Static Margin Requirements

For Small Remotely Piloted

Aircraft Based on the Masters Degree Thesis of Mark Peters (Purdue, Aero & Astro, May 1996)

AAE451 Fall 2008

Table 6.1 Definition of Airplane Classes

MIL–F–8785C	Examples	Civilian Equivalent	Examples
Class I Small, light airplanes such as: * Light utility * Primary trainer * Light observation	* Cessna T–41 * Beech T–34C * Rockwell OV–10A	Very Light Aircraft (VLA) and FAR 23 category airplanes	* Cessna 210 * Piper Tomahawk * Edgeley Optica
Class II Medium weight, low–to–medium maneuverability airplanes such as: * Heavy utility / search and rescue * Light or medium transport / cargo / tanker * Early warning / electronic counter–measures / airborne command, control or communications relay * Anti–submarine * Assault transport * Reconnaissance * Tactical Bomber * Heavy Attack * Trainer for Class II	* Fairchild C–26A/B * Fairchild C–123 * Grumman E–2C * Boeing E–3A * Lockheed S–3A * Lockheed C–130 * Fairchild OA–10 * Douglas B–60 * Grumman A–6 * Beech T–1A	FAR 25 category airplanes	* Boeing 737, * Airbus A 320 * McDD MD–80
Class III Large, heavy, low–to–medium maneuverability airplanes such as: * Heavy transport / cargo / tanker * Heavy bomber * Patrol / early warning / electronic counter–measures / airborne command, control or communications relay * Trainer for Class III	* McDD C–17 * Boeing B–52H * Lockheed P–3 * Boeing E–3D * Boeing TC–135	FAR 25 category airplanes	* Boeing 747, * Airbus 340, * McDD MD–11
Class IV High maneuverability airplanes such as: * Fighter / interceptor * Attack * Tactical reconnaissance * Observation * Trainer for Class IV	* Lockheed F–22 * McDD F–15E * McDD RF–4 * Lockheed SR–71 * Northrop T–38	FAR 23 aerobatic category airplanes	* Pitts Special, * Sukhoi Su–26M

Table 6.2 Definition of Flight Phase Categories

MIL–F–8785C	Suggested Civilian Equivalent: VLA, FAR 23 and FAR 25
Non–terminal Flight Phases	
Category A: Those non–terminal flight phases that require rapid maneuvering, precision tracking or precise flight path control. Included in this category are:	
a) Air–to–air combat (CO)	None
b) Ground attack (GA)	None
c) Weapon delivery/launch (WD)	None
d) Aerial recovery (AR)	None
e) Reconnaissance (RC)	Observation, Pipeline spotting and monitoring
f) In–flight refuelling (receiver) (RR)	None as yet
g) Terrain following (TF)	None
h) Anti–submarine search (AS)	Fish spotting
i) Close formation flying (FF)	Air–show demonstrations
Category B: Those non–terminal flight phases that are normally accomplished using gradual maneuvers and without precision tracking, although accurate flight–path control may be required. Included in this category are:	
→ a) Climb (CL) ←	Various climb segments
→ b) Cruise (CR) ←	Various cruise segments
→ c) Loiter (LO) ←	Flight in holding pattern
d) In–flight refuelling (tanker) (RT)	None as yet
e) Descent	Various descent segments
f) Emergency descent (ED)	Emergency descent
g) Emergency deceleration (DE)	None
h) Aerial delivery (AD)	Parachute drop
Terminal Flight Phases	
Category C: Terminal flight phases are normally accomplished using gradual maneuvers and usually require accurate flight path control. Included in this category are:	
a) Takeoff (TO)	Various takeoff segments
b) Catapult takeoff (CT)	None
c) Approach (PA)	Various approach segments
d) Wave–off / go–around (WO)	Aborted approach
e) Landing (L)	Various landing segments

Table 6.8 Flight Path Requirements

MIL–F–8785C:		FAR 23, FAR 25, VLA:
Level I:	$d\gamma/dV_P \leq 0.06$deg/knot	No requirement
Level II:	$d\gamma/dV_P \leq 0.15$deg/knot	No requirement
Level III:	$d\gamma/dV_P \leq 0.24$deg/knot	No requirement

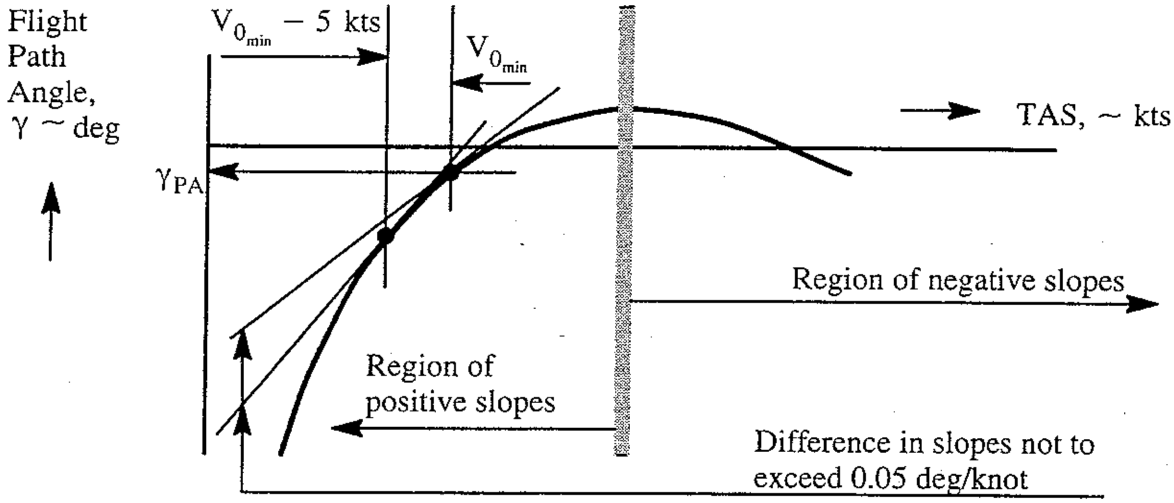


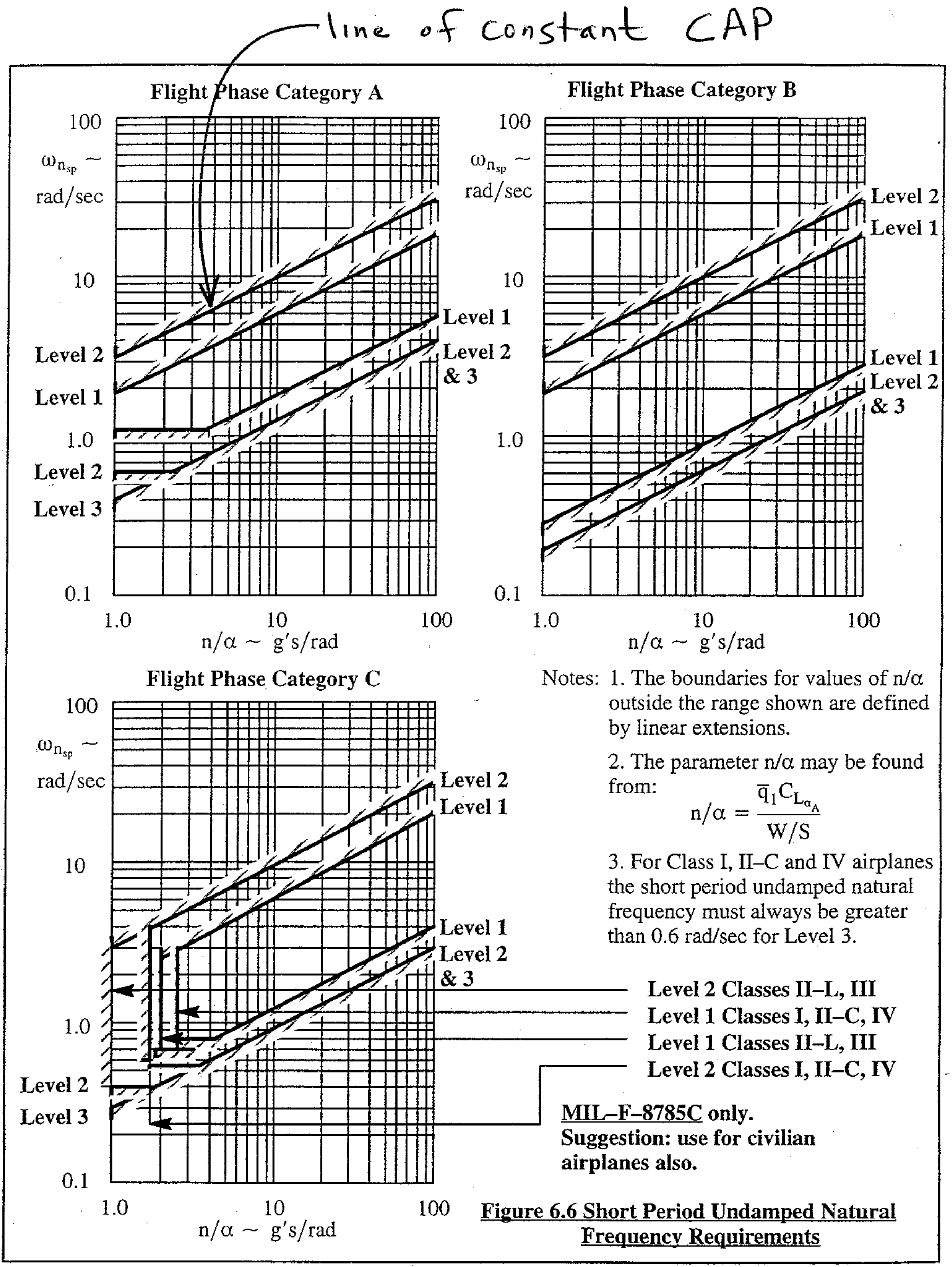
Figure 6.5 Illustration of Flight Path Stability Requirement

The speed, $V_{0_{min}}$ is defined as the minimum operating speed of the airplane during final approach. For military aircraft that speed is typically $1.15V_{s_{PA}}$ for carrier–based aircraft and $1.20V_{s_{PA}}$ for land–based aircraft. For civilian aircraft that speed is typically $1.30V_{s_{PA}}$.

Flight path stability may be predicted with the generalized trim analysis method presented in Section 4.6. The slope $d\gamma/dV_P$ is a component of the r.h.s. matrix in Eqn (4.232).

6.3.4 SHORT PERIOD FREQUENCY AND DAMPING

MIL–F–8785C requires the (**equivalent**) short period undamped natural frequency, $\omega_{n_{sp}}$, of the short period mode to be within the limits shown in Figure 6.6 for three Flight Phase Categories. Although the FAR/VLA requirements do not set specific limits on $\omega_{n_{sp}}$, common design practice is to adopt the military requirements. Reference 6.2 is a recent replacement specification for MIL–F–8785C of Reference 6.1. In MIL–STD–1797A, there appears a requirement for a so–called Control Anticipation Parameter (CAP). This parameter and its relationship to airplane maneuver margin is discussed in Sub–section 6.3.5.



Notes: 1. The boundaries for values of n/α outside the range shown are defined by linear extensions.

2. The parameter n/α may be found from:

$$n/\alpha = \frac{\overline{q}_1 C_{L_{\alpha_A}}}{W/S}$$

3. For Class I, II–C and IV airplanes the short period undamped natural frequency must always be greater than 0.6 rad/sec for Level 3.

MIL–F–8785C only.
Suggestion: use for civilian airplanes also.

Figure 6.6 Short Period Undamped Natural Frequency Requirements

The (**equivalent**) short period damping ratio, ζ_{sp} , must be within the limits presented in Table 6.9. For airplanes which do not require stability augmentation systems to meet the requirements of Figure 6.6 and Table 6.9 the word **equivalent** should be omitted.

The FAR/VLA requirements of References 6.3 and 6.4 merely require the short period oscillation to be heavily damped. It is considered good design practice to use the military requirement for civilian airplanes.

The word '**equivalent**' refers to highly augmented airplanes **only**. In such airplanes, an 'equivalent' short period frequency is achieved with the help of a feedback system. The dynamic characteristics of the feedback system (including its actuator dynamics, sensor dynamics and computational delays) give rise to the term 'equivalent' frequency.

Table 6.9 Short Period Damping Ratio Limits
MIL–F–8785C

	Category A and C Flight Phases			Category B Flight Phases		
Level	Minimum		Maximum	Minimum		Maximum
Level 1*	0.35	$\leftarrow \zeta_{sp} \rightarrow$	1.30	0.30	$\leftarrow \zeta_{sp} \rightarrow$	2.00
Level 2	0.25	$\leftarrow \zeta_{sp} \rightarrow$	2.00	0.20	$\leftarrow \zeta_{sp} \rightarrow$	2.00
Level 3	0.15 * *	$\leftarrow \zeta_{sp} \rightarrow$	no maximum	0.15 *	$\leftarrow \zeta_{sp} \rightarrow$	no maximum

* For VLA, FAR 23 and FAR 25 : ζ_{sp} must be heavily damped

** For altitudes above 20,000 ft this requirement may be reduced if approved by the procuring activity

It is seen in Table 6.9 that damping ratios larger than 1.0 are admitted. A damping ratio larger than 1.0 indicates that the short period mode has degenerated into two, stable real roots.

The short period damping requirements apply with the cockpit–flight–controls–fixed and with the cockpit–flight–controls–free. The controls–fixed case applies to airplanes with irreversible flight control systems as well as to airplanes with reversible flight controls, while the pilot keeps the cockpit controllers in a fixed position. The controls–free case applies to airplanes with reversible flight controls, while the pilot does not touch the corresponding cockpit controls. In the latter case, the oscillatory characteristics of the freely oscillating flight control system must be accounted for. The methods of Chapter 5 deal only with the controls–fixed case. Appendix D contains a mathematical model which accounts for freely oscillating flight controls: controls–free.

Figure 6.7 illustrates the significance of the short period flying quality requirements in the s–plane. The designer must see to it that the short period poles are located in the allowable areas.

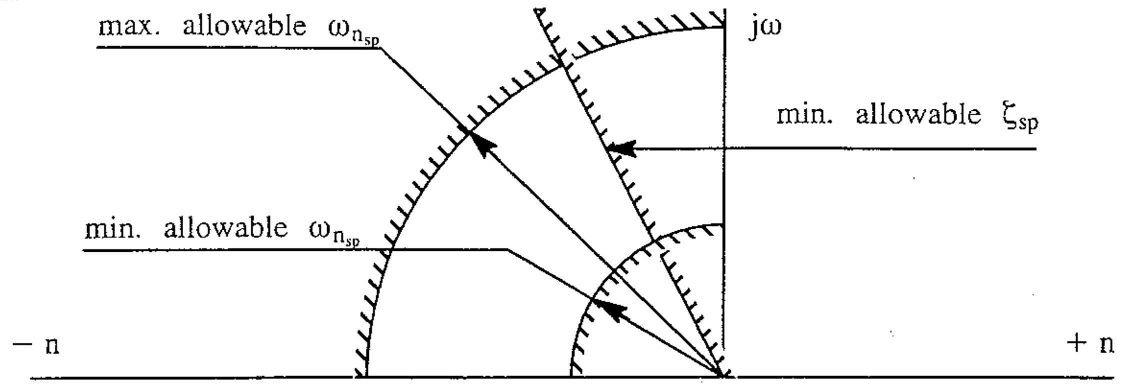


Figure 6.7 Acceptable Areas in the s–Plane for Short Period Roots

6.3.5 CONTROL ANTICIPATION PARAMETER

MIL–STD–1797A (Reference 6.2) contains a requirement that airplanes must stay within a minimum and maximum range of values of the so–called Control Anticipation Parameter (CAP) over a range of allowable short period damping ratios. For highly augmented airplanes, this requirement has in fact replaced the short period undamped natural frequency and damping ratio requirements of Figure 6.6 and Table 6.9. For non–augmented airplanes the author recommends continued use of Figure 6.6 and Table 6.9.

In preliminary design it is acceptable to use the following equation to estimate the control anticipation parameter (CAP):

$$\mathrm{CAP} = \frac{\omega_{n_{sp}}{}^2}{n_\alpha} \qquad \frac{\left(\frac{rad}{sec}\right)^2}{g/rad} = \frac{rad^2}{sec^2}\,\frac{rad}{g} = \frac{1}{sec^2} \tag{6.3}$$

where: $\omega_{n_{sp}}$ is the undamped natural frequency of the short period mode

$n_\alpha = \partial n/\partial\alpha$ which is also referred to as the gust– or load–factor–sensitivity of an airplane.

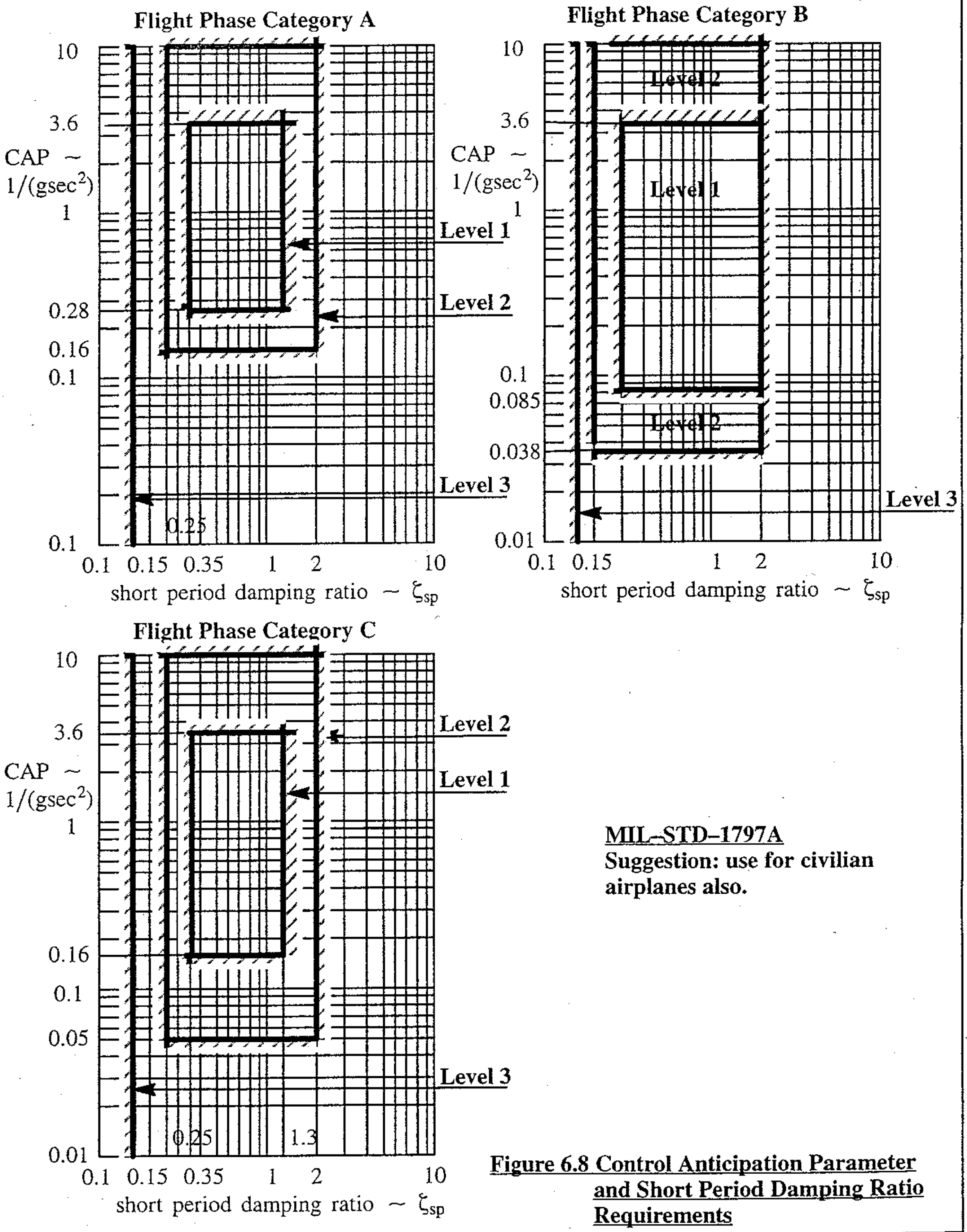
Figure 6.8 shows how allowable CAP–values are related to the short period damping ratio for various categories of Flight Phases and to handling quality Levels. It is shown next, that the CAP is mathematically related to the following quantities:

- * maneuver margin (See: Section 4.3)
- * overall airplane length
- * wing m.g.c.
- * dimensionless radius of gyration about the Y–axis.

According to Eqn (5.61), the following approximation holds for $\omega_{n_{sp}}{}^2$:

$$\omega_{n_{sp}}{}^2 = \frac{Z_\alpha M_q}{U_1} - M_\alpha \tag{6.4}$$

By partially differentiating Eqn (4.91) with respect to angle–of–attack it is found that:



MIL-STD-1797A
Suggestion: use for civilian airplanes also.

Figure 6.8 Control Anticipation Parameter and Short Period Damping Ratio Requirements

$$n_\alpha \approx \frac{\bar{q}_1 C_{L_\alpha}}{(W/S)} \qquad \frac{lbf}{ft^2}\,\frac{ft^2}{lbf} = ND \Rightarrow \left.\frac{g}{rad}\right) \text{— both ND} \tag{6.5}$$

The dimensional derivatives in Eqn (6.4) are defined in Table 5.1 as:

$$Z_\alpha \approx \frac{-\bar{q}_1 S C_{L_\alpha}}{m} \tag{6.6}$$

$$M_q = \frac{\bar{q}_1 S \bar{c}^2 C_{m_q}}{2 I_{yy} U_1} \tag{6.7}$$

$$M_\alpha \approx \frac{\bar{q}_1 S \bar{c} C_{m_\alpha}}{I_{yy}} \tag{6.8}$$

It should be recalled from Eqn (3.39) that the dimensionless derivatives C_{m_α} and C_{L_α} are related to the non–dimensional distance between airplane c.g. and a.c. in the following manner:

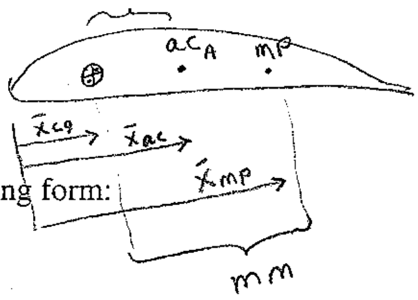
$$C_{m_\alpha} = C_{L_\alpha}(\bar{x}_{cg} - \bar{x}_{ac_A}) \tag{6.9}$$

By substituting Eqns (6.4) through (6.9) in Eqn (6.3) and by rearranging, the reader is asked to show that:

$$CAP = \frac{W\bar{c}}{I_{yy}}\underbrace{\Big(\overbrace{-\bar{x}_{cg} + \bar{x}_{ac_A}}^{sm} \underbrace{- \frac{g\varrho S\bar{c}C_{m_q}}{4W}}_{x_{mp}}\Big)}_{mm} \tag{6.10}$$

From Chapter 4, Eqn (4.121) it is recognized that the maneuver point of an airplane can be written as follows:

$$\bar{x}_{MP} = \bar{x}_{ac_A} - \frac{g\varrho S\bar{c}C_{m_q}}{4W} \tag{6.11}$$



Therefore, Eqn (6.10) can be cast in the following form:

$$CAP = \frac{W\bar{c}}{I_{yy}}(\overbrace{\bar{x}_{MP} - \bar{x}_{cg}}) = \frac{W\bar{c}}{I_{yy}}MM \tag{6.12}$$

where: MM is the so–called maneuver margin of an airplane.

The pitching moment of inertia, I_{yy}, is related to the following airplane design parameters:

- weight, W
- overall length,
- non–dimensional radius of gyration, $\bar{R}_y$
- mean geometric chord, $\bar{c}$

in accordance with:

$$I_{yy} = \left(\frac{L^2 W \overline{R}_y^2}{4g}\right) \tag{6.13}$$

By combining Eqns (6.12) and (6.13) it is found that:

$$CAP = \frac{4\bar{c}gMM}{L^2 \overline{R}_y^2} \tag{6.14}$$

The minimum and maximum allowable CAP values from MIL–STD–1797A can therefore be translated into a minimum and maximum allowable maneuver margin for any airplane, as long as airplane geometric size (as expressed by $\bar{c}$ and L) and its longitudinal mass distribution (as expressed by $\overline{R}_y$) are known. Equation (6.12) suggests that for very large airplanes the minimum acceptable maneuver margin will increase relative to that required for smaller airplanes to maintain some minimum acceptable CAP value.

6.4 LATERAL–DIRECTIONAL FLYING QUALITY REQUIREMENTS

The lateral–directional flying quality requirements of References 6.1 through 6.3 have been copied and included verbatim as Appendices A and B in Part VII of Reference 6.5. This section contains a summary of only those requirements which should be addressed during the early stages of design analysis of new airplanes. The reader should consult the actual regulations before considering a flying quality assessment to be completed. In several instances, when the FAR/VLA regulations do not contain specific requirements a statement of 'Civilian Equivalent' is included. The author suggests these civilian equivalents requirements as sound design practice only.

6.4.1 LATERAL–DIRECTIONAL CONTROL FORCES

The lateral–directional control force requirements, which are summarized here, apply to airplanes equipped with conventional stick– or wheel–type cockpit controllers. The regulations deal with many specific situations which are summarized as follows:

* 6.4.1.1 Roll control forces
* 6.4.1.2 Directional control forces with asymmetric loadings
* 6.4.1.3 Directional and roll control forces with one engine inoperative

6.4.1.1 Roll Control Forces

The stick or wheel control forces required to obtain the roll performance of Sub–section 6.4.6 may not be greater than those listed in Table 6.10, nor may these forces be less than the control system break–out forces plus:

for Level 1:	1/4 the values of Table 6.10
for Level 2:	1/8 the values listed in Table 6.10
for Level 3:	zero.

DEVELOPMENT OF A LIGHT UNMANNED AIRCRAFT FOR THE DETERMINATION OF FLYING QUALITIES REQUIREMENTS

A Thesis
Submitted to the Faculty

of

Purdue University

by

Mark E. Peters

In Partial Fulfillment of the
Requirements for the Degree

of

Master of Science

May 1996

Seems to drag its tail. Gentle maneuvers required. Aicraft lost in crash during this flight. Pilot #1 rating: 4. $w_{n_{sp}} = 9.87$ $\zeta_{sp} = 0.7894$ $T_{\theta_2} = .1321$ $n_\alpha = 18.81$

11.4 The Determination of Level 1 Boundaries for Light Unmanned Aircraft

This section outlines boundaries for level 1 flying qualities requirements for light unmanned aircraft. The first data analysis compares the pilot ratings collected to the current manned flying qualities requirements. This is done by over plotting the flight test data on the charts from chapter 9. Only Category B flight phases are considered.

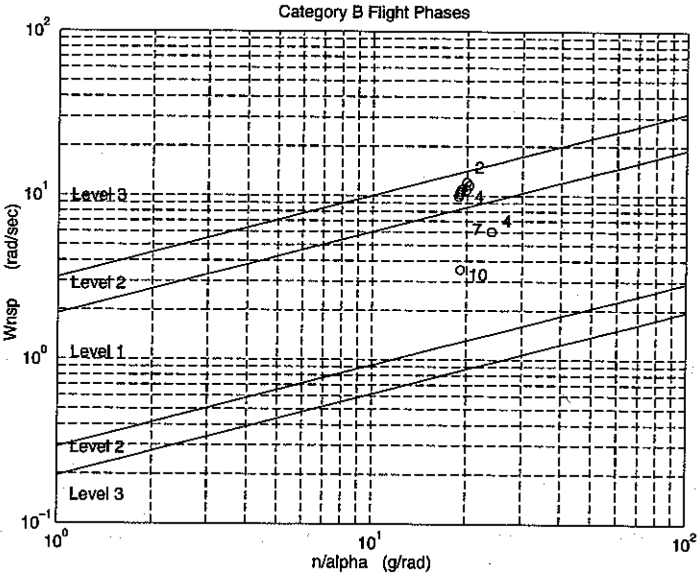


Figure 11.1 Category B $w_{n_{sp}}$ and n_α Requirements

From the data shown it can be clearly seen that the pilot of the light unmanned aircraft wanted much faster dynamics than what was necessary larger manned aircraft. Figure 11.1 shows $w_{n_{sp}}$ vs n_α. The flight data was clustered in the upper right half corner. This data cluster straddles the lower boundary between level 1 and level 2 flying qualities. Points in this cluster either scored a pilot rating of 2 or 4. One data point, the 8oz ballast condition, is of particular interest. ($w_{n_{sp}} = 10.7 rad/sec$ $n_\alpha = 19.1 g/rad$). This point scored a pilot rating of 2 and 4 during two different tests. The reason for the difference is attributed to the amount of turbulence present. The pilot rating of 2 was assigned during a calm period. During this test a 12oz

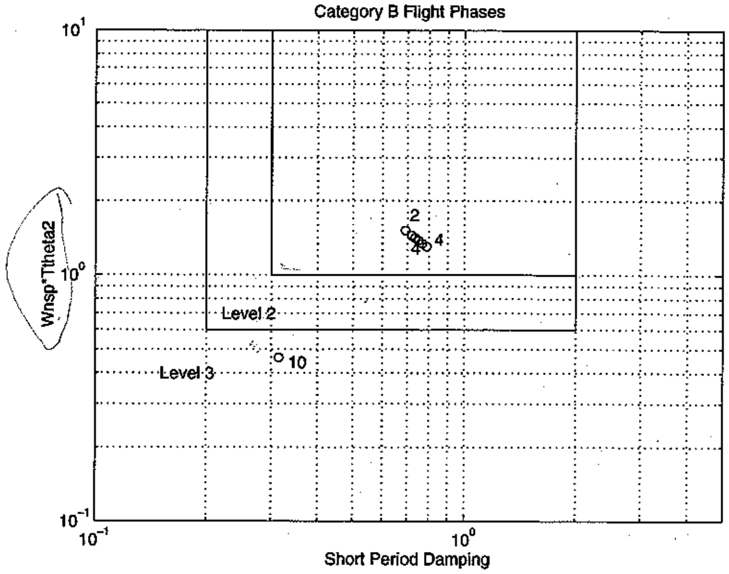


Figure 11.2 Category B ζ_{sp} Requirements

ballast weight was required to score a pilot rating of 4. Points above 8oz always were rated at level 1 regardless of turbulence. Because the 8 oz position seems to straddle the two boundaries, it is assigned as the dividing point between level 1 and level 2 flying qualities.

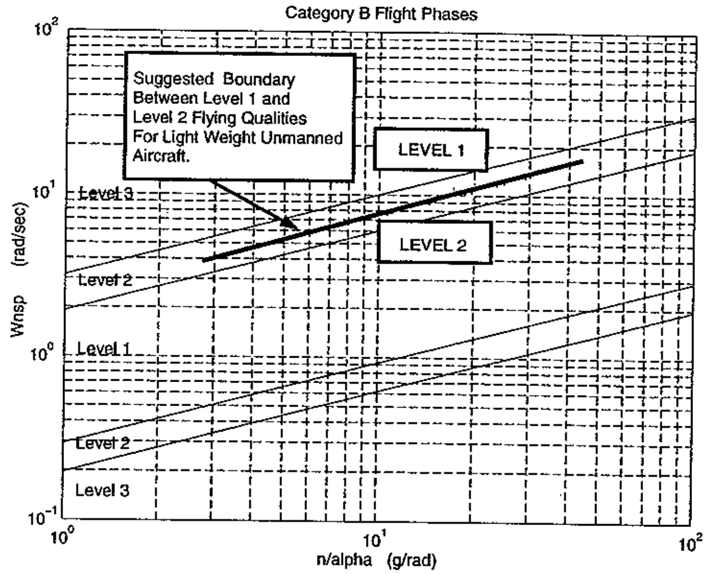
Using the 8oz loading point as a divider it is possible to draw a new boundary. The boundaries on fig 11.1 are drawn using a constant value for the ratio of the natural frequency squared divided by the load factor gradient, $\frac{w_{n_{sp}}^2}{n_\alpha}$. The manned aircraft lower boundary for Level 1 flying qualities uses 0.085 for $\frac{w_{n_{sp}}^2}{n_\alpha}$. From the 8 oz loading condition, a value of 5.92 is calculated for $\frac{w_{n_{sp}}^2}{n_\alpha}$. A new line can then be defined.

$$\frac{w_{n_{sp}}^2}{n_\alpha} = 5.92 \qquad [11.1]$$

$$log \frac{w_{n_{sp}}^2}{n_\alpha} = 2\ log\ w_{n_{sp}} - log\ n_\alpha = 0.77 \qquad [11.2]$$

$$log\ w_{n_{sp}} = \frac{1}{2}\ log\ n_\alpha + \frac{0.77}{2} \qquad [11.3]$$

Figure 11.3 shows that the light aircraft Level 1 flying qualities overlap the upper Level 2 and Level 3 flying qualities for the manned aircraft.



← lines of constant CAP

Figure 11.3 Category B Proposed Light Unmanned Aircraft Requirements

Figure 11.2 show the relationship between $w_{n_{sp}}T_{\theta_2}$ and ζ_{sp}. From figure 11.2 it can be seen that the data points seem to straddle a boundary for Level 1 and Level 2 flying qualities. This suggests that the boundary for $w_{n_{sp}}T_{\theta_2}$ should be shifted upward to 1.38 from 1.0. Figure 11.4 shows this boundary drawn. Again, the 8 oz ballast condition is used as the Level 1 / Level 2 border.

Note that none of the data points clustered around the 8oz flight condition demonstrate a damping ratio which is anything out of the ordinary. All points fit well within the Category B ζ_{sp} requirements. This is unfortunate because no precise conclusion can be made about the range of acceptable damping ratio.

11.5 The Exploration of Level 2 Boundaries for Light Unmanned Aircraft

There is only one data point which could be used as a divider between Level 1 and Level 2 flying qualities. It represents the flight condition where destabilizing rate feedback was used without any ballast. It was fortunate that both pilots had the opportunity to rate this flight condition. While pilot #1 rated it as a 4, pilot # 2

```
disp(' '); disp('Start Here <<<<<<<<<<<<<<<<<<<<<<<<'); format compact
disp('INPUTS')
aircraft='MPX5'     % Name of the aircraft
W = 19.2            % Weight of Airplane [lbf]
S_w = 1350/144      % Surface area of wing [ft^2]
altitude= 607       % Trim altitude [ft]
c_w = 15/12         % Mean aerodynamic chord of the wing [ft]
Iyy = 1.10          % Airplane moment of inertia about y-axis [slug-ft^2]
g=32.17             % accel of gravity (ft/sec^2)
Xref = 0.25*c_w     % Distance from the leading edge of the wing mean↙
aerodynamic chord
% to the arbitrary moment reference point. The equivalent force system
% for the aerodynamic force system is given about this point.
% Measured as positive aft, starting from the leading edge of the mean aero.↙
chord. [ft]
Xcg =  0.25*c_w        % Distance from the leading edge of the wing mean↙
aerodynamic chord
% to the center of gravity.
% Measured as positive aft, starting from the leading edge of the mean aero.↙
chord. [ft]
% CL_alpha
CL_alpha= 4.84
% Cm_alpha where Xref is the moment reference point.
Cm_alpha= -1.13
% Cm_q  where Xref is the moment reference point.
Cm_q= -11.9


disp(' '); disp('OUTPUTS')
rho=rhofun(altitude)             % Air density at altitude [slug/ft^3]
Xbarcg=Xcg/c_w;;   % XbarCG,  nondimensional, measured aft from leading edge↙
of wing mean aerodynamic chord.
%Xbarcg=.15
Xbarref=Xref/c_w;  % XbarRef, nondimensional, measured aft from leading edge↙
of wing mean aerodynamic chord.
Xbarac=-Cm_alpha/CL_alpha+Xbarref;
StaticMargin=Xbarac-Xbarcg;
stringA=['C.G. location,               Xbarcg= ',num2str(Xbarcg),' (fraction↙
of chord)'];
stringB=['Aerodynamic center location, Xbarac= ',num2str(Xbarac),' (fraction↙
of chord)'];
stringC=['Static Margin      (Xbarac-Xbarcg) = ',num2str(StaticMargin),'↙
(fraction of chord)'];
disp(stringA); disp(stringB); disp(stringC);
string4=['Typically 0.05 to 0.50 of the reference chord.'];
disp(string4)
disp('NOTE: static margin above is relative the the c.g.')

disp('See Roskam 421 book pages 431-434 for discussion of Control↙
Anticipation Parameter (CAP).')
ManeuverMargin=StaticMargin-g*rho*S_w*c_w*Cm_q/(4*W)   % in fractions of↙
wing chord (eqn 6.10 p. 433)
% StickFixedNeutralPoint=Xbarcg+StaticMargin % Same as Aerodynamic center
ManeuverPoint=Xbarac-g*rho*S_w*c_w*Cm_q/(4*W)   %Eqn 6.11 p.433
CAP=W*c_w*ManeuverMargin/Iyy
```

```
MinimumCAP=5.92 % This is the requirement from Mark Peters' thesis
disp(' ')
disp('According to Mark Peters'' MS thesis Level 1 flying qualities ')
disp(['for small model airplanes requires that CAP>',num2str↙
(MinimumCAP),'.']);
disp('For this to happen the static margin (SM) must be greater than or↙
equal to the following')
SMminimum=MinimumCAP*Iyy/(W*c_w)+g*rho*S_w*c_w*Cm_q/(4*W)  % in fractions of↙
wing chord
disp('and the cg must be forward of the following point')
disp('(expressed in fractions of the Cbar meaured aft from the leading edge↙
of the wing).')
MostAftCG=Xbarac-SMminimum
if(CAP>MinimumCAP)
    disp('This aircraft meets Level 1 flying qualities for CAP')
else
    disp('This aircraft does not meet Level 1 flying qualities for CAP.')
end
```

```
Start Here <<<<<<<<<<<<<<<<<<<<<<<<
INPUTS
aircraft =
MPX5
W =
   19.2000
S_w =
    9.3750
altitude =
   607
c_w =
    1.2500
Iyy =
    1.1000
g =
   32.1700
Xref =
    0.3125
Xcg =
    0.3125
CL_alpha =
    4.8400
Cm_alpha =
   -1.1300
Cm_q =
  -11.9000

OUTPUTS
rho =
    0.0023
C.G. location,                Xbarcg= 0.25 (fraction of chord)
Aerodynamic center location, Xbarac= 0.48347 (fraction of chord)
Static Margin      (Xbarac-Xbarcg) = 0.23347 (fraction of chord)
Typically 0.05 to 0.50 of the reference chord.
NOTE: static margin above is relative the the c.g.
See Roskam 421 book pages 431-434 for discussion of Control Anticipation↙
Parameter (CAP).
ManeuverMargin =
    0.3699
ManeuverPoint =
    0.6199
CAP =
    8.0698
MinimumCAP =
    5.9200

According to Mark Peters' MS thesis Level 1 flying qualities
for small model airplanes requires that CAP>5.92.
For this to happen the static margin (SM) must be greater than or equal to↙
the following
SMminimum =
    0.1349
and the cg must be forward of the following point
(expressed in fractions of the Cbar meaured aft from the leading edge of the↙
wing).
MostAftCG =
```

```
    0.3485
This aircraft meets Level 1 flying qualities for CAP
```

Mark Peters CAP Reqt.

$C_{m_\alpha} = -1.13$ /rad p 37

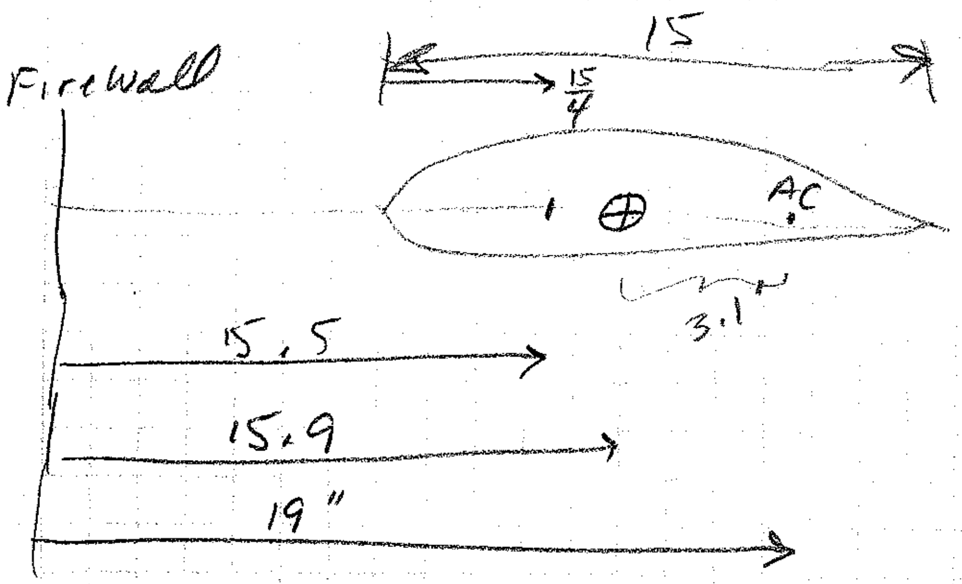
$C_{L_\alpha} = 4.84$ /rad "

X_{cg} = ? 15.9"

$X_{AC_{Aircraft}}$ = 19"

$X_{c/4}$ = 15.5"

C = 15"



$C_{m_\alpha} = -C_{L_\alpha}$ (Static Margin)

Static Margin $= -\frac{C_{m_\alpha}}{C_{L_\alpha}} = - -\frac{1.13}{4.84} = .2335$

<u>23%c</u>

$$\begin{array}{ll} X_{ac} & 19.0 \\ X_{cg} & \underline{15.9} \\ & 3.1 \end{array}$$

$$\frac{3.1}{15} = .2067 \longleftrightarrow .2335$$

Level 1 - 2 boundary

$$\text{CAP} = \frac{\omega_{n_{sp}}^2}{n_\alpha} = 5.92 = \left(\frac{\text{rad}}{\text{sec}}\right)^2 \frac{\text{rad}}{g} = \frac{\text{rad}^3}{g\ \text{sec}^2} = \frac{1}{g\ \text{sec}^2}$$

Peters p 153

↗ Roskam 421 p 431

Level 1 Region

$$\frac{\omega_{n_{sp}}}{n_\alpha} > 5.92$$

$\omega_{n_{sp}}$ ← rad/sec

n_α } $\frac{g}{\text{rad}}$

Roskam 421 p 334

$$\boxed{\omega_{n_{sp}}} \approx \sqrt{\frac{Z_\alpha M_q}{U_1} - M_\alpha} \approx \sqrt{-M_\alpha} = \sqrt{-\frac{C_{m_\alpha} \bar{q} S\bar{c}}{I_{yy}}}$$

$$n_\alpha = \frac{\bar{q}\, C_{L_\alpha}}{W/S}$$

stick fixed Neutral pt ↓

$$\boxed{\text{CAP}} = \frac{W \bar{c}}{I_{yy}} \Big(\underbrace{-\bar{X}_{cg} + \bar{X}_{ac_A}}_{\text{static margin}} - \underbrace{\frac{g \rho S \bar{c} C_{m_q}}{4W}}_{\approx .05} \Big)$$

Can be ignored

ignore C_{m_q}

$$CAP = \frac{W\bar{c}}{I_{yy}}\left(sm\right)$$

$$5.92 =$$

$$sm = \frac{I_{yy}}{W\bar{c}}\left(\overset{5.92}{CAP}\right)$$

$I_{yy} = 1.1$ slug ft
p 214

$W = 19.2$ lbf

$$= \frac{1.1}{(19.2)\left(\frac{15}{12}\right)}\ 5.92$$

$$12\times(.003819)(5.92) = (.0226)\ 12 = \underline{\underline{.2713}}$$

For Level 1 FQ
sm > .27

$$S = \frac{1350\ in^2}{144\ in^2/ft^2} = \quad ft^2$$

$\rho = .002378$ slug/ft³

$g = 32.2$ ft/sec²

$\bar{c} = \frac{15}{12}$ ft

-11.91 p 96

$C_{m_q} = -10.57$
p 34

$$CAP = \frac{W\bar{c}}{I_{yy}}\left(\underbrace{\underbrace{\left[\bar{X}_{ac_A} - \bar{X}_{cg}\right]}_{SM} - \frac{g\rho S\bar{c}}{4W}C_{m_q}}_{MM}\right)$$

$$CAP = \frac{W\bar{c}}{I_{yy}}MM$$

$$MM = \frac{I_{yy}}{W\bar{c}}CAP = SM - \frac{g\rho S\bar{c}}{4W}C_{m_q}$$

$$SM = \frac{I_{yy}}{W\bar{c}}CAP + \frac{g\rho S\bar{c}}{4W}C_{m_q}$$

According to Mark Peters For Level 1 Flying Qualities $CAP \geq 5.92$

To meet this requirement

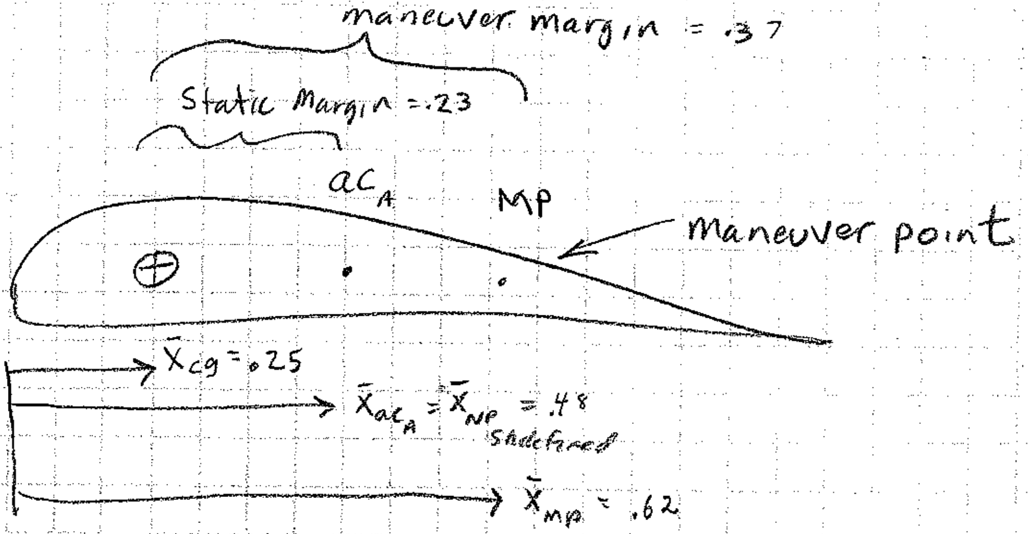
$$SM \geq \frac{I_{yy}}{W\bar{c}}(5.92) + \frac{g\rho S\bar{c}}{4W}C_{m_q}$$

$SM = \bar{X}_{ac_A} - \bar{X}_{cg}$

S

$\bar{X}_{cg} = \bar{X}_{AC} - SM_{min}$

421 F07



.48
.13
.35