Error Analysis

AAE 490A/AT490F

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Error Analysis

What follows is an annotated version of the results of running a MATLAB script called CLerror_analysis.m. This script and related functions can be found on the course web site.

http://roger.ecn.purdue.edu/~andrisan/Courses/AAE490A_S2003/Index.html

Start here

```
% This is a MATLAB script to study error analysis
% of two types,
% absolute errors and root-mean square (rss) errors.
% The script assumes that lift coefficient is to be
computed
% from measured values of weight (W), air density (rho),
% speed (v), and wing area (S).
%
% CL=W/(.5*pho*v^2S)
%
% x(1)=W=weight in pounds
% x(2)=rho=air density in slugs per ft^3
% x(3)=v=speed in ft/sec
% x(4)=S=wing area in ft^2
%
% Unfortunately, the measured values of
% these four quantities are all slightly in error.
% These errors mean that the
% computed values for lift coefficient will also be in
error.
% This script studies the relationship between the errors
% in the four inputs (W, rho, v S) and the error in the
output (CL).


Wtrue = 5500


rhotrue =0.0023769


vKnots =70


vtrue =118.15
```

Strue =199.2

CLtrue =1.6644

The individual input errors in the four quantities used to compute CL are given below.

error = [7 1e-05 0.1 0.1]

The individual contributions to the output error from these four input errors (errors in W, rho, v and S) can be examined to see which input creates the biggest errors in the output (CL).

Output_error(i)=absolute value of [(partial of CL wrt x(i))*(error in x(i))]

Numerical values of these output errors are given below.
err_vec = [0.0021183 0.0070023 0.0028175 0.00083553]

From these output errors, the total error in the output CL can be computed in two ways.

The absolute error as printed out above (abs_error) is defined as

abs_error=sum(error(i)) for i=1,2,3,4

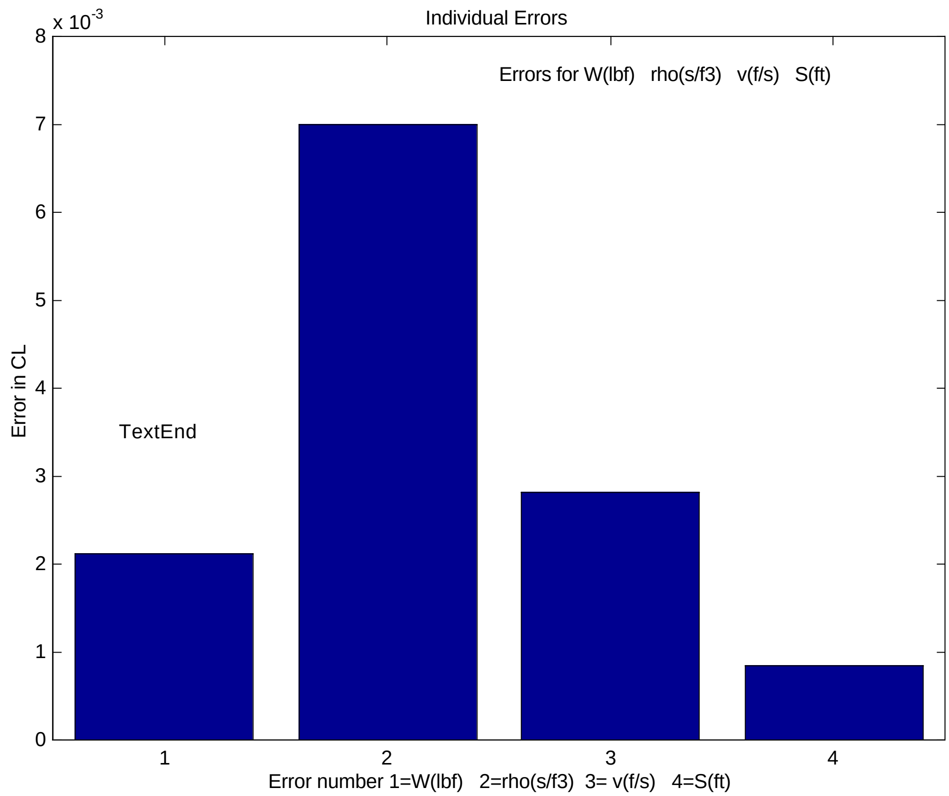
The numerical value of the absolute error is
abs_error = 0.012774

The root-sum square error as printed out above (rss_error) is defined as

rss_error=sqrt(sum(error(i)^2)) for i=1,2,3,4

The numerical value of the root-sum square error is
rss_error =0.0078839

The four output errors are plotted below. Their relative size is very apparent in the plot below.

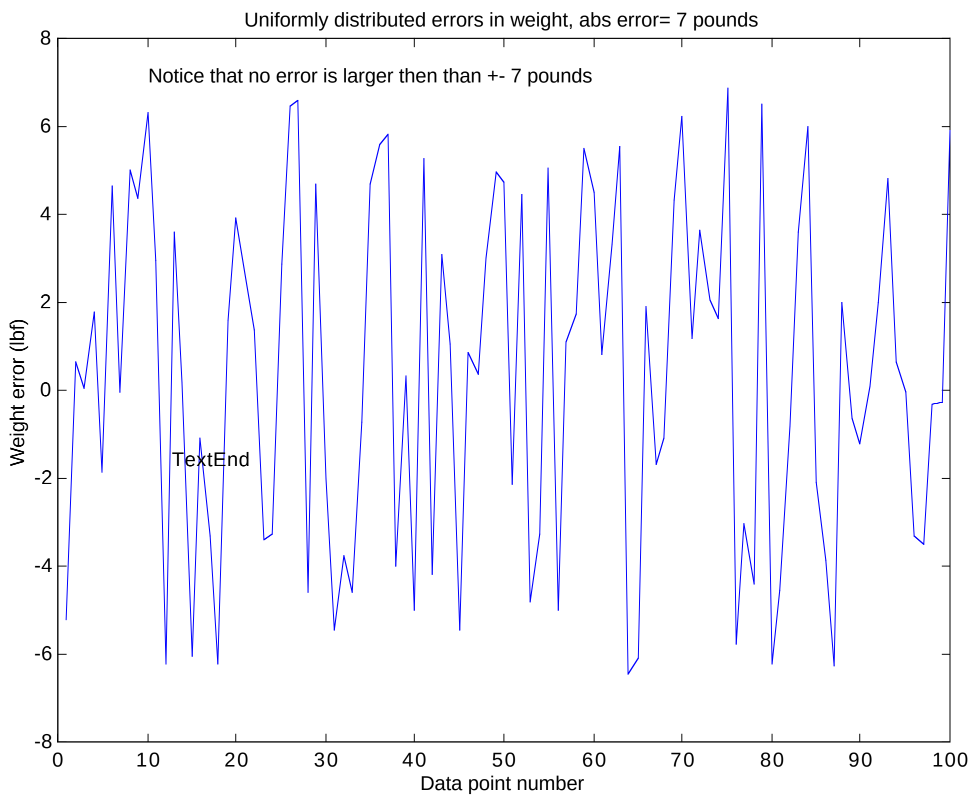


Notice how the error due to rho (air density) dominates the total output error (absolute error and rss error. This suggests that it is worthwhile to try to reduce this input error due to rho.

Let's simulate measurements that have errors on them. To do this, we have to assume an error model for the measurement errors.

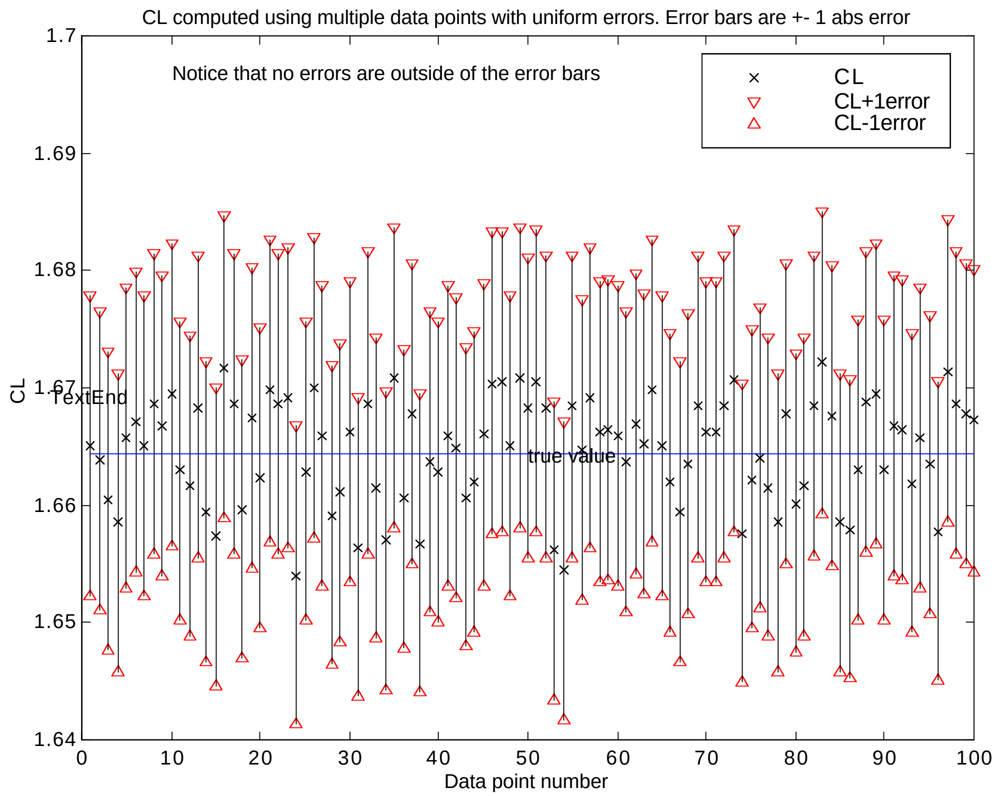
```
% Analysis using absolute errors
% Assume that  weight, density, speed and wing area are all
% measured with errors.
% Assume that the errors are uniformly distributed around
% the true values plus and minus the absolute error.
% The vector error is interpreted as the +- (absolute)
errors
% of a uniformly distributed random variable.
echo off
```

Plotted below is a random sample of errors in weight that are uniformly distributed around zero and lie in the range +1 7 pounds.



We have also generated uniformly distributed errors in rho, v, and S, with +- bounds of 1e-05, 0.1, and 0.1.

Based on this noisy (random) measurements of W, rho, v and S we can compute CL and determine the absolute error bounds for CL.

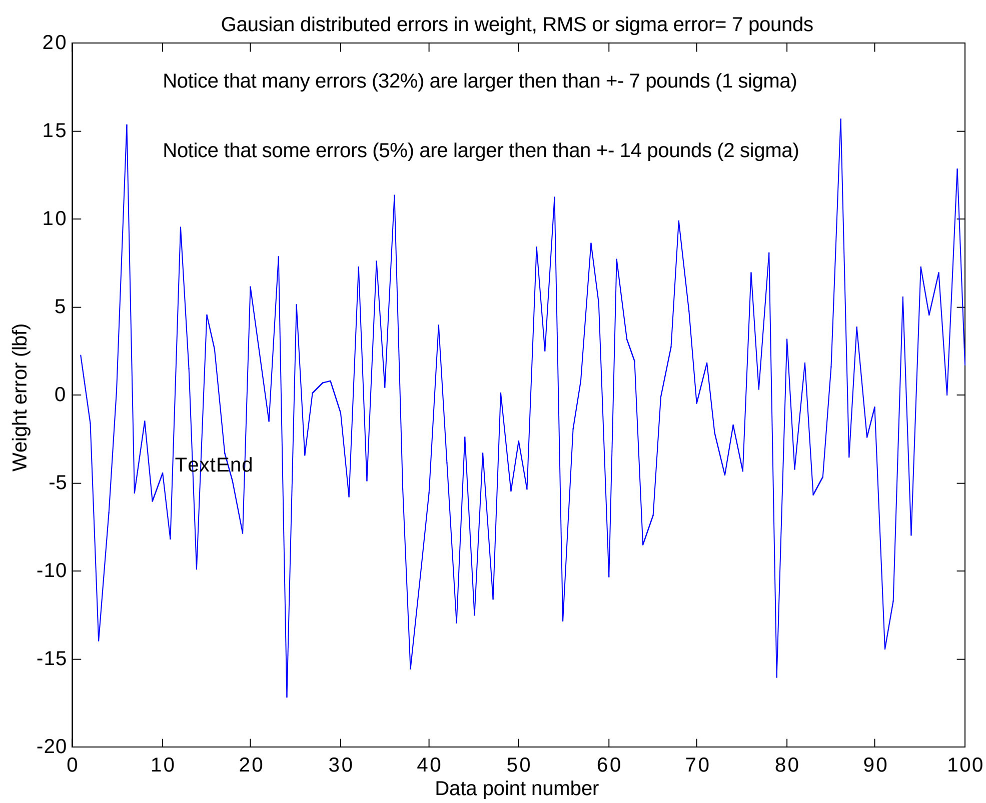


Notice above that the computed value of CL +- the total absolute error form a vertical line for each data point. The vertical line always brackets the true value of CL. This is a desirable property of absolute error analysis. A disadvantage of absolute error analysis is that it tends to produce larger error bounds then does the root-mean square error analysis.

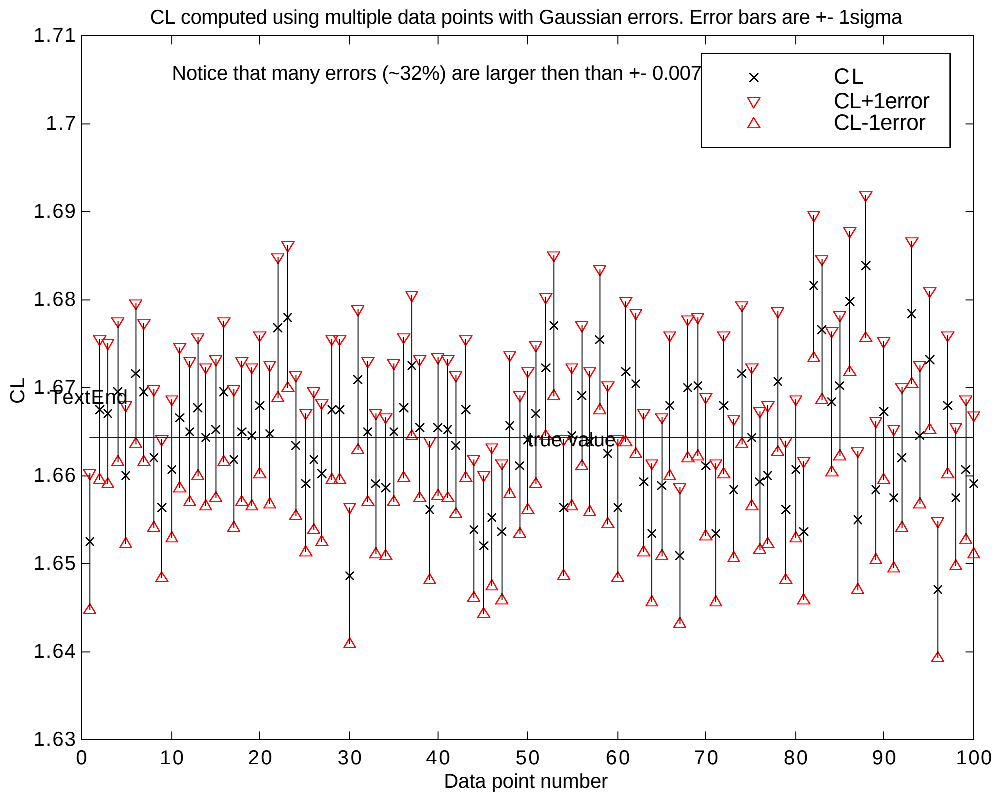
In a similar way we can simulate measurements that have errors on them of a different type. To do this, we have to assume a different error model for the measurement errors.

```
% Analysis using Gaussian errors
% Assume that  weight, density, speed and wing area are all
% measured with errors.
% Assume that the errors are Gaussian distributed around
% the true values with a sigma as specified
% with the array error.
% The vector error is interpreted as the
% sigma or root-mean square of a
% Gaussian distributed random variable.
»
```

Plotted below is a random sample of errors in weight that follow the Gaussian (normal) distribution. These values are centered around zero and have an root-mean square (rss or sigma) of 7 pounds.

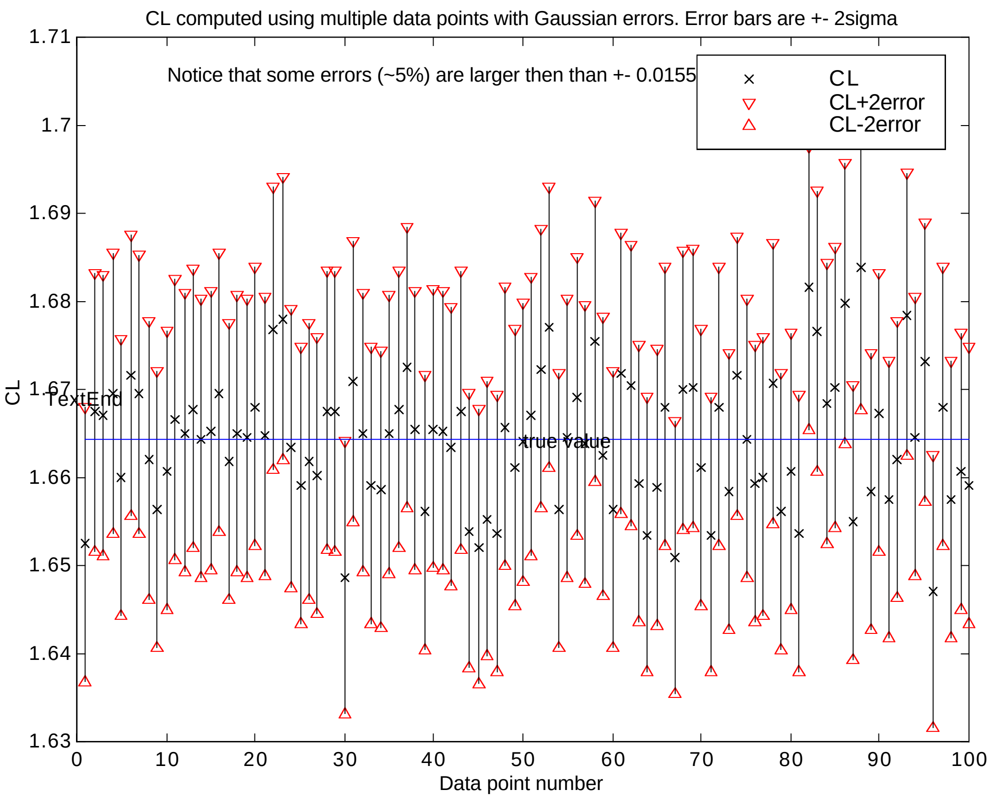


We have also generated gaussian distributed errors in rho, v, and S, with sigma errors of 1e-05, 0.1, and 0.1. Based on this noisy (random) measurements of W, rho, v and S we can compute CL and determine the RSS (sigma) bounds for CL.



Notice above that the computed value of CL +- 1 sigma form a vertical line for each data point. The vertical line bracket the true value of CL about 68% of the time.

A better bound is to take CL +- 2 sigma, as shown below. The vertical line bracket the true value of CL about 95% of the time.



When using a root-mean square (Gaussian) error model, it is generally preferable to use +- 2 sigma error bounds around computed data points.

Students should always compute error bars around experimentally determined quantities. The error model they use (absolute errors or root-mean square errors) may be decided on a problem-by-problem basis but must be clearly described when documenting results.