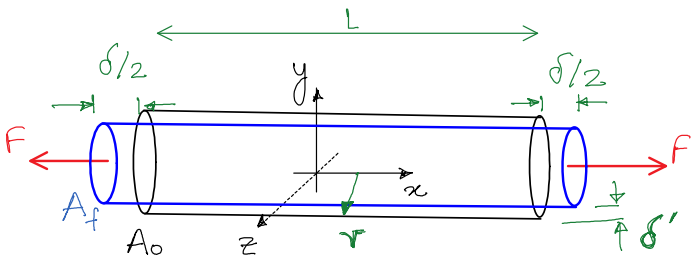
$u(5) - u(0) = -0.4\ mm \Rightarrow \boxed{\delta_A = -0.4\ mm}$

($u(5) = \delta_A$, $u(0) = \delta_C = 0$)

Strain Energy

When we load a material and deform it (within the elastic range), the material behaves as a "spring" and stores energy called elastic strain energy.

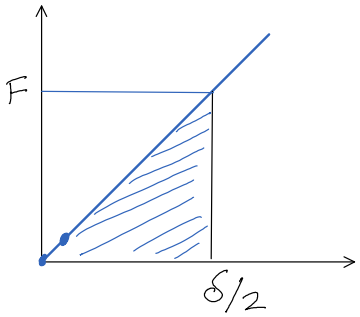
This strain energy per unit volume of material is called strain energy density.



For example, work done by each *F*: $\Delta W = \frac{1}{2} F\left(\frac{\delta}{2}\right)$

Total work done by both *F* forces: $2 * \Delta W = \frac{1}{2} F \delta$

This is stored as strain energy in the whole bar: $U = \frac{1}{2} F \delta$



Dividing by the volume of the bar, we get strain energy density:

$$u = \frac{U}{Vol} = \frac{1}{2} \frac{F}{A_o} \cdot \frac{\delta}{L_o} = \boxed{\frac{1}{2} \bar{\sigma} \bar{\epsilon}}$$

Note also:

Average Normal Strain (in Longitudinal direction) $\bar{\epsilon}_{LONG} = \frac{\delta}{L}$

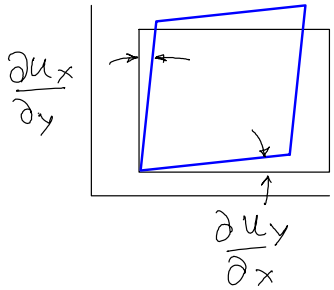
Average Normal Strain (in Lateral direction) $\bar{\epsilon}_{LAT} = -\frac{\delta'}{r}$

We define **Poisson's Ratio**: $\boxed{\nu = -\frac{\bar{\epsilon}_{LAT}}{\bar{\epsilon}_{LONG}}}$

(material property similar to Young's modulus *E*)

Finally, for **shear**: $\bar{\tau} = G \bar{\gamma}$

(for pure shear) $\bar{\gamma}$

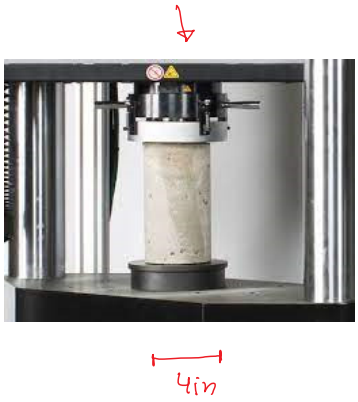


G: Shear modulus

$G = \frac{E}{2(1+\nu)}$

Example:

In a uniaxial compression test, a cylindrical concrete sample of height 10 in. and diameter 4 in. is subjected to a load of 20 kips.
The reduction in height of the cylinder is measured to be 0.004 in and the sample diameter expands by 0.0004 in.



Compute:
a) Average Young's modulus E of the sample
b) Average Poisson's ration ν of the sample
c) Strain energy density stored in the sample after deformation

a) Young's Modulus $E = \dfrac{\bar{\sigma}}{\bar{\epsilon}_{LONG}} = \dfrac{F}{A} \cdot \dfrac{L}{\delta}$

$A = \pi (2)^2 \text{ in}^2$

$= \dfrac{20 \text{ Kips}}{\pi (2)^2 \text{ in}^2} \cdot \dfrac{10 \text{ in}}{0.004 \text{ in}} \cong 3.978 \times 10^6 \text{ psi}$

$\cong 3\,978 \text{ ksi}$

b) $\nu = \dfrac{-\bar{\epsilon}_{LAT}}{\bar{\epsilon}_{LONG}} = \dfrac{0.0004 \text{ in}}{4 \text{ in}} \cdot \dfrac{10 \text{ in}}{0.004 \text{ in}} = \boxed{0.25}$

$-1 \leq \nu \leq 0.5$

c) $U = \frac{1}{2} F \delta = \frac{1}{2} \times 20 \text{ Kips} \times 0.004 \text{ in} = 40 \text{ lb-in}$

$\bar{u} = \dfrac{U}{Vol} = \dfrac{1}{2} \dfrac{F}{\text{Area}} \dfrac{\delta}{\text{Height}} = \frac{1}{2} \bar{\sigma} \bar{\epsilon} = \dfrac{40 \text{ lb-in}}{\pi (2)^2 \text{ in}^2 \times 10 \text{ in}} = 0.3182 \dfrac{\text{lb-in}}{\text{in}^3}$

$\dfrac{\text{lb}}{\text{in}^2} = \text{psi}$