

Static Margin Requirements
for small Remotely Piloted
aircraft based on the
Masters Degree Thesis
of Mark Peters

AAE 451, Fall 2008

Table 6.1 Definition of Airplane Classes

MIL-F-8785C	Examples	Civilian Equivalent	Examples
<p>Class I Small, light airplanes such as:</p> <ul style="list-style-type: none"> * Light utility * Primary trainer * Light observation 	<ul style="list-style-type: none"> * Cessna T-41 * Beech T-34C * Rockwell OV-10A 	<p>Very Light Aircraft (VLA) and FAR 23 category airplanes</p>	<ul style="list-style-type: none"> * Cessna 210 * Piper Tomahawk * Edgeley Optica
<p>Class II Medium weight, low-to-medium maneuverability airplanes such as:</p> <ul style="list-style-type: none"> * Heavy utility / search and rescue * Light or medium transport / cargo / tanker * Early warning / electronic counter-measures / airborne command, control or communications relay * Anti-submarine * Assault transport * Reconnaissance * Tactical Bomber * Heavy Attack * Trainer for Class II 	<ul style="list-style-type: none"> * Fairchild C-26A/B * Fairchild C-123 * Grumman E-2C * Boeing E-3A * Lockheed S-3A * Lockheed C-130 * Fairchild OA-10 * Douglas B-60 * Grumman A-6 * Beech T-1A 	<p>FAR 25 category airplanes</p>	<ul style="list-style-type: none"> * Boeing 737, * Airbus A 320 * McDD MD-80
<p>Class III Large, heavy, low-to-medium maneuverability airplanes such as:</p> <ul style="list-style-type: none"> * Heavy transport / cargo / tanker * Heavy bomber * Patrol / early warning / electronic counter-measures / airborne command, control or communications relay * Trainer for Class III 	<ul style="list-style-type: none"> * McDD C-17 * Boeing B-52H * Lockheed P-3 * Boeing E-3D * Boeing TC-135 	<p>FAR 25 category airplanes</p>	<ul style="list-style-type: none"> * Boeing 747, * Airbus 340, * McDD MD-11
<p>Class IV High maneuverability airplanes such as:</p> <ul style="list-style-type: none"> * Fighter / interceptor * Attack * Tactical reconnaissance * Observation * Trainer for Class IV 	<ul style="list-style-type: none"> * Lockheed F-22 * McDD F-15E * McDD RF-4 * Lockheed SR-71 * Northrop T-38 	<p>FAR 23 aerobatic category airplanes</p>	<ul style="list-style-type: none"> * Pitts Special, * Sukhoi Su-26M

Table 6.2 Definition of Flight Phase Categories

MIL-F-8785C

Suggested Civilian Equivalent:
VLA, FAR 23 and FAR 25

Non-terminal Flight Phases

Category A: Those non-terminal flight phases that require rapid maneuvering, precision tracking or precise flight path control.

Included in this category are:

- | | |
|---|---|
| a) Air-to-air combat (CO) | None |
| b) Ground attack (GA) | None |
| c) Weapon delivery/launch (WD) | None |
| d) Aerial recovery (AR) | None |
| e) Reconnaissance (RC) | Observation, Pipeline spotting and monitoring |
| f) In-flight refuelling (receiver) (RR) | None as yet |
| g) Terrain following (TF) | None |
| h) Anti-submarine search (AS) | Fish spotting |
| i) Close formation flying (FF) | Air-show demonstrations |

Category B: Those non-terminal flight phases that are normally accomplished using gradual maneuvers and without precision tracking, although accurate flight-path control may be required.

Included in this category are:

- | | |
|---------------------------------------|---------------------------|
| → a) Climb (CL) ← | Various climb segments |
| → b) Cruise (CR) ← | Various cruise segments |
| → c) Loiter (LO) ← | Flight in holding pattern |
| d) In-flight refuelling (tanker) (RT) | None as yet |
| e) Descent | Various descent segments |
| f) Emergency descent (ED) | Emergency descent |
| g) Emergency deceleration (DE) | None |
| h) Aerial delivery (AD) | Parachute drop |

Terminal Flight Phases

Category C: Terminal flight phases are normally accomplished using gradual maneuvers and usually require accurate flight path control.

Included in this category are:

- | | |
|------------------------------|---------------------------|
| a) Takeoff (TO) | Various takeoff segments |
| b) Catapult takeoff (CT) | None |
| c) Approach (PA) | Various approach segments |
| d) Wave-off / go-around (WO) | Aborted approach |
| e) Landing (L) | Various landing segments |

Table 6.8 Flight Path Requirements

MIL-F-8785C:	FAR 23, FAR 25, VLA:
Level I: $dy/dV_P \leq 0.06 \text{deg/knot}$	No requirement
Level II: $dy/dV_P \leq 0.15 \text{deg/knot}$	No requirement
Level III: $dy/dV_P \leq 0.24 \text{deg/knot}$	No requirement

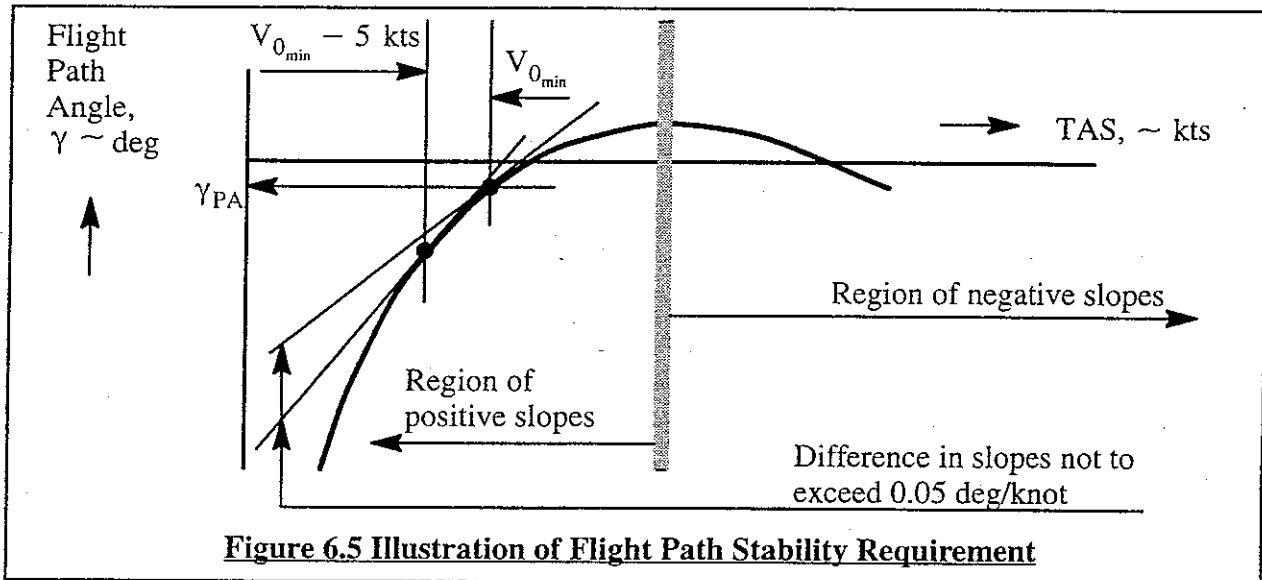


Figure 6.5 Illustration of Flight Path Stability Requirement

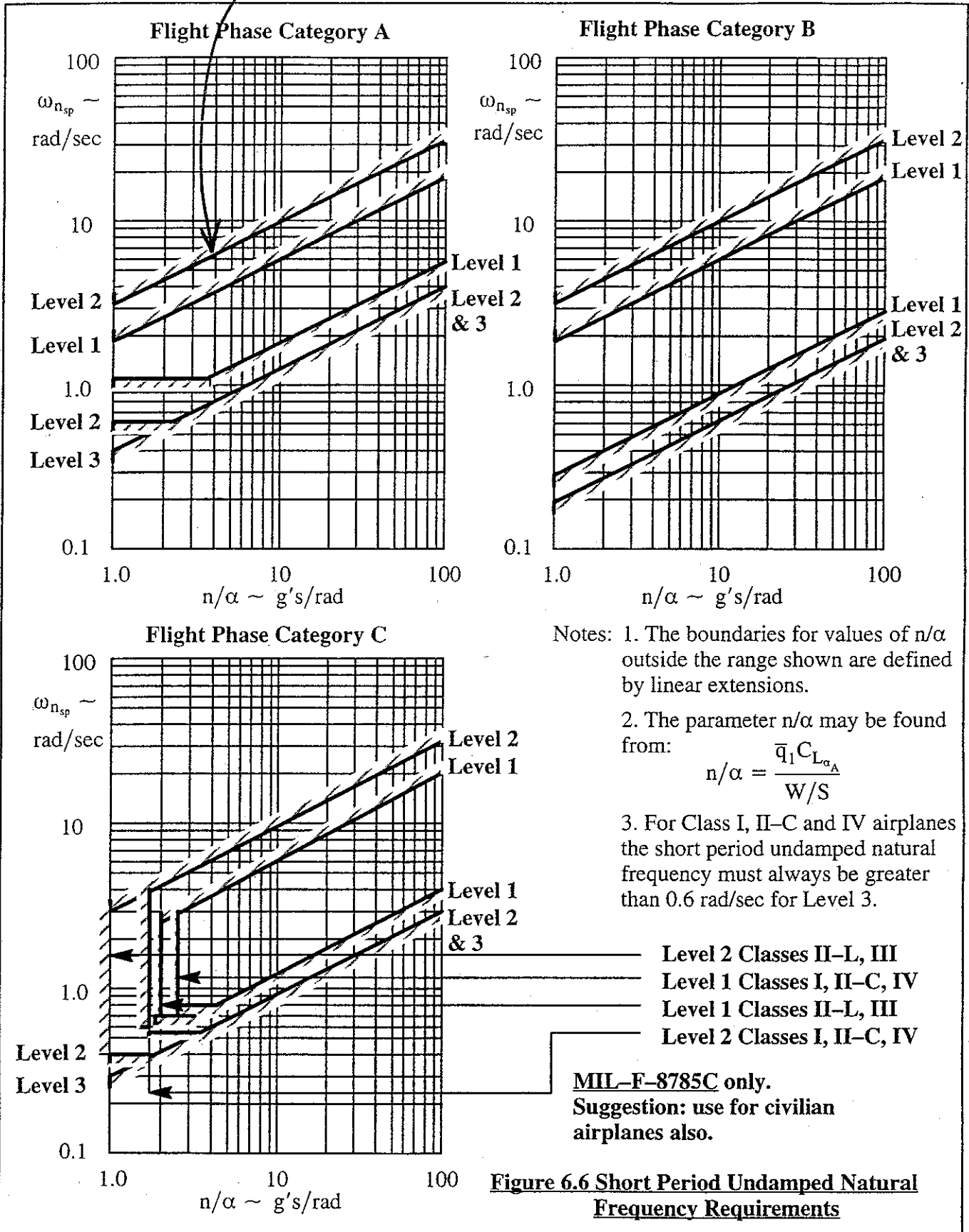
The speed, $V_{0_{min}}$ is defined as the minimum operating speed of the airplane during final approach. For military aircraft that speed is typically $1.15V_{SPA}$ for carrier-based aircraft and $1.20V_{SPA}$ for land-based aircraft. For civilian aircraft that speed is typically $1.30V_{SPA}$.

Flight path stability may be predicted with the generalized trim analysis method presented in Section 4.6. The slope dy/dV_P is a component of the r.h.s. matrix in Eqn (4.232).

6.3.4 SHORT PERIOD FREQUENCY AND DAMPING

MIL-F-8785C requires the (equivalent) short period undamped natural frequency, $\omega_{n_{sp}}$, of the short period mode to be within the limits shown in Figure 6.6 for three Flight Phase Categories. Although the FAR/VLA requirements do not set specific limits on $\omega_{n_{sp}}$, common design practice is to adopt the military requirements. Reference 6.2 is a recent replacement specification for MIL-F-8785C of Reference 6.1. In MIL-STD-1797A, there appears a requirement for a so-called Control Anticipation Parameter (CAP). This parameter and its relationship to airplane maneuver margin is discussed in Sub-section 6.3.5.

line of constant CAP



The (**equivalent**) short period damping ratio, ζ_{sp} , must be within the limits presented in Table 6.9. For airplanes which do not require stability augmentation systems to meet the requirements of Figure 6.6 and Table 6.9 the word **equivalent** should be omitted.

The FAR/VLA requirements of References 6.3 and 6.4 merely require the short period oscillation to be heavily damped. It is considered good design practice to use the military requirement for civilian airplanes.

The word '**equivalent**' refers to highly augmented airplanes **only**. In such airplanes, an 'equivalent' short period frequency is achieved with the help of a feedback system. The dynamic characteristics of the feedback system (including its actuator dynamics, sensor dynamics and computational delays) give rise to the term 'equivalent' frequency.

Table 6.9 Short Period Damping Ratio Limits				
MIL-F-8785C				
Level	Category A and C Flight Phases		Category B Flight Phases	
	Minimum	Maximum	Minimum	Maximum
Level 1*	0.35	$\leftarrow \zeta_{sp} \rightarrow$ 1.30	0.30	$\leftarrow \zeta_{sp} \rightarrow$ 2.00
Level 2	0.25	$\leftarrow \zeta_{sp} \rightarrow$ 2.00	0.20	$\leftarrow \zeta_{sp} \rightarrow$ 2.00
Level 3	0.15 **	$\leftarrow \zeta_{sp} \rightarrow$ no maximum	0.15 *	$\leftarrow \zeta_{sp} \rightarrow$ no maximum
* For VLA, FAR 23 and FAR 25: ζ_{sp} must be heavily damped				
** For altitudes above 20,000 ft this requirement may be reduced if approved by the procuring activity				

It is seen in Table 6.9 that damping ratios larger than 1.0 are admitted. A damping ratio larger than 1.0 indicates that the short period mode has degenerated into two, stable real roots.

The short period damping requirements apply with the cockpit-flight-controls-fixed and with the cockpit-flight-controls-free. The controls-fixed case applies to airplanes with irreversible flight control systems as well as to airplanes with reversible flight controls, while the pilot keeps the cockpit controllers in a fixed position. The controls-free case applies to airplanes with reversible flight controls, while the pilot does not touch the corresponding cockpit controls. In the latter case, the oscillatory characteristics of the freely oscillating flight control system must be accounted for. The methods of Chapter 5 deal only with the controls-fixed case. Appendix D contains a mathematical model which accounts for freely oscillating flight controls: controls-free.

Figure 6.7 illustrates the significance of the short period flying quality requirements in the s-plane. The designer must see to it that the short period poles are located in the allowable areas.

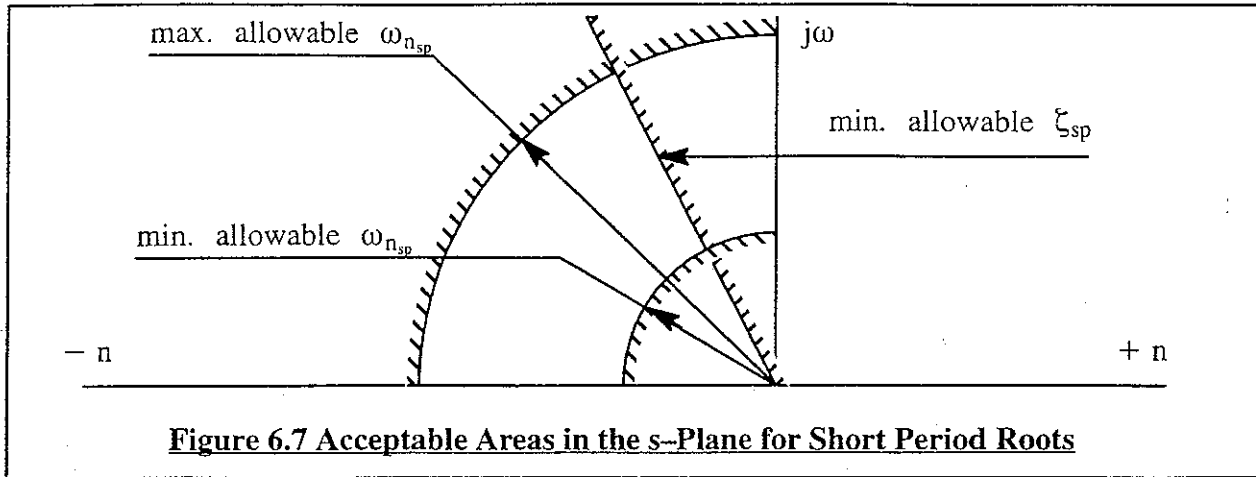


Figure 6.7 Acceptable Areas in the s-Plane for Short Period Roots

6.3.5 CONTROL ANTICIPATION PARAMETER

MIL-STD-1797A (Reference 6.2) contains a requirement that airplanes must stay within a minimum and maximum range of values of the so-called Control Anticipation Parameter (CAP) over a range of allowable short period damping ratios. For highly augmented airplanes, this requirement has in fact replaced the short period undamped natural frequency and damping ratio requirements of Figure 6.6 and Table 6.9. For non-augmented airplanes the author recommends continued use of Figure 6.6 and Table 6.9.

In preliminary design it is acceptable to use the following equation to estimate the control anticipation parameter (CAP):

$$CAP = \frac{\omega_{n_{sp}}^2}{n_\alpha} \quad \frac{\left(\frac{\text{rad}}{\text{sec}}\right)^2}{g/\text{rad}} = \frac{\text{rad}^2}{\text{sec}^2} \frac{\text{rad}}{g} = \frac{1}{\text{sec}^2} \quad (6.3)$$

where: $\omega_{n_{sp}}$ is the undamped natural frequency of the short period mode

$n_\alpha = \partial n / \partial \alpha$ which is also referred to as the gust- or load-factor-sensitivity of an airplane.

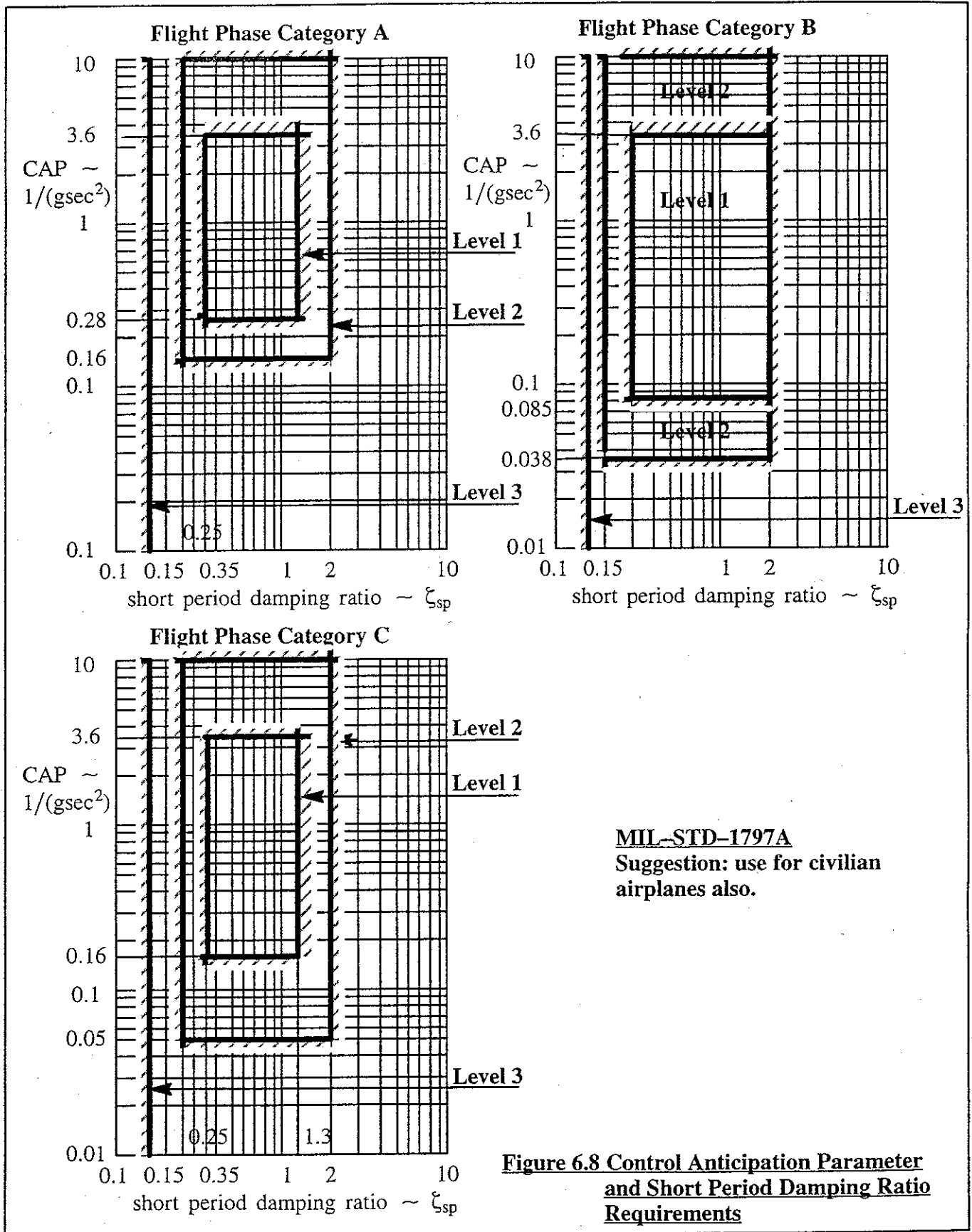
Figure 6.8 shows how allowable CAP-values are related to the short period damping ratio for various categories of Flight Phases and to handling quality Levels. It is shown next, that the CAP is mathematically related to the following quantities:

- * maneuver margin (See: Section 4.3)
- * wing m.g.c.
- * overall airplane length
- * dimensionless radius of gyration about the Y-axis.

According to Eqn (5.61), the following approximation holds for $\omega_{n_{sp}}^2$:

$$\omega_{n_{sp}}^2 = \frac{Z_\alpha M_q}{U_1} - M_\alpha \quad (6.4)$$

By partially differentiating Eqn (4.91) with respect to angle-of-attack it is found that:



$$n_\alpha \approx \frac{\bar{q}_1 C_{L_\alpha}}{(W/S)}$$

$$\frac{lb_f}{ft^2} \cdot \frac{ft^2}{lb_f} = ND \Rightarrow \frac{g}{rad} \quad \text{--- both ND} \quad (6.5)$$

The dimensional derivatives in Eqn (6.4) are defined in Table 5.1 as:

$$Z_\alpha \approx \frac{-\bar{q}_1 S C_{L_\alpha}}{m} \quad (6.6)$$

$$M_q = \frac{\bar{q}_1 S \bar{c}^2 C_{m_q}}{2I_{yy} U_1} \quad (6.7)$$

$$M_\alpha \approx \frac{\bar{q}_1 S \bar{c} C_{m_\alpha}}{I_{yy}} \quad (6.8)$$

It should be recalled from Eqn (3.39) that the dimensionless derivatives C_{m_α} and C_{L_α} are related to the non-dimensional distance between airplane c.g. and a.c. in the following manner:

$$C_{m_\alpha} = C_{L_\alpha} (\bar{x}_{cg} - \bar{x}_{ac_A}) \quad (6.9)$$

By substituting Eqns (6.4) through (6.9) in Eqn (6.3) and by rearranging, the reader is asked to show that:

$$CAP = \frac{W\bar{c}}{I_{yy}} \left(-\bar{x}_{cg} + \bar{x}_{ac_A} - \frac{gQ\bar{c}C_{m_q}}{4W} \right) \quad (6.10)$$

From Chapter 4, Eqn (4.121) it is recognized that the maneuver point of an airplane can be written as follows:

$$\bar{x}_{MP} = \bar{x}_{ac_A} - \frac{gQ\bar{c}C_{m_q}}{4W} \quad (6.11)$$

Therefore, Eqn (6.10) can be cast in the following form:

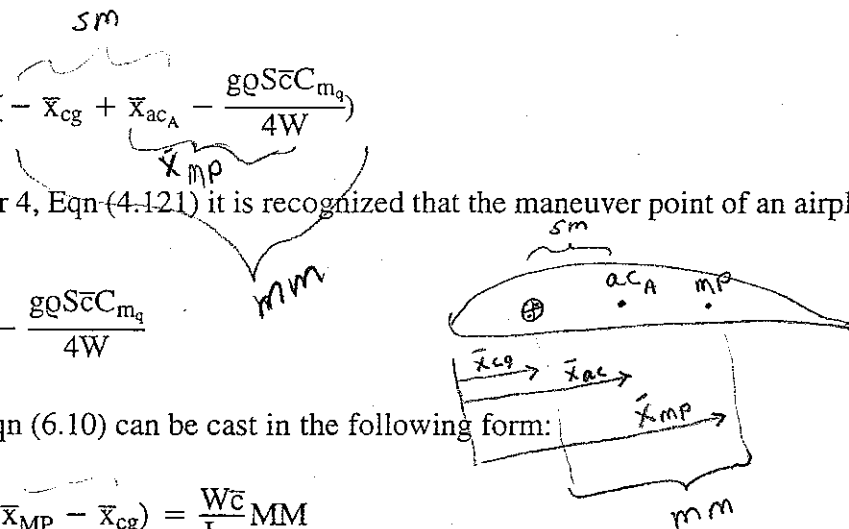
$$CAP = \frac{W\bar{c}}{I_{yy}} (\bar{x}_{MP} - \bar{x}_{cg}) = \frac{W\bar{c}}{I_{yy}} MM \quad (6.12)$$

where: MM is the so-called maneuver margin of an airplane.

The pitching moment of inertia, I_{yy} , is related to the following airplane design parameters:

- * weight, W
- * non-dimensional radius of gyration, \bar{R}_y
- * overall length,
- * mean geometric chord, \bar{c}

in accordance with:



$$I_{yy} = \left(\frac{L^2 W \bar{R}_y^2}{4g} \right) \quad (6.13)$$

By combining Eqns (6.12) and (6.13) it is found that:

$$CAP = \frac{4\bar{c}_g M M}{L^2 \bar{R}_y^2} \quad (6.14)$$

The minimum and maximum allowable CAP values from MIL-STD-1797A can therefore be translated into a minimum and maximum allowable maneuver margin for any airplane, as long as airplane geometric size (as expressed by \bar{c} and L) and its longitudinal mass distribution (as expressed by \bar{R}_y) are known. Equation (6.12) suggests that for very large airplanes the minimum acceptable maneuver margin will increase relative to that required for smaller airplanes to maintain some minimum acceptable CAP value.

6.4 LATERAL-DIRECTIONAL FLYING QUALITY REQUIREMENTS

The lateral-directional flying quality requirements of References 6.1 through 6.3 have been copied and included verbatim as Appendices A and B in Part VII of Reference 6.5. This section contains a summary of only those requirements which should be addressed during the early stages of design analysis of new airplanes. The reader should consult the actual regulations before considering a flying quality assessment to be completed. In several instances, when the FAR/VLA regulations do not contain specific requirements a statement of 'Civilian Equivalent' is included. The author suggests these civilian equivalents requirements as sound design practice only.

6.4.1 LATERAL-DIRECTIONAL CONTROL FORCES

The lateral-directional control force requirements, which are summarized here, apply to airplanes equipped with conventional stick- or wheel-type cockpit controllers. The regulations deal with many specific situations which are summarized as follows:

- * 6.4.1.1 Roll control forces
- * 6.4.1.2 Directional control forces with asymmetric loadings
- * 6.4.1.3 Directional and roll control forces with one engine inoperative

6.4.1.1 Roll Control Forces

The stick or wheel control forces required to obtain the roll performance of Sub-section 6.4.6 may not be greater than those listed in Table 6.10, nor may these forces be less than the control system break-out forces plus:

- | | |
|--------------|-------------------------------------|
| for Level 1: | 1/4 the values of Table 6.10 |
| for Level 2: | 1/8 the values listed in Table 6.10 |
| for Level 3: | zero. |

DEVELOPMENT OF A LIGHT UNMANNED AIRCRAFT FOR THE
DETERMINATION OF FLYING QUALITIES REQUIREMENTS

A Thesis

Submitted to the Faculty

of

Purdue University

by

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In Partial Fulfillment of the
Requirements for the Degree

of

Master of Science

May 1996

Figure 11.3 shows that the light aircraft Level 1 flying qualities overlap the upper Level 2 and Level 3 flying qualities for the manned aircraft.

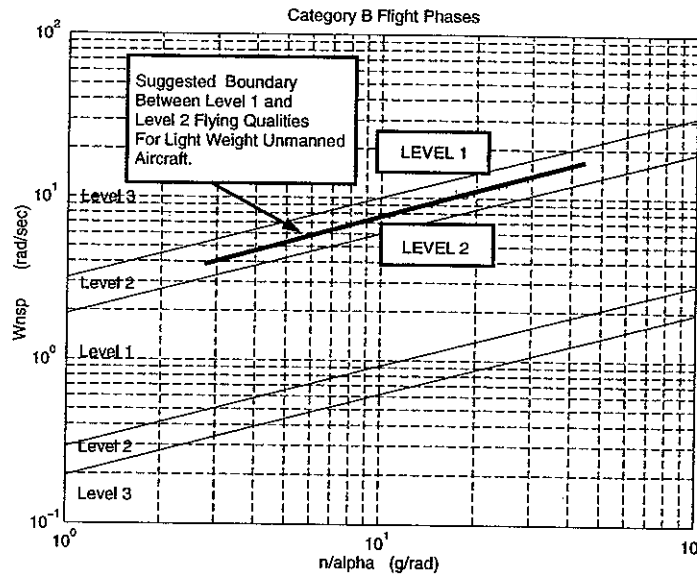


Figure 11.3 Category B Proposed Light Unmanned Aircraft Requirements

Figure 11.2 show the relationship between $w_{n_{sp}} T_{\theta_2}$ and ζ_{sp} . From figure 11.2 it can be seen that the data points seem to straddle a boundary for Level 1 and Level 2 flying qualities. This suggests that the boundary for $w_{n_{sp}} T_{\theta_2}$ should be shifted upward to 1.38 from 1.0. Figure 11.4 shows this boundary drawn. Again, the 8 oz ballast condition is used as the Level 1 / Level 2 border.

Note that none of the data points clustered around the 8oz flight condition demonstrate a damping ratio which is anything out of the ordinary. All points fit well within the Category B ζ_{sp} requirements. This is unfortunate because no precise conclusion can be made about the range of acceptable damping ratio.

11.5 The Exploration of Level 2 Boundaries for Light Unmanned Aircraft

There is only one data point which could be used as a divider between Level 1 and Level 2 flying qualities. It represents the flight condition where destabilizing rate feedback was used without any ballast. It was fortunate that both pilots had the opportunity to rate this flight condition. While pilot #1 rated it as a 4, pilot # 2

```

disp(' '); disp('Start Here <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<'); format compact
disp('INPUTS')
aircraft='MPX5'           % Name of the aircraft
W = 19.2                 % Weight of Airplane [lbf]
S_w = 1350/144           % Surface area of wing [ft^2]
altitude= 607           % Trim altitude [ft]
c_w = 15/12              % Mean aerodynamic chord of the wing [ft]
Iyy = 1.10              % Airplane moment of inertia about y-axis [slug-ft^2]
g=32.17                 % accel of gravity (ft/sec^2)
Xref = 0.25*c_w          % Distance from the leading edge of the wing mean
aerodynamic chord
% to the arbitrary moment reference point. The equivalent force system
% for the aerodynamic force system is given about this point.
% Measured as positive aft, starting from the leading edge of the mean aero.
chord. [ft]
Xcg = 0.25*c_w          % Distance from the leading edge of the wing mean
aerodynamic chord
% to the center of gravity.
% Measured as positive aft, starting from the leading edge of the mean aero.
chord. [ft]
% CL_alpha
CL_alpha= 4.84
% Cm_alpha where Xref is the moment reference point.
Cm_alpha= -1.13
% Cm_q where Xref is the moment reference point.
Cm_q= -11.9

disp(' '); disp('OUTPUTS')
rho=rhofun(altitude)    % Air density at altitude [slug/ft^3]
Xbarcg=Xcg/c_w;         % XbarCG, nondimensional, measured aft from leading edge
of wing mean aerodynamic chord.
%Xbarcg=.15
Xbarref=Xref/c_w;       % XbarRef, nondimensional, measured aft from leading edge
of wing mean aerodynamic chord.
Xbarac=-Cm_alpha/CL_alpha+Xbarref;
StaticMargin=Xbarac-Xbarcg;
stringA=['C.G. location,          Xbarcg= ',num2str(Xbarcg),' (fraction
of chord)'];
stringB=['Aerodynamic center location, Xbarac= ',num2str(Xbarac),' (fraction
of chord)'];
stringC=['Static Margin          (Xbarac-Xbarcg) = ',num2str(StaticMargin),'
(fraction of chord)'];
disp(stringA); disp(stringB); disp(stringC);
string4=['Typically 0.05 to 0.50 of the reference chord.'];
disp(string4)
disp('NOTE: static margin above is relative the the c.g.')

disp('See Roskam 421 book pages 431-434 for discussion of Control
Anticipation Parameter (CAP).')
ManeuverMargin=StaticMargin-g*rho*S_w*c_w*Cm_q/(4*W) % in fractions of
wing chord (eqn 6.10 p. 433)
% StickFixedNeutralPoint=Xbarcg+StaticMargin % Same as Aerodynamic center
ManeuverPoint=Xbarac-g*rho*S_w*c_w*Cm_q/(4*W) %Eqn 6.11 p.433
CAP=W*c_w*ManeuverMargin/Iyy

```

```
MinimumCAP=5.92 % This is the requirement from Mark Peters' thesis
disp(' ')
disp('According to Mark Peters'' MS thesis Level 1 flying qualities ')
disp(['for small model airplanes requires that CAP>',num2str
(MinimumCAP),'.']);
disp('For this to happen the static margin (SM) must be greater than or
equal to the following')
SMminimum=MinimumCAP*Iyy/(W*c_w)+g*rho*S_w*c_w*Cm_q/(4*W) % in fractions of
wing chord
disp('and the cg must be forward of the following point')
disp('(expressed in fractions of the Cbar meaured aft from the leading edge
of the wing).')
MostAftCG=Xbarac-SMminimum
if(CAP>MinimumCAP)
    disp('This aircraft meets Level 1 flying qualities for CAP')
else
    disp('This aircraft does not meet Level 1 flying qualities for CAP.')
end
```


0.3485

This aircraft meets Level 1 flying qualities for CAP

Mark Peters CAP Regt.

$$C_{m\alpha} = -1.13 / \text{rad}$$

p 37

$$C_{L\alpha} = 4.84 / \text{rad}$$

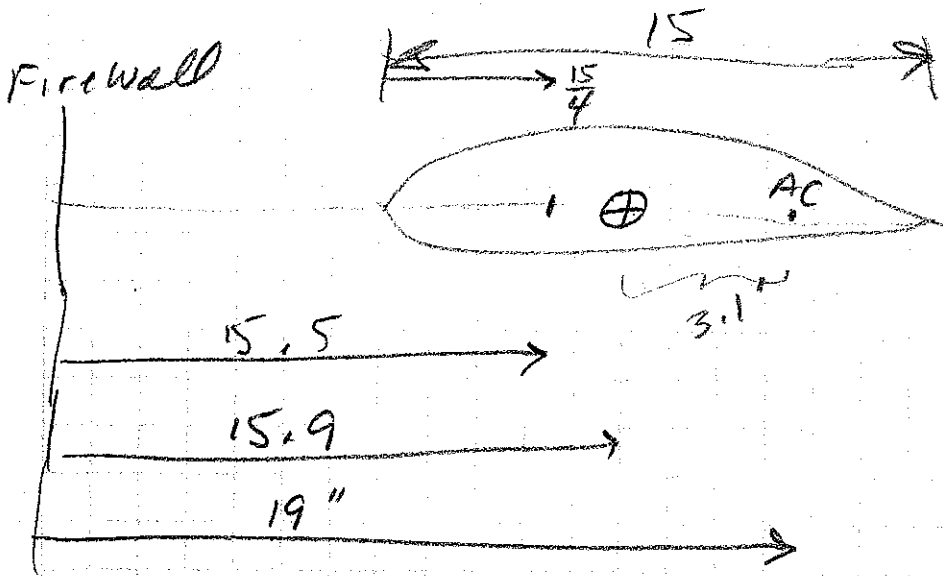
"

$$X_{cg} = ? \quad 15.9''$$

$$X_{AC \text{ Aircraft}} = 19''$$

$$X_{c/4} = 15.5''$$

$$C = 15''$$



$$C_{m\alpha}^{\circ} = -C_{L\alpha} \text{ (Static Margin)}$$

$$\text{Static Margin} = -\frac{C_{m\alpha}}{C_{L\alpha}} = -\frac{-1.13}{4.84} = \frac{1.2335}{23.904}$$

$$\begin{array}{r} X_{ac} \quad 19.0 \\ X_{cg} \quad 15.9 \\ \hline 3.1 \end{array}$$

$$\frac{3.1}{15} = 0.2067 \leftrightarrow 1.2335$$

Level 1 - 2 boundary

$$CAP = \frac{W_{nsp}^2}{\eta_\alpha} = 5.92 = \left(\frac{\text{rad}}{\text{sec}}\right)^2 \frac{\text{rad}}{g}$$

Peters p 153

$$= \frac{\text{rad}^3}{g \text{ sec}^2} = \frac{1}{g \text{ sec}^2}$$

FORM C APPROVED FOR USE IN PURDUE UNIVERSITY

CAP
→ 20km
421 p 431

Level 1 Region

$$\frac{W_{nsp}}{\eta_\alpha} > 5.92$$

← rad/sec

} $\frac{g}{\text{rad}}$

$$W_{nsp} \approx \sqrt{\frac{Z_\alpha M_q}{U_1} - M_\alpha}$$

Rostkam 421 p 334

$$\approx \sqrt{-M_\alpha} = \sqrt{-\frac{C_{m_\alpha} \bar{q} S c}{I_{yy}}}$$

$$\eta_\alpha = \frac{\bar{q} C_{L_\alpha}}{W/S}$$

stick fixed
Neutral Pt

$$CAP = \frac{W \bar{c}}{I_{yy}} \left(\underbrace{-\bar{X}_{cg} + \bar{X}_{ac}}_A \quad \underbrace{-\frac{g \rho S \bar{c} C_{m_q}}{4W}}_B \right)$$

static margin

can use 191100
≈ .05

$$\underline{19000 \text{ cmg}}$$

$$CAP = \frac{W \bar{c}}{I_{yy}} \text{ (sm)}$$

$$5.92 =$$

$$SM = \frac{I_{yy}}{W \bar{c}} \text{ (5.92 CAP)}$$

$$= \frac{1.1}{(19.2) \left(\frac{15}{12} \right)} 5.92$$

$$12 \times (0003819) (5.92) = (00226) 12 = \underline{\underline{0.2713}}$$

$$I_{yy} = 1.1 \text{ slug ft}^2 \text{ p 214}$$

$$W = 19.2 \text{ lbf}$$

For Level 1 FQ

$$SM > .27$$

$$S = \frac{1350 \text{ in}^2}{144 \text{ in}^2/\text{ft}^2} = \text{ft}^2$$

$$f = 0.002378 \text{ slug/ft}^3$$

$$g = 32.2 \text{ ft/sec}^2$$

$$\bar{c} = \frac{15}{12} \text{ ft}$$

$$C_{mg} = -11.91 \text{ p 90}$$

$$-10.57 \text{ p 34}$$

$$CAP = \frac{W \bar{c}}{I_{yy}} \left(\underbrace{\left[\bar{x}_{ac_A} - \bar{x}_{cg} \right]}_{SM} - \frac{g p S \bar{c}}{4W} C_{mq} \right)$$

MM

$$CAP = \frac{W \bar{c}}{I_{yy}} MM$$

$$MM = \frac{I_{yy}}{W \bar{c}} CAP = SM - \frac{g p S \bar{c}}{4W} C_{mq}$$

$$SM = \frac{I_{yy}}{W \bar{c}} CAP + \frac{g p S \bar{c}}{4W} C_{mq}$$

According to Mark Peters For Level 1 Flying Qualities $CAP \geq 5.92$

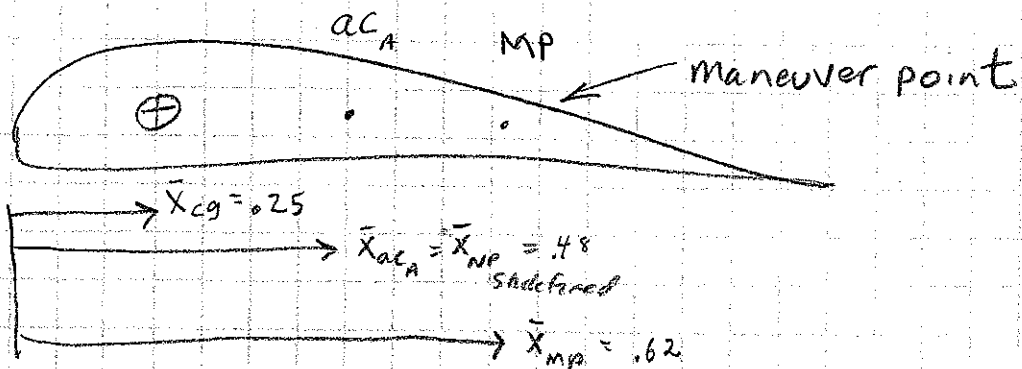
To meet this requirement

$$SM \geq \frac{I_{yy}}{W \bar{c}} (5.92) + \frac{g p S \bar{c}}{4W} C_{mq}$$

$$SM = \bar{x}_{ac_A} - \bar{x}_{cg}$$

$$\bar{x}_{cg} = \bar{x}_{ac} - SM_{min}$$

maneuver margin = .37
Static Margin = .23



WLL F17

4.13
3.5