Preliminary Weight Estimation (Electric Powered Aircraft)

Idea:

1. Consider how much energy is used in each mission phase.
2. Estimate ~~the~~ battery weight required to store that much energy
3. Use historical data to ~~see~~ determine weight of aircraft to carry that battery plus any payload

Weight Breakdown

W_e = empty weight (does not include payload or battery

W_B = battery weight

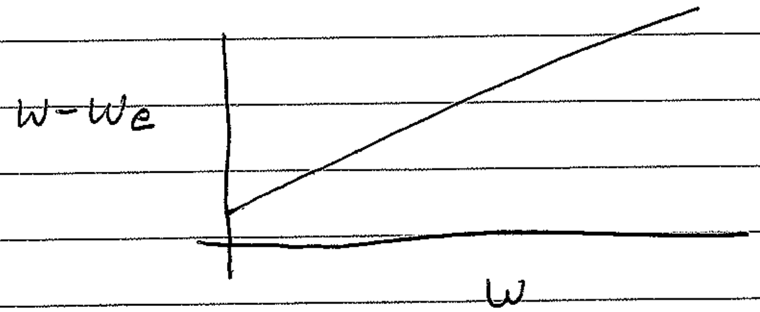
W_p = payload weight

W = total aircraft weight

$W = W_e + W_B + W_p \Rightarrow W - W_e = W_b + W_p$

Historical Data

$W - W_e = 0.2103 * \text{Weight} + 0.1243$ lbf



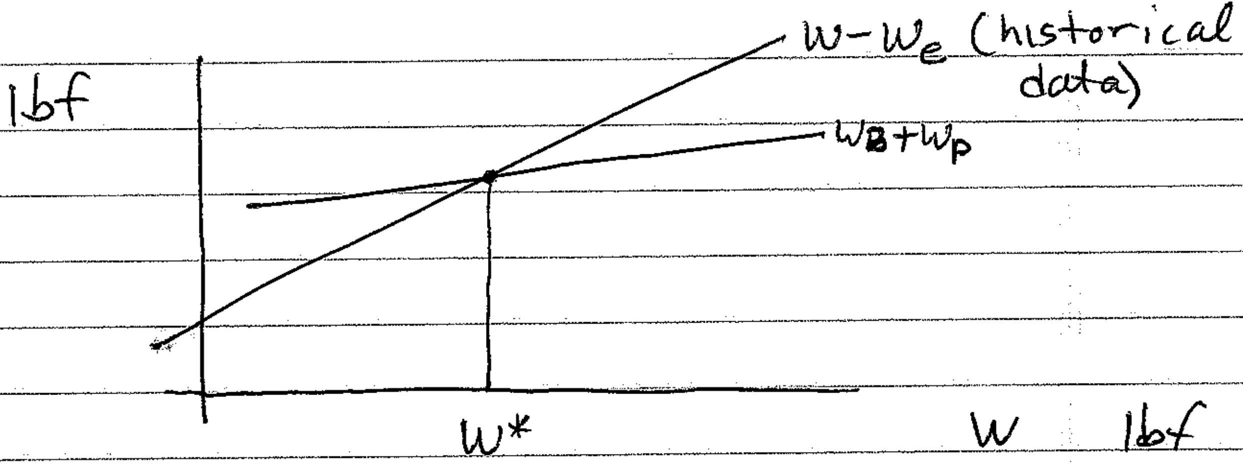
Solution Procedure

1. Given W_p
2. Estimate battery weight fraction $\frac{W_b}{W}$ based on mission analyses

~~4.~~ ~~Plot~~ ~~$W_B =$~~ Then $W_B = \left(\frac{W_B}{W}\right) W$

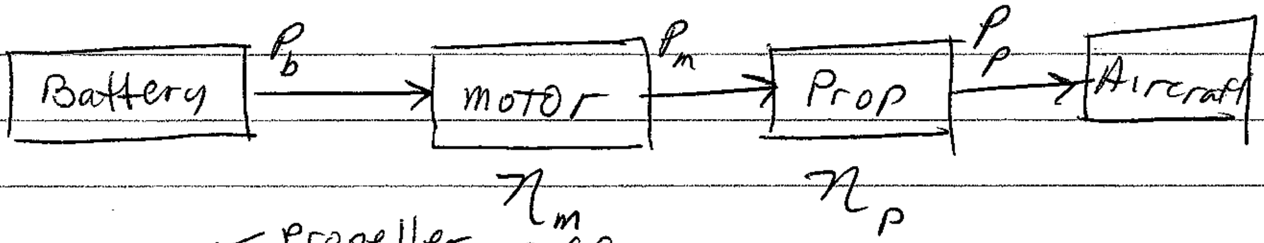
3. Plot $W_B + W_p = \left(\frac{W_B}{W}\right) W + W_p$

on same plot as historical data and look for intersection



W^* is the weight of the aircraft that carries that battery and payload.

Power Flow



$$P_p = \eta_p P_m \quad \leftarrow \text{propeller efficiency}$$

$$P_m = \eta_m P_b \cancel{= \eta_m \eta_p}$$

$$\uparrow \text{motor efficiency}$$

$$P_p = \eta_p \eta_m P_b$$

To get P_b out of the prop requires

$$P_b = \frac{P_p}{\eta_p \eta_m}$$

$$\text{Energy} = \text{Power} \cdot \text{time}$$

$$\text{ft lbf} = \left(\frac{\text{ft lbf}}{\cancel{\text{sec}}}\right)\left(\cancel{\text{sec}}\right)$$

$$\text{Energy (joule)} = E(\text{ft lbf})(1.356)$$

$$\text{Energy in battery (Joule)} = \frac{\overset{\text{ft lbf/sec}}{P_p}\ \overset{\text{sec}}{T}}{\eta_p \eta_m}\ 1.356$$

Mission Phases

Divide the mission into various flight phases and determine energy (and battery size) required for each mission segment

Segment	Enery Required
Takeoff	E_{TO}
Warm-up	E_{WU}
Climb	E_{CL}
Fly Straight (level flight)	E_{LF}
Fly turning	$\underline{E_{TU}}$

Total Energy required by the airplane

$$E_A = E_{TO} + E_W + E_{CL} + E_{LF} + E_{TU}$$

Total Energy required by the battery

$$E_B = \frac{E_A}{\eta_p \eta_m} \quad \text{(Joule)}$$

Energy Density of the battery

$$E_B = \rho_B \, W_B = \frac{\text{Joule}}{\text{lbf}} \, \text{lbf} \qquad \text{(Joule)}$$

$\rho_B \equiv$ energy density of the battery (Joule/lbf)

Now we can solve for battery weight

$$W_B = \frac{E_B}{\rho_B} = \frac{E_A}{\eta_p \eta_m \rho_B} = \frac{E_{TO} + E_{WU} + E_{CL} + E_{LF} + E_{TU}}{\eta_p \eta_m \rho_B}$$

However the individual energy components will be a function of total aircraft weight, which we don't know yet. So we will normalize by the unknown aircraft weight.

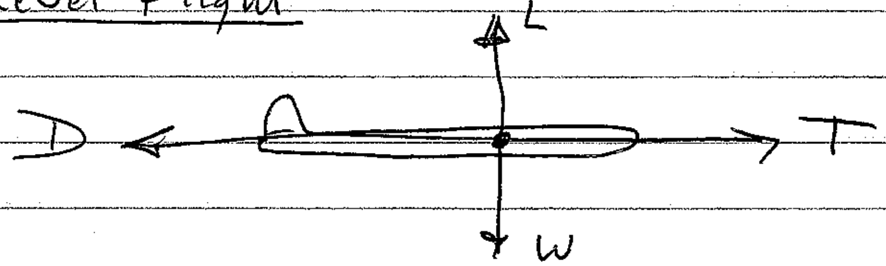
$$\frac{W_B}{W} = \frac{W_{B_{TO}}}{W} + \frac{W_{B_{WU}}}{W} + \frac{W_{B_{CL}}}{W} + \frac{W_{B_{LF}}}{W} + \frac{W_{B_{TU}}}{W}$$

We can use this in the procedure outlined on pages 1a and 1b to determine aircraft weight from historical data.

The problem boils down to finding the battery wt fractions for each mission segment.

Energy Consumption

Level Flight



Thrust = Drag $\qquad\qquad$ Lift = Weight

$T = D$ $\qquad\qquad$ $L = W$

$$T = D = \frac{L}{L/D} = \frac{W}{L/D}$$

Power expended

$$P = TV = \frac{VW}{L/D} \qquad \text{ft lbf/sec}$$

Energy used in level flight depends on the time used in level flight

$$E_{LF} = P\,T_{LF} = \left(\frac{VW}{L/D}\,T_{LF}\right)1.356 \qquad (\text{Joule})$$

This is the energy coming out of the propeller. There is, however energy (or power) loss across the motor and the propeller.

$$W_{B_{LF}} = 1.356 \frac{V\, W\, T_{LF}}{L/D\, \eta_p\, \eta_m\, e_B}$$

The Battery weight fraction for this Level flight mission segment is given by

$$\boxed{\frac{W_{B_{LF}}}{W} = \frac{1.356\, V\, T_{LF}}{L/D\, \eta_p\, \eta_m\, e_B}}$$

<u>For turning flight</u>

R = turn radius (ft)
ϕ = bank angle (rad)
$\phi = \tan^{-1}\left(\frac{V^2}{Rg}\right)$

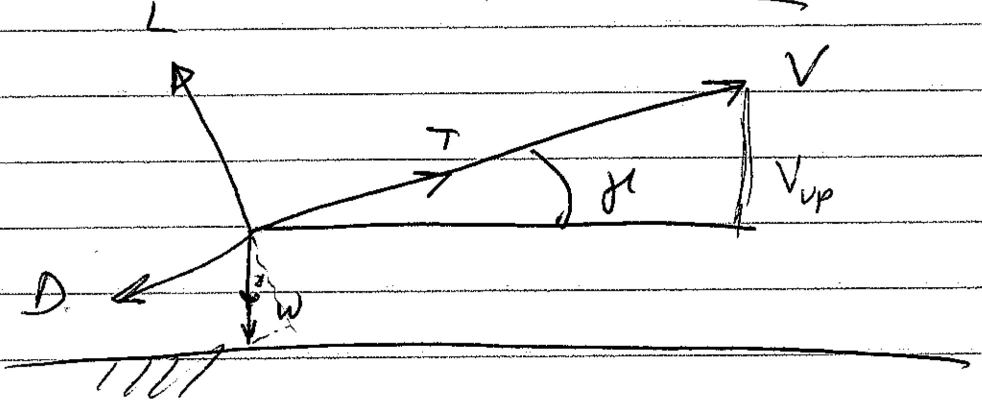
$g = 32.17$ ft/sec^2

In turning flight the aircraft pulls more g's. Therefore lift and drag go up, by a $1/\cos\phi$. The battery weight fraction for this case becomes

$$\frac{W_{B_{TU}}}{W} = \frac{1.356\, V\, T_{TU}}{L/D\, \eta_p\, \eta_m\, e_B}$$

where T_{TU} is the time (sec) in turning flight.

Climbing Flight



$L = W\cos\gamma \qquad\qquad +W\sin\gamma$

$W\sin\gamma + D = T = \frac{L}{L/D} = \frac{W\cos\gamma}{L/D} + W\sin\gamma$

Power $TV = \frac{VW\cos\gamma}{L/D} + VW\sin\gamma$

Energy $E_{cl} = TV\,T_{climb} = \frac{VW\cos\gamma}{L/D}\,T_{climb} + VW\sin\gamma\,T_{cli}$

$E_{CL} = VW\,T_{climb}\left(\frac{\cos\gamma}{L/D} + \sin\gamma\right)$ ft lbf

$E_{CL} = 1.356\,VW\,T_{climb}\left(\frac{\cos\gamma}{L/D} + \sin\gamma\right)$ Joule

$W_{B_{CL}} = \frac{1.356}{\eta_p\eta_m\rho_B}\,\frac{VW\,T_{climb}}{\cancel{E}}\left(\frac{\cos\gamma}{L/D} + \sin\gamma\right)$

Battery weight fraction for climb

$$\boxed{\frac{W_{B_{CL}}}{W} = \frac{1.356}{\eta_p\eta_m\rho_B}\,V\,T_{climb}\left(\frac{\cos\gamma}{L/D} + \sin\gamma\right)}$$

Time to climb to 20 feet

$V\sin\gamma = V_{up}\,T_{climb} = 20 \qquad T_{climb} = \frac{20}{V\sin\gamma}$

Take-off

The equations of motion for take-off can be integrated for a given wing loading and power loading. Combinations of W/S and W/hp that lead to successful take-offs can be determined. For each successful take off the battery weight fraction can be computed as follows

$$\underset{(\text{watts})}{\text{Motor output power}} = \underset{\left(\frac{\text{watts}}{\text{hp}}\right)}{745.7}\ \underset{(\text{hp})}{\overbrace{\text{motor output power}}^{\text{hp}}}$$

$$\underset{(\text{watts})}{\text{Battery Output power}} = \frac{\text{motor Output power (watts)}}{\text{Motor efficiency}}$$

Let T_{TO} denote time to take off

Battery energy to take off (E_{TO})

$$E_{TO} = T_{TO}\ \frac{745.7}{\eta_m}\ hp$$

Normalize by unknown aircraft weight

$$\frac{E_{TO}}{W} = T_{TO}\ \frac{745.7}{\eta_m}\ \boxed{\frac{hp}{W}}$$ inverse of power loading

Battery weight fraction for take-off

$$\boxed{\frac{W_{B_{TO}}}{W} = \frac{745.7\ T_{TO}}{\eta_m\ W/hp\ \rho_B}} \approx .002$$

The take2.m and Takeoff3.m scripts compute this.

WARM-UP $W_{B_{WU}}/W = N \cdot W_{B_{TO}}/W$ $N \approx 10$.

Summary

Battery Weight Fraction

$$\frac{W_B}{W} = \frac{W_{B_{TO}}}{W} + \frac{W_{B_{WU}}}{W} + \frac{W_{B_{CL}}}{W} + \frac{W_{B_{LF}}}{W} + \frac{W_{B_{TU}}}{W}$$

Weight of Battery

$$W_B = \left(\frac{W_B}{W}\right) W$$

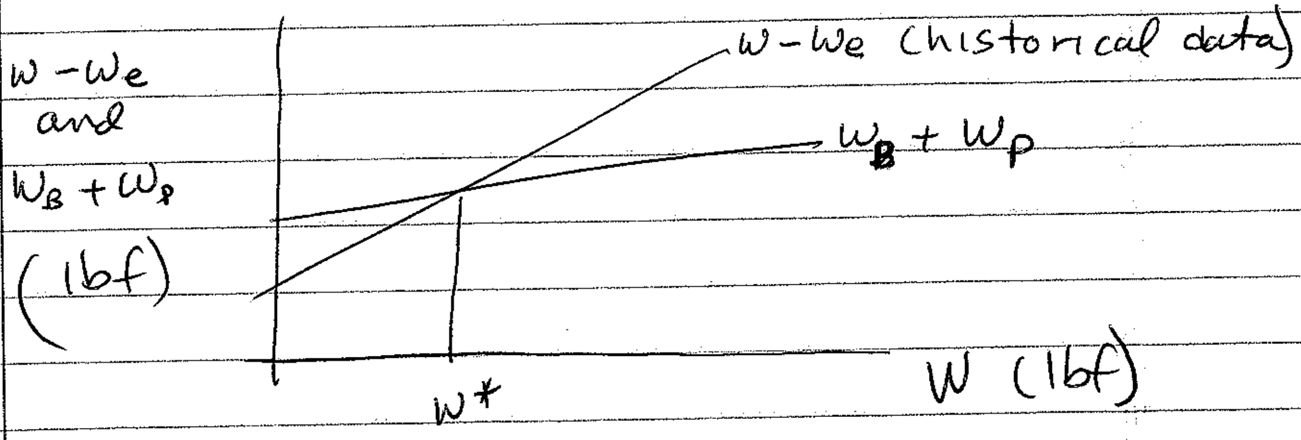
Weight of Battery + Payload

$$W_B + W_P$$

Weight of aircraft minus weight of empty aircraft from historical data

$$W - W_e$$

Preliminary weight estimate is the weight (W^*) where $W - W_e = W_B + W_P$



```
% FILE: Weight_3.m
% Preliminary weight estimator for electric powereed aircraft
% Revised 9/5/06
disp(' '); disp('>>>>>>>>>Start here<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<'); disp(' ')
disp(' This preliminary weight estimator is set up for an endurance mission.'); disp(' ');

% INPUTS
LoverDmax=12              % for fixed gear GA aircraft (Skyhawk) (See Raymer p. 22)
LoverD=.866*LoverDmax     % for loiter (See Raymer p. 22)
Vloiter=40                % ft/sec, Estimated loiter speed
ETAmotor=0.75
ETAprop= 0.60
Altitude=20                % altitude to fly at (ft)
R=50 % Turn radius at loiter from mission spec.
echo on
%RHOb=72900    % battery energy density for NiCad joule per pound
%RHOb=9.25E+04 % battery energy density for NiMH joule per pound
RHOb=2.39E+05 % battery energy density for Lithium polymer joule per pound
echo off
disp('Battery energy density, joules per pound')
EnduranceMIN=7
Wpayload=1 % payload weight pounds
g=32.17 % acceleration of gravity ft/sec^2
gamma=35/57.3 % climb angle (rad)
disp('From integration of eoms at takeoff, assume that the battery')
disp(' weight fraction is given below')
WtoperW=.002
disp('Warmup battery weight fraction is N times the takeoff weight fraction')
N=10 % Warmup battery weight fraction is N times the takeoff weight fraction
% END INPUTS

EnduranceSEC=EnduranceMIN*60
TimeLoiterStraight=EnduranceSEC/2   % Loiter time in straight flight (sec)
TimeLoiterTurn=EnduranceSEC/2       % Loiter time in turning flight (sec)

% For loiter in straight flight
WlsperW=Vloiter*1.356*TimeLoiterStraight/(ETAmotor*ETAprop*RHOb*LoverD)
```

```
% For loiter in turning flight
phi=atan(Vloiter*Vloiter/(R*g)) % bank angle in the turn (rad)
WltperW=Vloiter*1.356*TimeLoiterTurn/↙
(ETAmotor*ETAprop*RHOb*LoverD*cos(phi))

% For climbing flight
TimeClimb=Altitude/(Vloiter*sin(gamma)) % time to climb (sec)
WclimbperW=Vloiter*1.356*TimeClimb*(cos(gamma)/LoverD+sin(gamma))/↙
(ETAmotor*ETAprop*RHOb)

% For Takeoff
% See inputs
WtoperW

% For warm-up assume takeoff times aree about 3 sec and
% warm-up times are about 30 seconds.
disp('Assume that the warmup weight fraction is N times the ')
disp(' takeoff weight fraction.')
WwarmperW=N*WtoperW

% Assemble the complete battery weight fraction.
WbperW=WlsperW+WltperW+WclimbperW+WtoperW+WwarmperW

Weight=0:1:10; %weight in pounds
echo on
WminusWe=.2103*Weight+.1243; % formula for historical data↙
(pounds)
echo off
disp('NOTE: Your weight estimate will only be as good as the↙
historical')
disp(' data represented in the equation above')
Wbattery=WbperW*Weight;
WbplusWpay=Wbattery+Wpayload;
plot(Weight,WminusWe,Weight,WbplusWpay)
xlabel('Weight~lbf')
ylabel('W-We and Wb+Wp~lbf')

% Determination of aircraft weight
delta=WminusWe-WbplusWpay;
% YI = INTERP1(X,Y,XI)
Waircraft=interp1(delta,Weight,0);
y=.2103*Waircraft+.1243;
string1=['Estimated aircraft weight is ',num2str(Waircraft),'↙
pounds.'];
disp(' '); disp(string1)
text2(.25,.2,['            ',string1])
```

```
title('Weight estimation using historical weight data')
legend('Historical data','Estimated weight')
hold on; plot(Waircraft,y,'o'); hold off
Wb=WbperW*Waircraft;
string2=['Estimated battery weight is ',num2str(Wb),' pounds.'];
disp(string2)
text2(.25,.15,['          ',string2])
string2=['Payload weight is ',num2str(Wpayload),' pounds.'];
disp(string2)
text2(.25,.1,['          ',string2])
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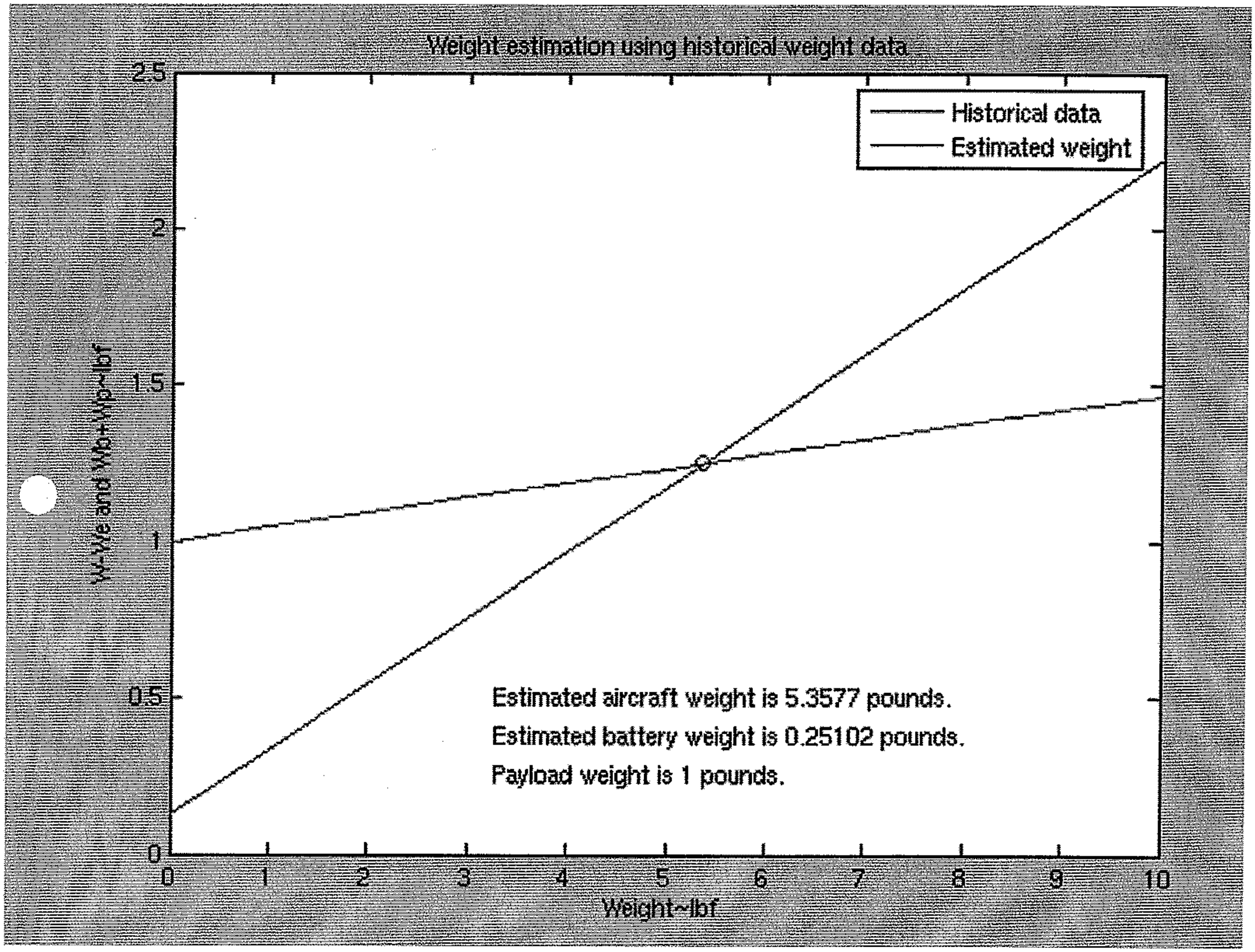
Weight estimation using historical weight data

Estimated aircraft weight is 5.3577 pounds.

Estimated battery weight is 0.25102 pounds.

Payload weight is 1 pounds.