AAE 565 Adendum to Notes on Phugoid Trim Analysis

D. An algorithm for solution of the 2 nonlinear equations of equilibrium

$$f_1(\bar{x},\bar{u}) = T\cos(\phi_T+\alpha) - W\sin\gamma - D$$

$$f_2(x,u) = L\cos\gamma + T\sin(\phi_T+\alpha+\gamma) - W - D\sin\gamma$$

where $\sin\gamma = \dot{h}/V$

Define two specified variables α and T as variables that can be set by the pilot. He sets α with his pitch stick (push/pull), and he sets T with his throttle/engine controls.

$$\bar{u} = \begin{bmatrix} \alpha \\ T \end{bmatrix}$$

Define two unknown variables V and $\dot{h}$

$$\bar{x} = \begin{bmatrix} V \\ \dot{h} \end{bmatrix}$$

<u>Given</u> $\bar{u}$, find $\bar{x}$ to make $\bar{f} = \bar{0}$
where vector $\bar{f}$ is a nonlinear function of $\bar{x}$

An algorithm for finding the zeros of a nonlinear vector function

Given $\bar{u}$
Find $\bar{x}$ that makes $\bar{f} = \bar{0}$
This is the equilibrium condition
where $\bar{u}$, $\bar{x}$ and $\bar{f}$ are vectors.

An Algorithm given u and a guess for x, x_0
Use Taylor series expansion about x_0

$$\bar{f}(\bar{x} + \bar{\Delta x}, \bar{u}) = \bar{f}(\bar{x}_0, \bar{u}) + \left.\frac{\partial \bar{f}}{\partial x}\right|_{x_0, u} \bar{\Delta x} + \underbrace{hot}_{ignore}$$

we want to be zero. ∴ Solving for Δx

$$\bar{\Delta x} = - \left.\frac{\partial \bar{f}}{\partial \bar{x}}\right|_{\bar{x}_0, \bar{u}}^{-1} \bar{f}(\bar{x}_0, \bar{u})$$

update $\bar{x}$

$$\bar{x}_{NEW} = \bar{x}_{OLD} + \bar{\Delta x} = \bar{x}_{OLD} - \left.\frac{\partial \bar{f}}{\partial \bar{x}}\right|_{\bar{x}_{OLD}, \bar{u}} \bar{f}(\bar{x}_{OLD}, \bar{u})$$

But since we are using linearization, let's not take all of the $\Delta \bar{x}$.
Let's add in $\alpha \Delta x$ where $\alpha < 1$ (is a scalar).
Let $\alpha = .25$ and let's use at least 4 iterations around the loop.

The algorithm:

$$\bar{x}_{OLD} = \bar{x}_0$$

$$\bar{f}_{OLD} = \bar{f}(\bar{x}_{OLD}, \bar{u})$$

is $\bar{f}_{OLD}$ small enough?
{ yes → done
{ no → continue

$$J = \left.\frac{\partial \bar{f}}{\partial \bar{x}}\right|_{x_{OLD}, u} = \text{Jacobian Matrix}$$

$$\bar{x}_{New} = \bar{x}_{OLD} - \alpha \left.\frac{\partial \bar{f}}{\partial \bar{x}}\right|_{x_{OLD}, u}^{-1} \bar{f}_{OLD}$$

$$\bar{x}_{OLD} = \bar{x}_{New}$$

F. Flight Path Stability
Re: Military Specification on Flying Qualities

MIL-F-8785C

3.2.1.3 Flight-path stability. Flight-path stability is defined in terms of flight-path-angle change where the airspeed is changed by the use of pitch control only (throttle setting not changed by the crew). For the landing approach Flight Phase, the curve of flight-path angle versus true airspeed shall have a local slope at $V_{o_{min}}$ which is negative or less positive than:

a. Level 1 ----- 0.06 degrees/knot "Good"

b. Level 2 ----- 0.15 degrees/knot

c. Level 3 ----- 0.24 degrees/knot. "Bad"

The thrust setting shall be that required for the normal approach glide path at $V_{o_{min}}$. The slope of the curve of flight-path angle versus airspeed at 5 knots slower than $V_{o_{min}}$ shall not be more than 0.05 degrees per knot more positive than the slope at $V_{o_{min}}$, as illustrated by: