Introduction to Performance and Flying Qualities Flight Testing

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Chapter 4
Standard Atmosphere

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Standard Atmosphere

Introduction

The performance of any flying machine will be greatly influenced by the atmosphere in which it is flying. This is true even for space shuttles, space probes etc. . . since at some point in their journey they will have to cross the atmosphere of the planet from which they are blasting off or re-entering.

The earth's atmosphere is a constantly changing system. The pressure and temperature variations are a function of altitude, geographical location around the earth, season, time of day etc. . . Since these atmospheric conditions cannot be duplicated at will to provide the exact environment in which a flight takes place, then a "standard atmosphere" must be devised to provide a common basis to relate all flight test, wind tunnel results, aircraft design and general performance.

The standard atmosphere will give mean values of pressure, temperature and density as a function of altitude. These values come from mathematical models supported by experimental data obtained from weather balloons and rocket probes.

There are several models of "standard atmosphere" published by various agencies. Minor differences based on mathematical constants used to calculate changes above 100,000 ft. may account for the variation between the models published, however, the differences are negligible in the region of the atmosphere in which modern aircraft fly.

Altitude Definitions

Although everyone considers altitude as the distance above the earth, more precise definitions must be used in setting up the mathematical models which will define the standard atmosphere.

Geometric Altitude

The geometric altitude is defined as the actual height of an object or aircraft above sea level. Consider a helicopter that takes off from sea level, climbs to a certain height and hovers at that constant height. A tape measure is dropped from the helicopter to sea level. The height of the helicopter indicated by the tape defines its geometric or "tapeline" altitude.

Absolute Altitude

To measure the absolute altitude of a helicopter, its height must be taken with respect to the center of the earth (Figure 4.1).

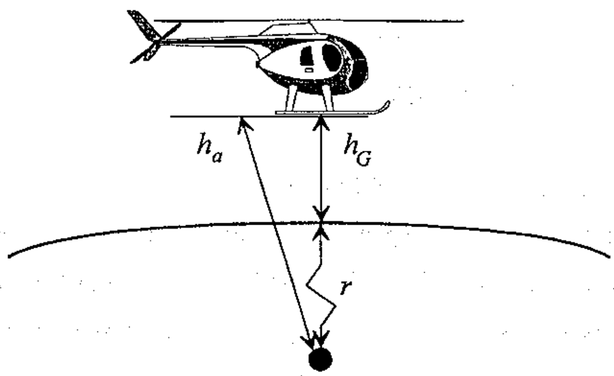

Figure 4.1 Geometric and Absolute Altitude

Absolute altitude is therefore the geometric (or tapeline) altitude plus 'r', radius of the earth. Since the radius varies around the earth, so will the absolute altitude of an aircraft.

$$h_a = h_G + r$$

Absolute altitude is important in space flights because the local acceleration due to gravity varies as a function of the absolute altitude, h_a. Newton's law of gravitation says that 'g' varies inversely as the square of the distance form the earth's center. The term g_o denotes standard sea level acceleration (g_o = 32.17 ft/sec^2) while g denotes gravitational acceleration at altitude. Although the earth's radius is not constant, it is approximately 20.9 × 10^6 ft at 35° latitude, sea level.

$$g = g_o\left(\frac{r}{h_a}\right)^2$$

or

$$g = g_o\left(\frac{r}{r + h_G}\right)^2 \tag{4.1}$$

This variation in 'g' with altitude must be taken into account when building mathematical models of the standard atmosphere. Note that $g = .99g_o$ at 100,000ft, $.999g_o$ at 10,000ft, and $.9999g_o$ at 1000 feet above sea level.

Geopotential Altitude

Because the gravitational acceleration changes as a function of height, there exists a requirement to define yet another altitude, the geopotential altitude. Potential energy is the product of true weight and tapeline altitude ($W \cdot h_G$). If an object's sea level weight is used instead ($W_{SL} = mg_o$), it will not need to go as high to obtain the same potential energy. This lower height is the geopotential height, Figure 4.2. In actual practice, however, the difference between geopotential and geometric height is insignificant unless dealing with space mechanics.

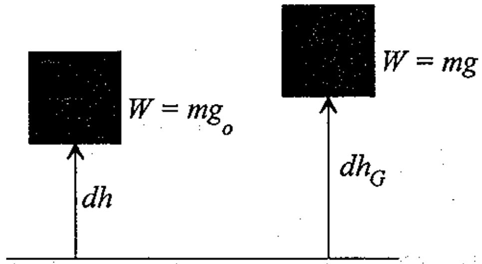

Figure 4.2

Therefore,

$$dhg_o = dh_G g \cdot$$

where:

- h: geopotential height
- h_G: geometric height
- g_o: gravitational constant: 32.17 ft/sec^2
- g: acceleration due to gravity

$$dh = dh_G \frac{g}{g_o}$$

Division of the Atmosphere

Figure 4.3 shows the layers of the atmosphere. There are five major divisions of the atmosphere: the troposphere, the stratosphere, the mesosphere, the thermosphere and the exosphere.

The troposphere is the closest region of the atmosphere surrounding the earth. It extends from the surface to approximately 28,000 feet at the poles and to 56,000 feet at the equator. This is the region which will be of most interest to us since it is the region in which most of the weather phenomenon occur and where most aircraft fly. The upper region of the troposphere is the tropopause. The height of this "pause" varies with seasons and also with latitudes.

The second division of the atmosphere is the stratosphere. It extends from the tropopause to approximately 14 miles outward. Depending on the latitude of flight, many aircraft do venture into the lower regions of the stratosphere. With the advent of new technology, many aircraft will someday fly on a routine basis well within the stratosphere.

One of the major characteristics of the lower stratosphere is that the temperature remains a constant. Although strong winds may be present at these high altitudes, they are generally constant in speed. The ever changing weather patterns we witness close to the earth do not occur in the stratosphere.

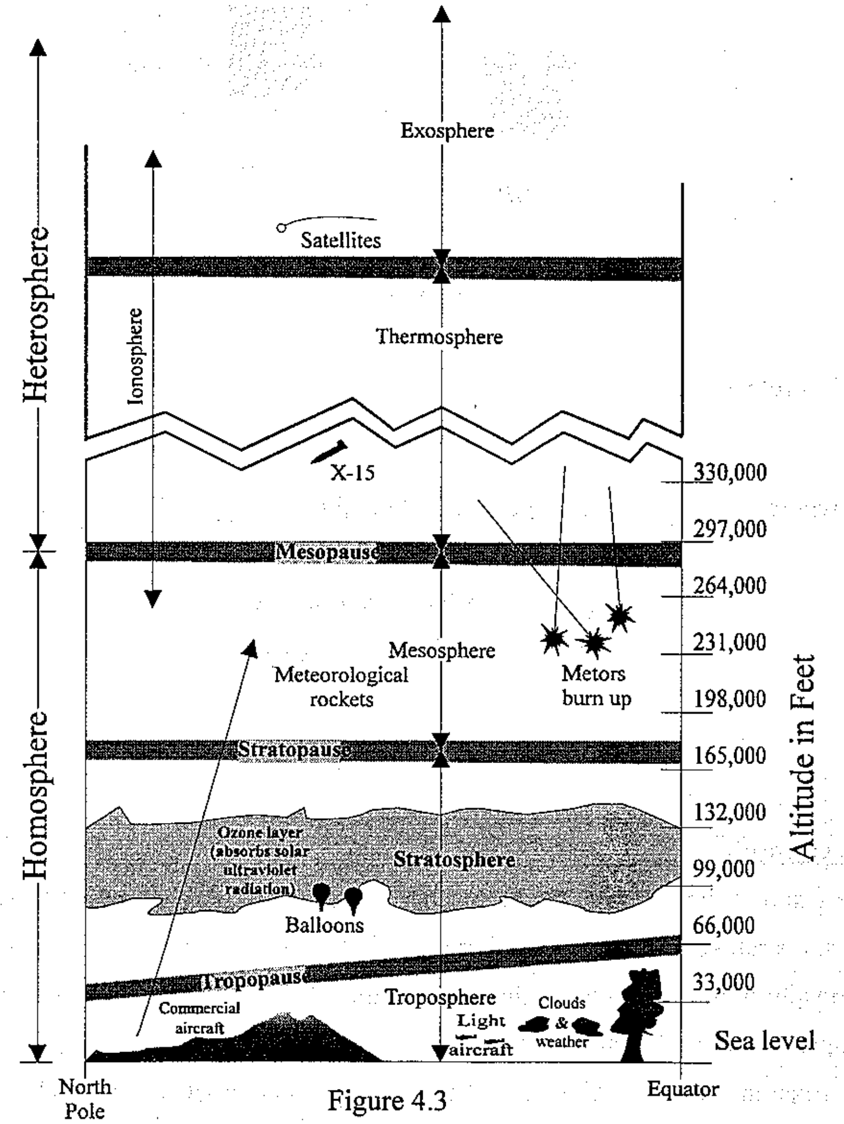

Figure 4.3

The composition of the earth's atmosphere per volume is approximate 78% nitrogen, 21% oxygen, 0.4% water vapor and a trace of other rare gases such as argon, carbon dioxide etc. . .

Standard Atmosphere Assumptions

1. The air is dry:

   With only 0.4% of water vapor per volume in standard air, this is a very reasonable assumption.

2. The air is a perfect gas.

   A perfect gas is one in which intermolecular forces are negligible. This is true up to an altitude of about 290,000 ft. This will also be true of the flow of air surrounding an aircraft at subsonic speed as well as supersonic speed. The air will therefore obey the equation of state for a perfect gas:

$$P = \rho R T$$

R is a gas constant which varies with the type of gas. For air, $R = 1716 \frac{lb.ft}{(slug)(^oR)}$. Note that in some books, the equation of state may read:

$$P = \rho g R T$$

In this case, $R = 53.3 \frac{lb.\sec^2}{(slug)(^oR)}$ and multiplying:

$$32.2 \frac{ft}{\sec^2} \times 53.3 \frac{lb.\sec^2}{(slug)(^oR)} = 1716 \frac{lb.ft}{(slug)(^oR)}$$

3. Gravitational Field Decreases with Altitude $g = g_o \left(\frac{r}{r + h_G} \right)^2$

   Newton's gravitational law leading to the definition of geopotential altitude has already been discussed.

4. Hydrostatic Equilibrium Exists

   One basic law of physics states that the air will always flow from a region of high pressure to a region of low pressure. Why then, does the air surrounding the earth not flow outward into space and leave the earth in a void? Gravity.

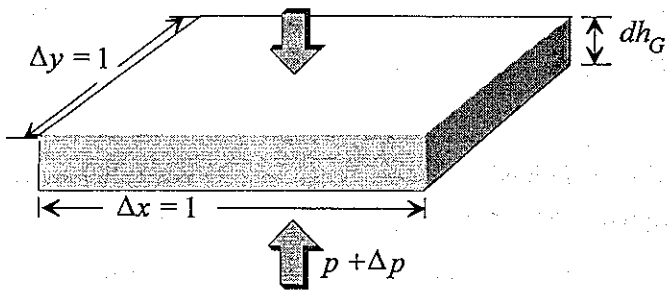

Figure 4.4 Hydrostatic Equilibrium

Considering a small cube of air of Δx and $\Delta y = 1$. If the cube of air is in equilibrium, i.e., not moving up, down, nor sideways, then the force holding the cube down must be equal to the weight of that element of air:

$$\Delta p \, (\Delta x \cdot \Delta y) = - W$$

The weight of the element of air is equal to it's mass times the local acceleration, i.e., $(m \cdot g)$, and since the mass is the density, ρ, per unit volume, then:

$$\Delta p \, (\Delta x \cdot \Delta y) = - \rho g \Delta h_G (\Delta x \cdot \Delta y)$$

which simplifies to:

$$\Delta p = - \rho g \Delta h_G$$

geometric

This hydrostatic equation applies to any fluid of density ρ, i.e., air as well as water etc.. In order to make the equation useful in our aerodynamics calculations, some basic assumptions must be made: in layers of the atmosphere where aircraft generally fly, the variation of g with altitude is negligible. Therefore, by convention, g was considered to be a constant through the atmosphere and equal to g_o. To keep the equations consistent, since

$$dhg_o = dh_G g \qquad (4.2)$$

the **hydrostatic equation** can now be written in its useful form:

$$\boxed{\Delta p = -\rho g_o \Delta h} \qquad (4.4)$$

geopotential

where h is the geopotential altitude. Geopotential altitude is a fictitious altitude which takes into consideration the change in local acceleration with altitude. Since $g \approx g_o$ then the geopotential altitude is the same as geometric altitude in lower layers of atmosphere.

Standard Atmosphere Equations

The three cornerstones of the standard atmosphere are accepted standard values for sea level pressure (P_o), temperature (T_o) and the variation of temperature with altitude (lapse rate). They are all based on experimental data:

$P_o = 2116.22\ \text{lb/ft}^2 = 14.696\ \text{lb/in}^2 = 29.921\ \text{in}\ Hg = 1013\ hP_a(mb) = 760\ mm\ Hg$

$T_o = 288.15\ °K = 518.67\ °R = 58.67\ °F = 15°C$

The standard lapse rate is defined as $^{-}1.9812°C$ ($^{-}3.57°F$) per 1,000 geopotential feet up to an altitude of 36,088 feet (11,000 m). The standard temperatue is a constant 216.65K between 36,088 ft and 65,616 ft (20,000 m) above this altitude, the temperature begins to actually increase at about +.3°C per 1,000 ft, but these altitudes are outside the scope of this text. Using these cornerstones, the hydrostatic equation, and the perfect gas equation, a relationship can be derived to calculate peressure and density at any altitude.

Ratios

Most standard atmosphere tables will present the data in terms of ratios instead of carrying absolute values.

$$\delta\ (\text{delta}) \equiv \text{pressure ratio} \equiv \frac{\text{pressure at altitude}}{\text{pressure sea level std day}} \equiv \frac{P_a}{P_o}$$

$$\theta\ (\text{theta}) \equiv \text{temperature ratio} \equiv \frac{\text{temperature at altitude}}{\text{temperature sea level std day}} \equiv \frac{T_a}{T_o}$$

$$\sigma\ (\text{sigma}) \equiv \text{density ratio} \equiv \frac{\text{density at altitude}}{\text{density sea level std day}} \equiv \frac{\rho_a}{\rho_o}$$

Subscripted with a "o" is a fixed value that never changes

Recall the perfect gas equation:

$$P = \rho RT$$

From this relation, standard day sea level density is calculated as:

$$\rho_o = 0.002377 \text{ slgs/ft}^3 = 0.07647 \text{ lbm/in}^3 = 1.2284 \text{ kg/m}^3$$

The perfect gas equation applies at any condition. By dividing this relationship at some altitude by the same relation at sea level, we get:

$$\frac{P}{P_o} = \frac{\rho RT}{\rho_o RT_o} \text{ or } \delta = \sigma\theta \text{ or } \sigma = \frac{\delta}{\theta}$$

Dividing the hydrostatic equation by the perfect gas equation and integrating between various limits of pressure, temperature, and altitudes yields the standard atmosphere pressure, temperature, and density equations.

For geopotential altitudes below 36,089 feet, the equations, in terms of the ratios defined, are as follows:

$$\delta = \frac{P_a}{P_o} = (1 - k_1 H)^{k_2} \tag{4.5}$$

$$\theta = \frac{T_a}{T_o} = (1 - k_1 H) \tag{4.6}$$

$$\sigma = \frac{\rho_a}{\rho_o} = (1 - k_1 H)^{k_2 - 1} \tag{4.7}$$

where:

$$k_1 = \frac{L}{T_o} = 6.87559 \times 10^{-6} ft^{-1}$$

and:

$$k_2 = \frac{g_o}{RL} = 5.2559$$

where R is the gas constant
L is the temperature lapse rate
T is the temperature in degrees Rankine

The k_1 and k_2 constants come from the integration process mentioned above.

For geopotential altitudes above 36,089 feet the temperature is constant to 65,616 feet. The equations are as follows:

$$T_a = -56.5°C = 216.65K = 389.99R \tag{4.8}$$

$$\delta = \frac{P_a}{P_o} = 0.223358e^{-k_3(H-36089)} \quad (4.9)$$

$$\sigma = \frac{\rho_a}{\rho_o} = 0.29707e^{-k_3(H-36089)} \quad (4.10)$$

where:

$$k_3 = \frac{G}{RT_a} = 4.80614 \times 10^{-5} geopotential\ foot$$

This model is depicted graphically below and is tabulated in appendix B.

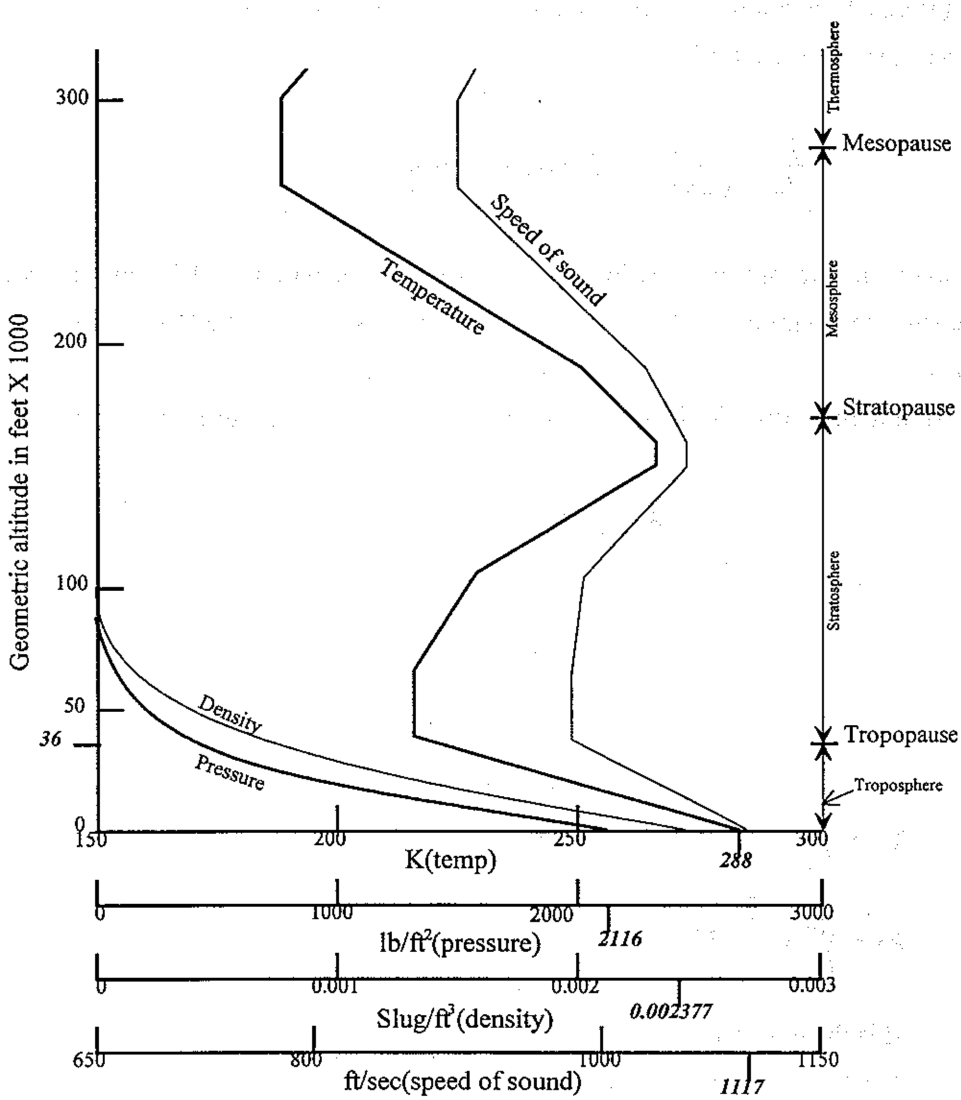

Figure 4.5 Geometric Altitude vs Temperature, Pressure, Density, Speed of Sound

Pressure Altitude

To make use of the standard atmosphere tables in flight testing aircraft, the absolute static pressure of the air is of primary importance. The static pressure of the air at any altitude results from the mass of air supported above that level. At standard sea level, the static pressure is 29.92 in Hg. A simple way of measuring the pressure at any altitude is by the use of the altimeter with the setting in the

Kohlsman window set to standard sea level pressure. Since the altimeter is calibrated according to the standard atmosphere pressure variation equation with altitude, with the setting to 29.92 in. Hg., the indicated altitude read directly off the altimeter is pressure altitude. This may not reflect the *true geometric* altitude of the aircraft above mean sea level (MSL), but true altitude is not important in most flight test, pressure altitude is. Pressure altitude can always be duplicated by flying the same indicated altitude as long as the barometric setting is 29.92 in. Hg.

Temperature Altitude

Temperature altitude is defined as the altitude, on a standard day, at which the test day temperature would be found. Temperature altitude can be readily found by using a corrected outside air temperature gauge and the standard atmosphere table.

Density Altitude

Density altitude is the altitude, on a standard day, at which the test day density would be found. Because aircraft are not equipped with "density gauges", the test day density must be calculated. This is done knowing test day pressure (P_a), temperature (T_a) and the perfect gas law ($P_a = \rho_a R T_a$). Although all flight testing is performed with the altimeter set to the standard day sea level pressure, i.e., flying a pressure altitude, density altitude is very important to the performance of an aircraft, especially in the take-off and climb phases.

density altitude:

$$\rho = \frac{P}{RT_{test}}$$

or using the ratio equation:

$$\sigma = \frac{\delta}{\theta} = \frac{\delta T_o}{T_{test}}$$

where:

δ is the pressure ratio at the indicated pressure altitude, obtained directly from the atmospheric tables

T_o is the standard sea level temperature in °Kelvin or °Rankine

T is the test day temperature at the test altitude in °Kelvin or °Rankine