CE-270 Introductory Structural Mechanics

- Arun Prakash

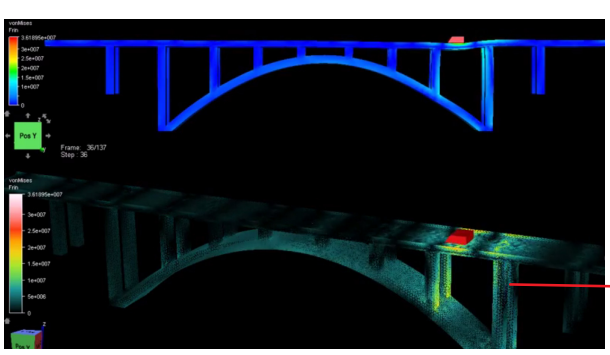
The BIG Picture

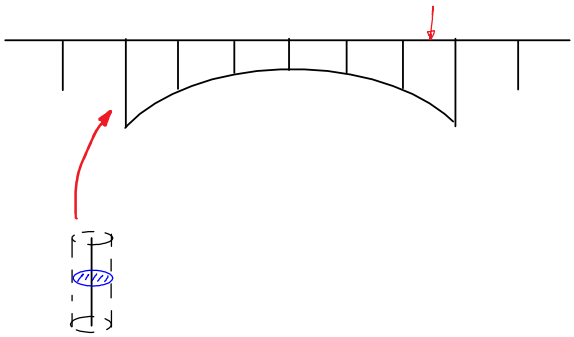
- What is Mechanics?
  - Mechanics is study of *how things work*: how anything works, how the world works!
    People ask: "Do you understand the mechanics of _____?" It could be:
    Do you understand the mechanics of this building/bridge? (How loads are being carried?)
    Do you understand the mechanics of heat transfer?
    Do you understand the mechanics of how this automobile works?
    Do you understand the mechanics of flight?
    Do you understand fracture mechanics / wave mechanics / geo-mechanics / thermo-mechanics / electro-mechanics / celestial mechanics / quantum mechanics etc. etc.
    In that sense, mechanics is almost synonymous with Physics. However, mechanics is really a branch of Physics.
  - Within the context of Civil Engineering and Structural Engineering, mechanics is typically used to mean:
    - Rigid body mechanics (Statics / Dynamics),
    - Mechanics of materials / Solid Mechanics,
    - Fluid mechanics,
    - Structural mechanics

    Based on Newtonian Mechanics.

- Structural Mechanics
  - Builds upon continuum (solid) mechanics
  - Make assumptions regarding the displacement field within individual structural members
  - Reduce the "number" of unknowns (dimensionality of the problem)

  Examples: Beam theory, plate theory, shell theory





http://youtu.be/IgfWjCvCSuk

- Overall objectives of this course
  - To be able to analyze how different types of structural components carry loads i.e. members with axial loads, members that bend, members with torsion, and combinations.
  - To be able to determine stresses and strains in these types of structural components.
  - Conduct experiments to evaluate the theoretically modeled behavior of these members

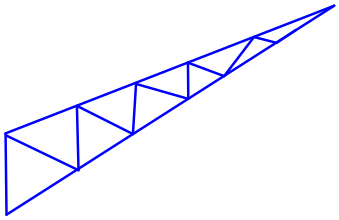
Review: Equilibrium

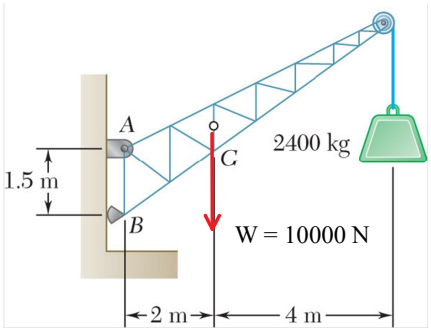
The free body diagram is a depiction of a body/structure (or a sub-part of a larger structure) showing **all the external forces** acting on it.

Steps in drawing a FBD of a body:

- Choose and draw the body (with dimensions). Carefully define its boundaries.
- Imagine the body in its current state and how it interacts with its surroundings.
- Draw ALL the external forces acting on the body (including self-weight).
- Any unknown forces acting on the body (required to keep it in equilibrium) must also be drawn.

Example:



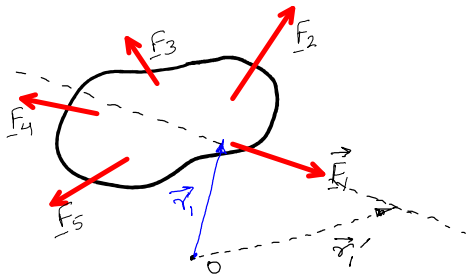


Equilibrium: A body is said to in equilibrium if the vector sum of ALL forces and all their moments taken about any (and all) points is zero.

In 3D:

$$\sum \vec{F} = 0$$

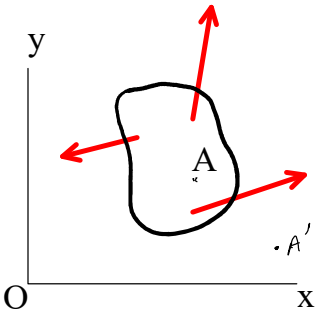
$$\sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F} \right) = 0$$



6 equations of equilibrium for each FBD:

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

In 2D (x-y plane), note: $F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$



3 equations of equilibrium for each FBD:

(A) $\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$

OR (B) $\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0$

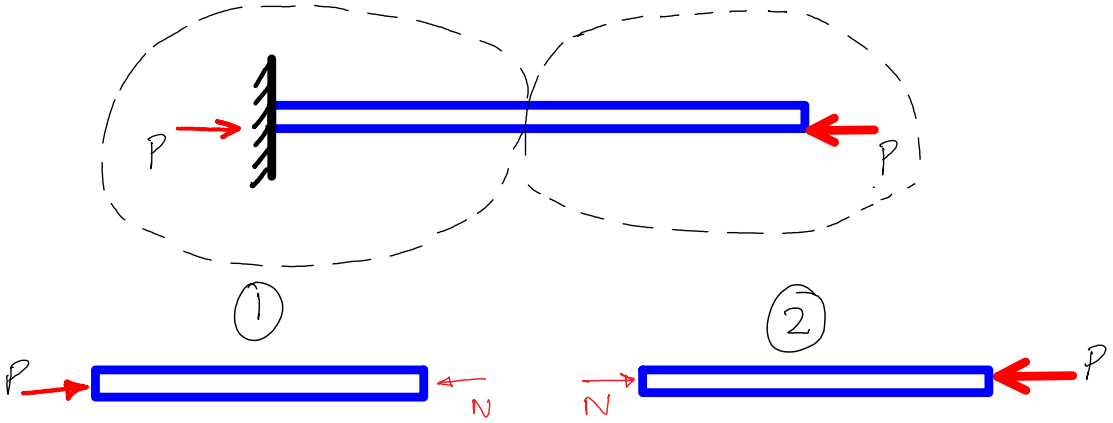
(where the line AB is not perpendicular to the x-axis)

OR (C) $\sum M_A = 0 \quad \sum M_B = 0 \quad \sum M_C = 0$

(where A, B and C are not all along the same line)

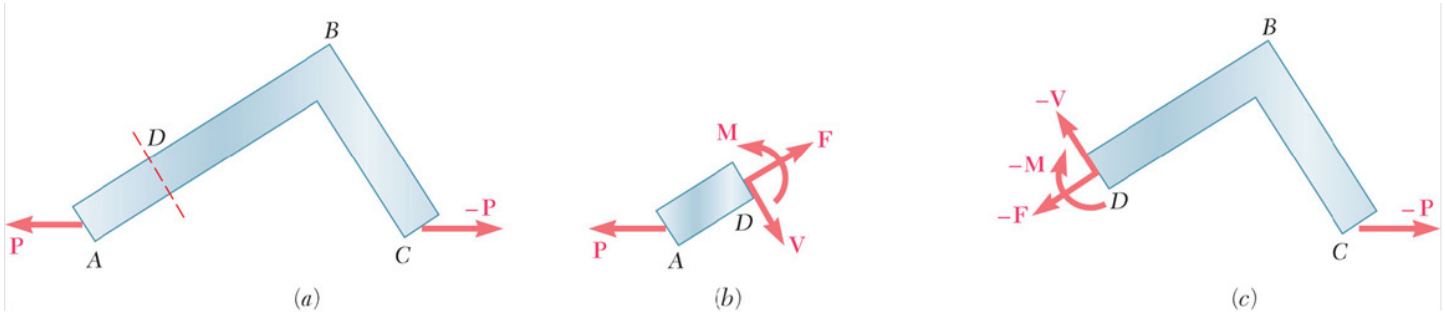
Internal Forces

To do this, we imagine two (or more) sub-parts of the beam as shown.



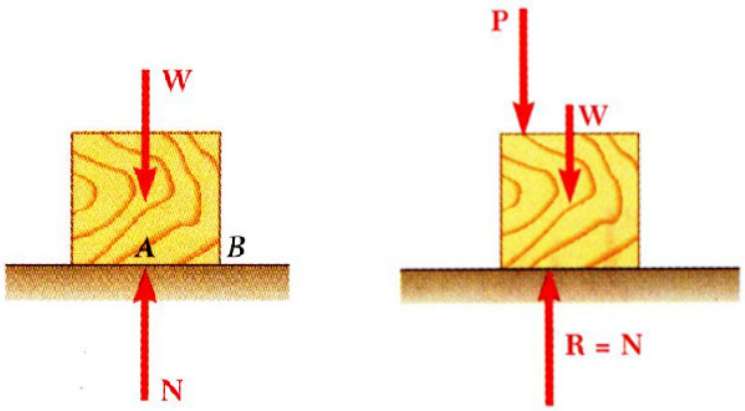
Internal forces are always equal and opposite and act on different sub-parts of the body.

Multi-force members can carry additional types of **internal forces** such as **shear** and **bending moment** in addition to **tension/compression**.



Note: Internal forces are shown as **resultants** of distributed forces.

Example:



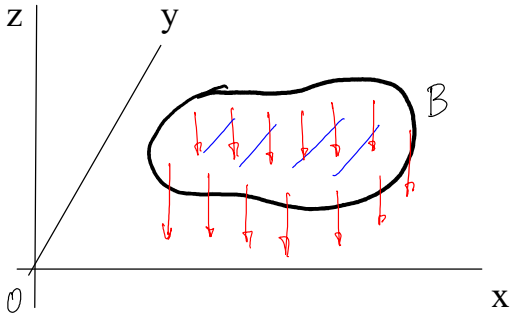
Distributed Forces

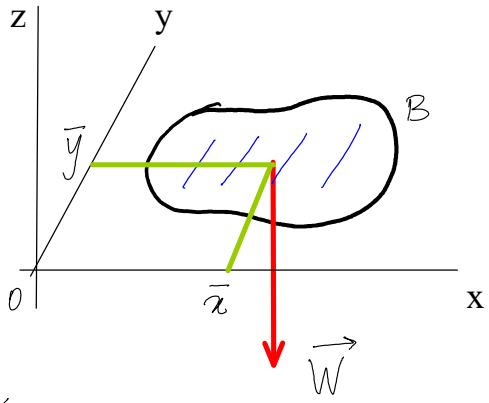
What are distributed forces?

- Forces that act on a body per unit length, area or volume.
- They are not discrete forces that act at specific points. Rather they act over a continuous region.

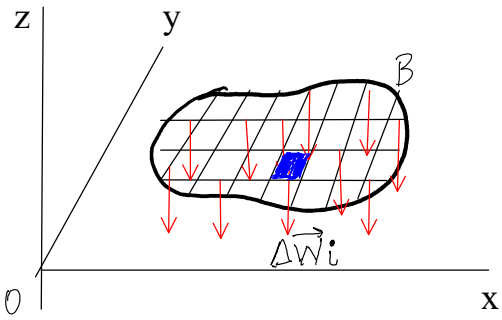
Example: Gravity pulls each and every particle of a body vertically downwards.

Center of Gravity: Location of the equivalent single force that replaces all the distributed forces.

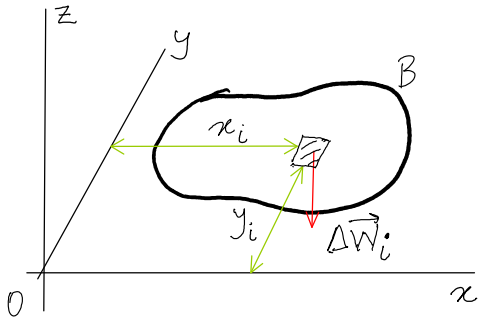




Finding Center of Gravity: Let the location be $(\bar{x}, \bar{y})$.



$i = 1, 2 \dots N$



Resultant Equivalent Force (Weight): $W = \int dW = \int (\rho g t)\, dA$

density, gravity, Thickness

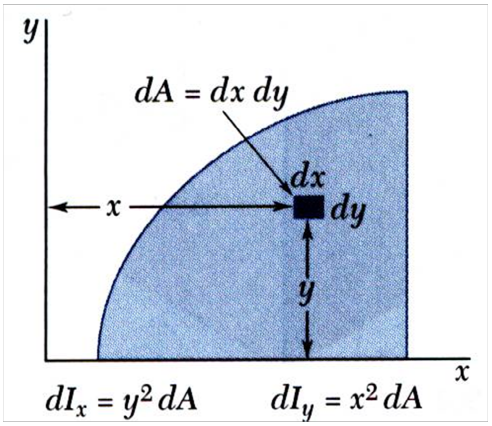
Centroid:

$$\bar{x} = \frac{\int x\, dW}{W}$$

$$\bar{y} = \frac{\int y\, dW}{W}$$

First Moment of Area

$$Q_x = \int y\,dA = \bar{y}\,A \qquad Q_y = \int x\,dA = \bar{x}\,A$$

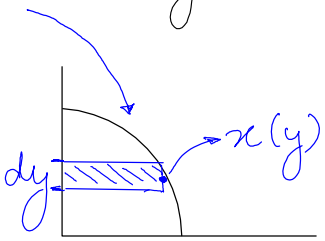


Second Moment of area

$$I_x = \int y^2 dA \qquad I_y = \int x^2 dA$$

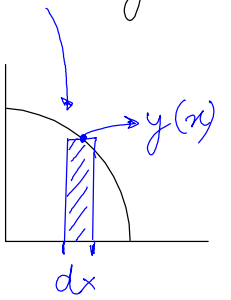
For I_x, choose $dA = x\,dy$

$$I_x = \int y^2 (x\,dy)$$



For I_y, choose $dA = y\,dx$

$$I_y = \int x^2 (y\,dx)$$



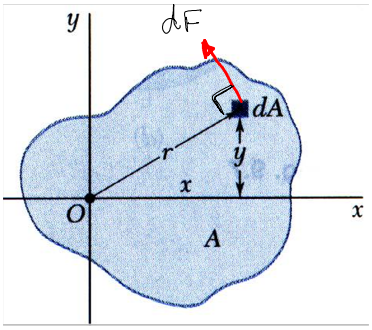
Polar moment of Inertia

$$J_0 = \int r^2 dA$$

$(r^2 = x^2 + y^2)$

$$J_0 = \int (x^2+y^2)\,dA = \underbrace{\int x^2 dA}_{I_y} + \underbrace{\int y^2 dA}_{I_x}$$

$$\Rightarrow \quad J_0 = I_y + I_x$$



Radius of gyration (***k***) is the distance from the axis where an area can be thought of as being concentrated for purposes of evaluating the second moment of area / moment of inertia.

$I_x = k_x^2 A$	$k_x = \sqrt{\frac{I_x}{A}}$	Radius of gyration about x-axis
$I_y = k_y^2 A$	$k_y = \sqrt{\frac{I_y}{A}}$	Radius of gyration about y-axis
$J_O = k_O^2 A$	$k_O = \sqrt{\frac{J_O}{A}}$	Polar Radius of gyration

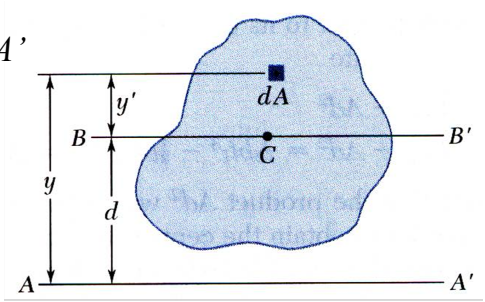
Analogous to $Q_x = \bar{y}A$, $Q_y = \bar{x}A$

Note: $k_O^2 = k_x^2 + k_y^2$

Parallel Axis Theorem

Consider moment of inertia I of an area A with respect to the axis AA'

$$I = \int y^2 dA$$



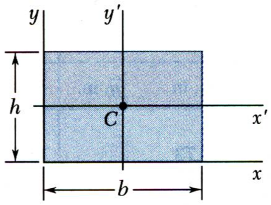
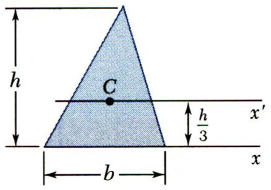
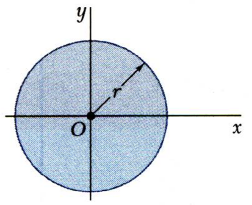
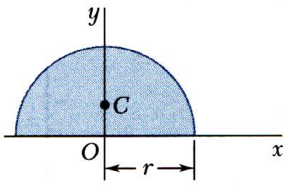
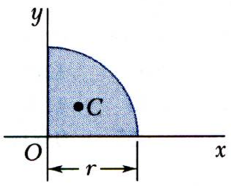
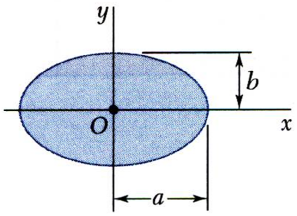
This moment of inertia can be calculated using the moment of inertia about the **centroidal axis**:

$$I_{AA'} = \int (y' + d)^2 dA = \underbrace{\int y'^2 dA}_{I_{BB'}} + 2d \underbrace{\int y' dA}_{Q_x} + d^2 \underbrace{\int dA}_{A}$$

$Q_x = 0$ IMP

$$\boxed{I = \bar{I} + Ad^2}$$

only if BB' is a centroidal axis

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ (+ $\frac{bh^3}{4}$) $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3 b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$