

Trim Analysis Using Phugoid Equations of Motion

Objectives

1. To illustrate the importance of aircraft lift and drag equations in generating trim maps, i.e. the relationships between angle of attack (α), Thrust (T), Velocity (V), and flight path angle (γ) in steady nonaccelerating flight.
2. To draw important conclusions from the trim maps:
 - a. The meaning and derivation of the “Flight Path Stability” criterion (a static stability criterion);
 - b. The meaning of the “frontside” and the “backside” of the power required curve;
 - c. “Trim angle of attack controls trim speed;”
 - d. “Trim throttle sets trim thrust which in turn controls trim vertical velocity.” Since vertical velocity produced a flight path angle (γ) the throttle is herein renamed the “**gammaron**.”
 - e. Trim elevator controls trim angle of attack, which in turn controls trim speed. Thus, the elevator is herein renamed the “**speederator**.”
 - f. The oversimplified intuitive relationship between elevator and elevation (pull up on the elevator to go up).
 - g. The oversimplified intuitive relationship between throttle and speed (increase throttle setting to speed up).
 - h. The meaning of the backside control strategy.
 - i. Use trim elevator to control trim speed.
 - ii. Use trim throttle to control trim vertical velocity.
 - iii. This works on **BOTH** the frontside and on the backside of the power required curve.
 - iv. Use of this strategy on the backside of the power required curve (slow speed flight) is critical.

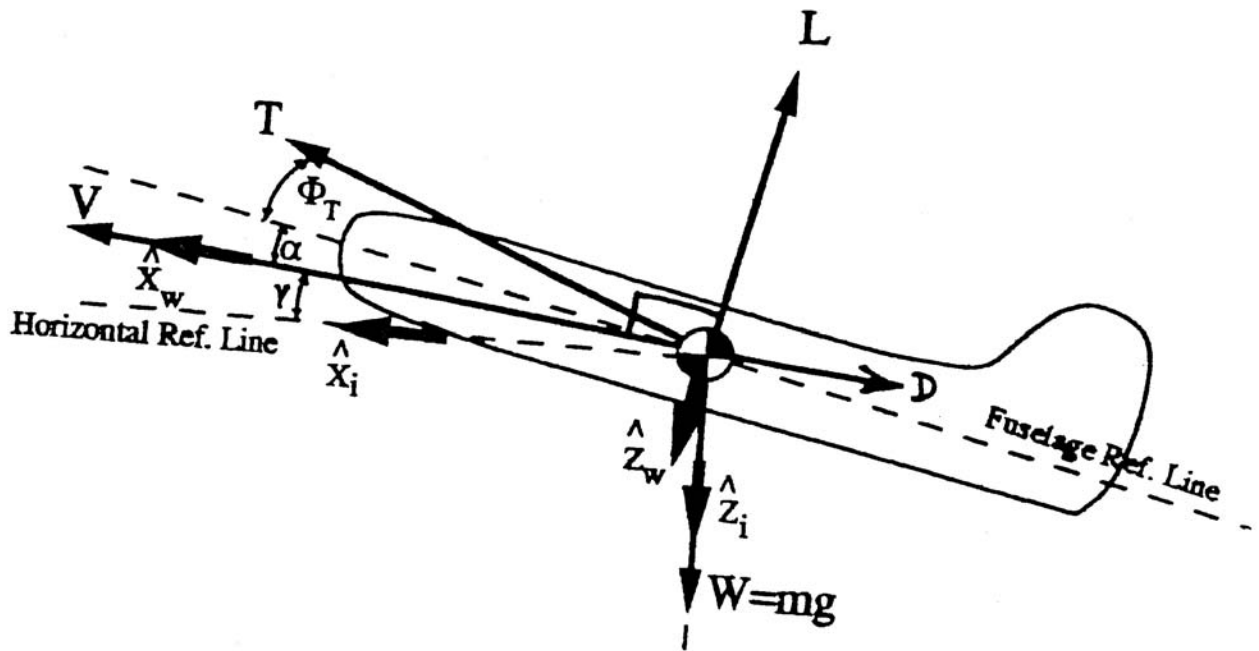
- i. The meaning of the frontside control strategy.
 - i. Use trim elevator to control trim vertical velocity.
 - ii. Use trim throttle to control trim speed.
 - iii. This works **ONLY** on the frontside of the power required curve.
 - iv. Use of this strategy on the backside of the power required curve will likely lead to stall and crash.

3. To illustrate procedure for solving nonlinear algebraic equations.

Phugoid Equations of Motion

A. Force Analysis

1. Free Body Diagram



2. Force Resultant

a. Wind Axis Unit Vectors

$$\begin{aligned}
 F &= T \cos(\phi_T + \alpha) \hat{X}_w - T \sin(\phi_T + \alpha) \hat{Z}_w \\
 &\quad - D \hat{x}_w - L \hat{Z}_w \\
 &\quad + mg \cos \gamma \hat{Z}_w - mg \sin \gamma \hat{x}_w \\
 &= \hat{x}_w [T \cos(\phi_T + \alpha) - D - mg \sin \gamma] \\
 &\quad + \hat{Z}_w [-T \sin(\phi_T + \alpha) - L + mg \cos \gamma]
 \end{aligned}$$

b.

Inertial Axis Unit Vectors

$$\begin{aligned}
F &= T \cos(\phi_T + \alpha + \gamma) \hat{x}_i - T \sin(\phi_T + \alpha + \gamma) \hat{Z}_i \\
&\quad - D \cos \gamma \hat{x}_i + D \sin \gamma \hat{Z}_i \\
&\quad - L \sin \gamma \hat{x}_i - L \cos \gamma \hat{Z}_i \\
&\quad + mg \hat{z}_i \\
&= \hat{x}_i [T \cos(\phi_T + \alpha + \gamma) - D \cos \gamma - L \sin \gamma] \\
&\quad + \hat{Z}_i [-T \sin(\phi_T + \alpha + \gamma) + D \sin \gamma - L \cos \gamma + mg]
\end{aligned}$$

B. Kinematic Analysis

1. Wind Axis Unit Vectors

$$\bar{V} = v \hat{x}_w$$

$$\begin{aligned}
\frac{{}^i d\bar{v}}{dt} &= \frac{{}^w d\bar{v}}{dt} + {}^i \omega^w \times \bar{v} = x_w [\dot{v}] \\
&\quad + y_w [0]
\end{aligned}$$

$${}^i \omega^w = +\dot{\gamma} \hat{y}_w + Z_w [-\dot{\gamma} v]$$

$$\therefore \frac{{}^i d\bar{v}}{dt} = \dot{v} \hat{x}_w - \dot{\gamma} v \hat{Z}_w$$

2. Inertial Axis Unit Vectors

$$\bar{V} = \dot{x} \hat{x}_i - \dot{h} \hat{Z}_i$$

$$\frac{{}^i d\bar{v}}{dt} = \ddot{x} \hat{x}_i - \ddot{h} \hat{Z}_i$$

C. Dynamic Analysis

$$m \frac{{}^i d\bar{v}}{dt} = \bar{F}$$

1. Wind Axis Unit Vectors

$$\hat{x}_w \Rightarrow m\dot{v} = T \cos(\phi_T + \alpha) - D - mg \sin \gamma \quad (1)$$

$$\hat{Z}_w \Rightarrow -m\dot{\gamma}v = -T \sin(\phi_T + \alpha) - L + mg \cos \gamma \quad (2)$$

2. Inertial Axis Unit Vectors

$$\hat{x}_i \Rightarrow m\ddot{x} = T \cos(\phi_T + \alpha + \gamma) - D \cos \gamma - L \sin \gamma \quad (3)$$

$$Z_i \Rightarrow -m\ddot{h} = -T \sin(\phi_T + \alpha + \gamma) + D \sin \gamma - L \cos \gamma + mg \quad (4)$$

3. Choice of Equations to Use

Each of the equations of motion (1)-(4) are equally valid. You can integrate any two of them. You might have to use the relationships

$$\sin \gamma = \frac{\dot{h}}{v}$$

$$\tan \gamma = \frac{\dot{h}}{\dot{x}}$$

Choosing equations (1) and (4)

$$m\dot{v} = T \cos(\phi_T + \alpha) - D - mg \sin \gamma \quad (1)$$

$$+ m\ddot{h} = +T \sin(\phi_T + \alpha + \gamma) - D \sin \gamma + L \cos \gamma - mg \quad (4)$$

4. Notice that in the earlier analyses that we called

$$F_v = T \cos(\phi_T + \alpha) - D - mg \sin \gamma$$

$$F_h = T \sin(\phi_T + \alpha + \gamma) - D \sin \gamma + L \cos \gamma - mg$$

so

$$m\dot{v} = F_v$$

$$m\ddot{h} = F_h$$

5. In doing problems of this type you should always perform steps A, B, and C. Your choice of unit vectors is arbitrary. Pick your favorite set of equations of motion to use. We will use equations for \dot{v} and $\dot{\gamma}$ in subsequent analysis

$$\begin{aligned} v_{\dot{}} &= (\text{Thrust}/\text{mass})\cos(\text{PhiT}+\alpha) - \text{Drag}/\text{mass} - g\sin(\text{gam}); \\ \gamma_{\dot{}} &= (\text{Thrust}/(\text{mass}\cdot v))\sin(\text{PhiT}+\alpha) + \text{Lift}/(\text{mass}\cdot v) - (g/v)\cos(\text{gam}); \end{aligned}$$

where

$$\begin{aligned} \text{Thrust} &= 550 \cdot \text{Bhp} \cdot \text{EtaP} / v; \\ \text{Cl} &= \text{Cl0} + \text{Clalpha} \cdot \alpha; \\ \text{Cd} &= \text{Cd0} + K \cdot \text{Cl} \cdot \text{Cl}; \\ \text{Drag} &= \text{HalfRhoS} \cdot v \cdot v \cdot \text{Cd}; \\ \text{Lift} &= \text{HalfRhoS} \cdot v \cdot v \cdot \text{Cl}; \end{aligned}$$

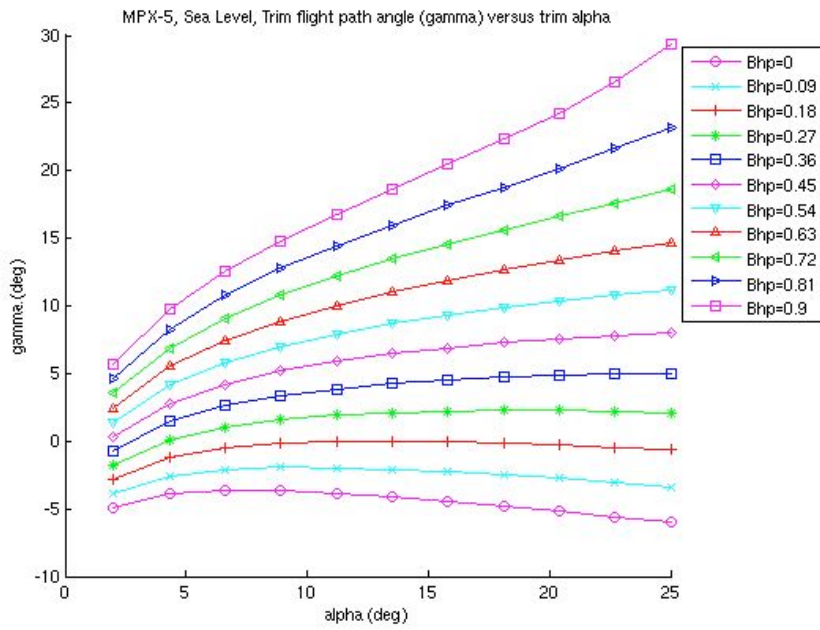
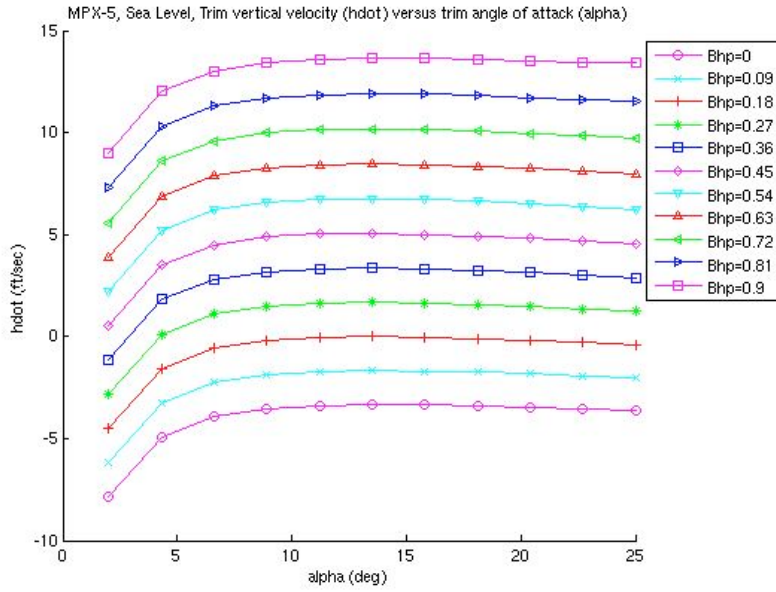
6. For the MPX5 vehicle

$$\begin{aligned} S &= 9.375 && \% \text{ wing area, ft}^2 \\ \text{HalfRhoS} &= 0.5 \cdot \text{Rho} \cdot S; \\ W &= 19.2 && \% \text{ weight, lbf} \\ \text{mass} &= W/g; && \% \text{ mass} \\ \text{PhiT} &= 0; && \% \text{ Thrust inclination, rad} \\ \text{Cd0} &= 0.015 && \% \text{ parasite drag (non-dim)} \\ K &= 0.068 && \% \text{ square ther in drag polar (non-dim)} \\ \text{Cl0} &= 0.10257 && \% \text{ lift coef at zero alpha (non-dim)} \\ \text{Clalpha} &= 2.9842 && \% \text{ lift curve slope, per rad} \\ \text{BhpMax} &= 1 && \% \text{ maximum horsepower} \\ \text{EtaP} &= 0.65 && \% \text{ Propeller efficiency} \end{aligned}$$

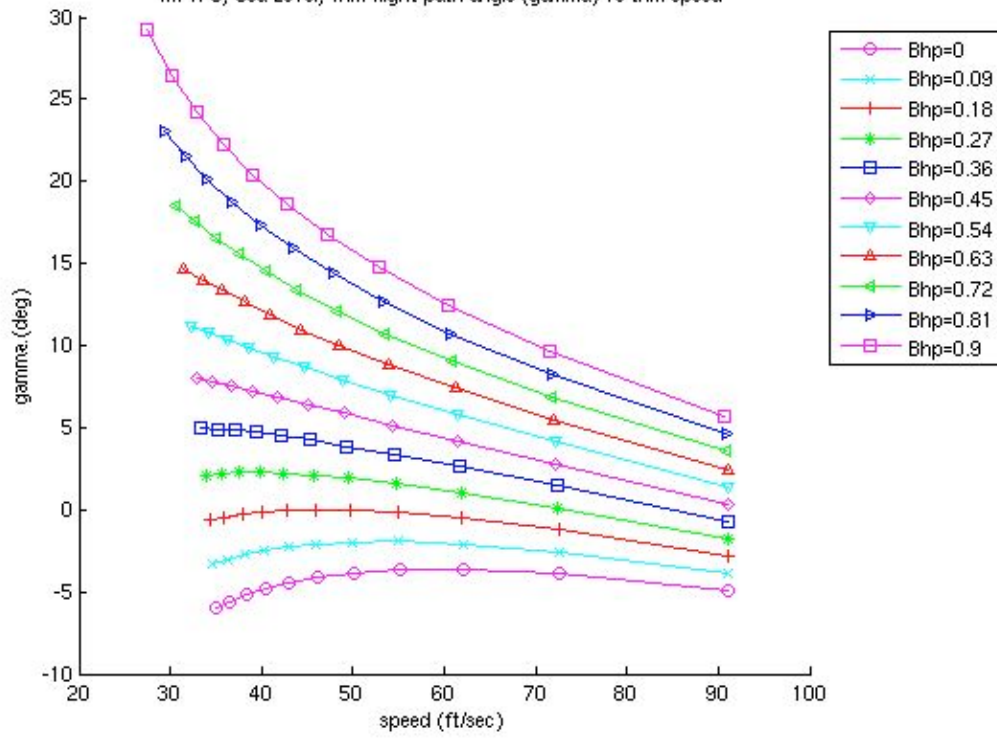
Additional parameters for the MPX-5 are as follows

$$\begin{aligned} \text{Clde} &= 0.48562 \\ \text{Cm0} &= -0.03 \\ \text{Cma} &= -1.0491 \\ \text{Cmde} &= -2.3176 \\ \text{Xcgbar} &= 0.25 \end{aligned}$$

Trim Maps



MPX-5, Sea Level, Trim flight path angle (gamma) vs trim speed



MPX-5, Sea Level, Power Required Curve (for level flight)

