AAE 451
Propeller Analysis Theory

Professor John Sullivan

ADVANCED THEORETICAL TREATMENTS
OF
PROPELLER AERODYNAMICS

INTRODUCTION

Propellers are currently the most efficient means of aircraft propulsion , generally operating in the 80-90% efficiency range as shown in Fig. 1. Propellers achieve this high efficiency by imparting a small change in pressure (or velocity) over a large area rather than a large change in velocity over a small area as in a turbojet. The results of a simple momentum theory (McCormick) show the ideal efficency is

$$\eta_i = \frac{2}{1 + \sqrt{1 + \frac{T}{1/2\rho V_\infty^2 A}}}$$

The thrust of the propeller is

$$T = A\Delta p$$

In order to increase the efficiency at constant thrust, T, and forward velocity, V_∞, the disc area, $A = \pi R^2$, must be increased and the change in pressure, Δp, over the disc used must decrease.

Although the one dimensional momentum theory (also called actuator disc theory) is useful for preliminary considerations, the efficiency predicated is overestimated because of the neglect of many real effects in this simplified theory.

The losses in efficiency of the propeller can be broken into two general catagories: induced losses and profile losses. The induced losses are caused by the finite size and finite number of blades of the propeller and can be broken into an axial loss and a rotational loss. The profile drag is composed of viscous skin friction and form drag and because of the high relative tip Mach number, compressible drag. The portion that each effect contributes to the total propeller loss is:

INDUCED	AXIAL	15%
	ROTATIONAL	50%
PROFILE	VISCOUS	20%
	COMPRESSIBLE	15%
		100%

These numbers are dependent on the propeller loading, number of blades and tip speed. The effect of blade number and loading on the induced losses is shown in Fig. 2.

In order to accurately predict propeller losses , advanced analytical techniques have been developed and the purpose of the lectures is to describe these methods.

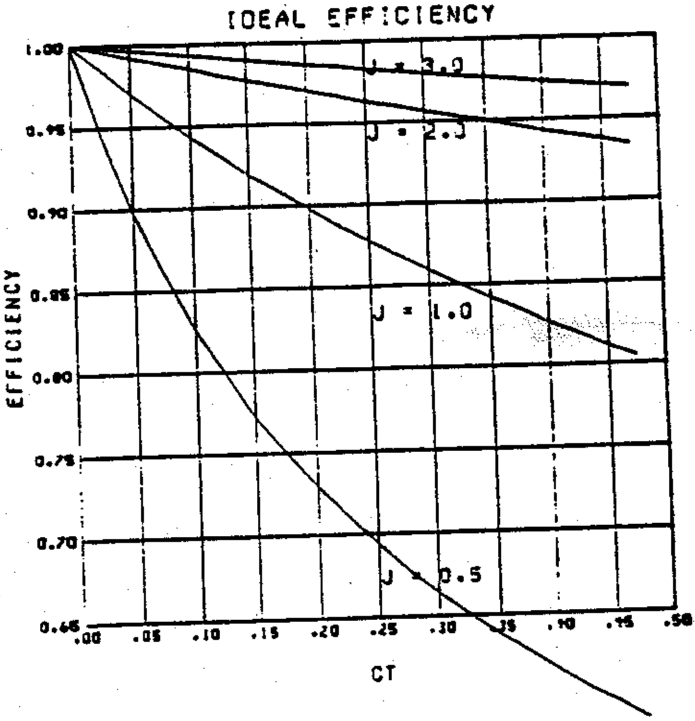


Figure 1 a

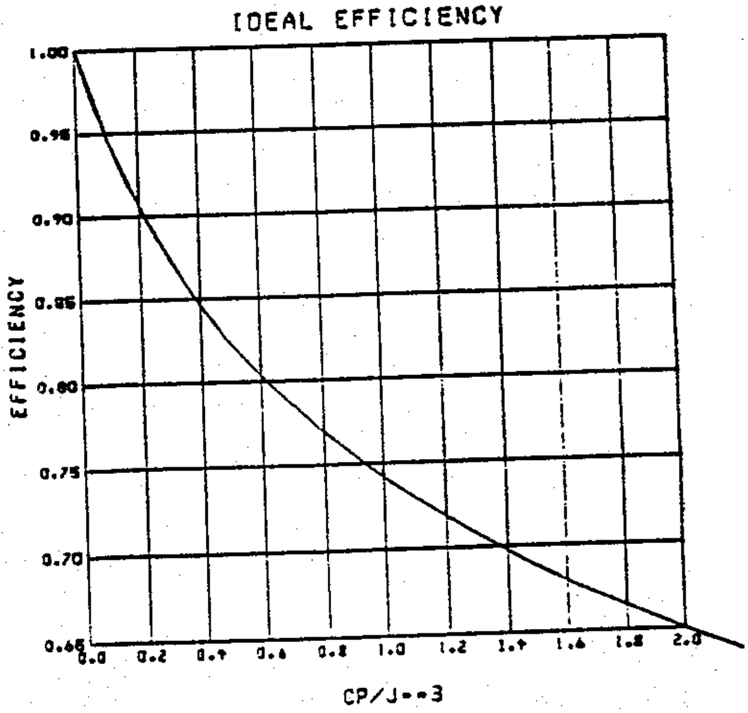
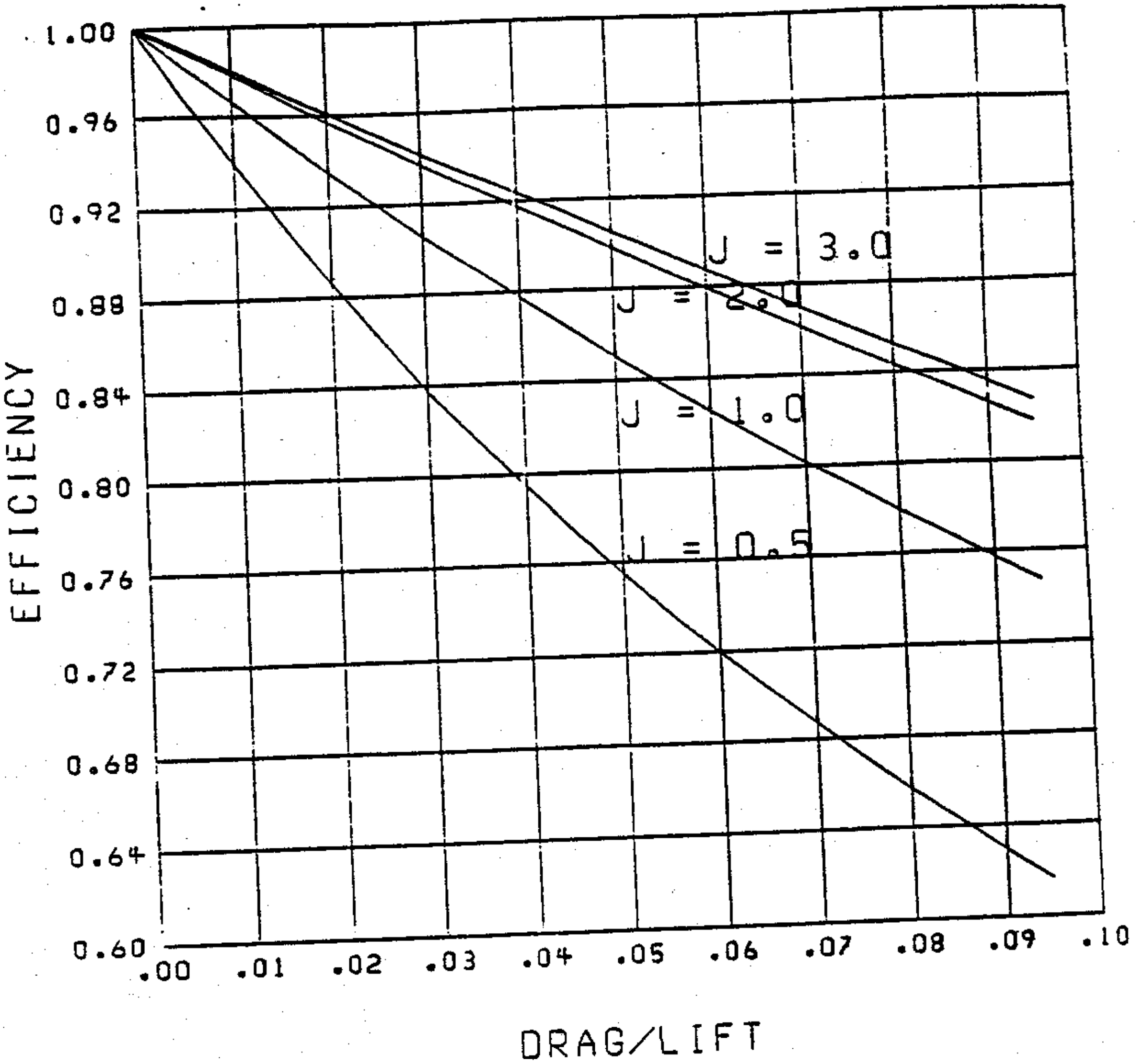


Figure 1 b



Effect of Viscous Drag on Propeller Efficiency
(Drag is constant along the blade)

Figure 1c

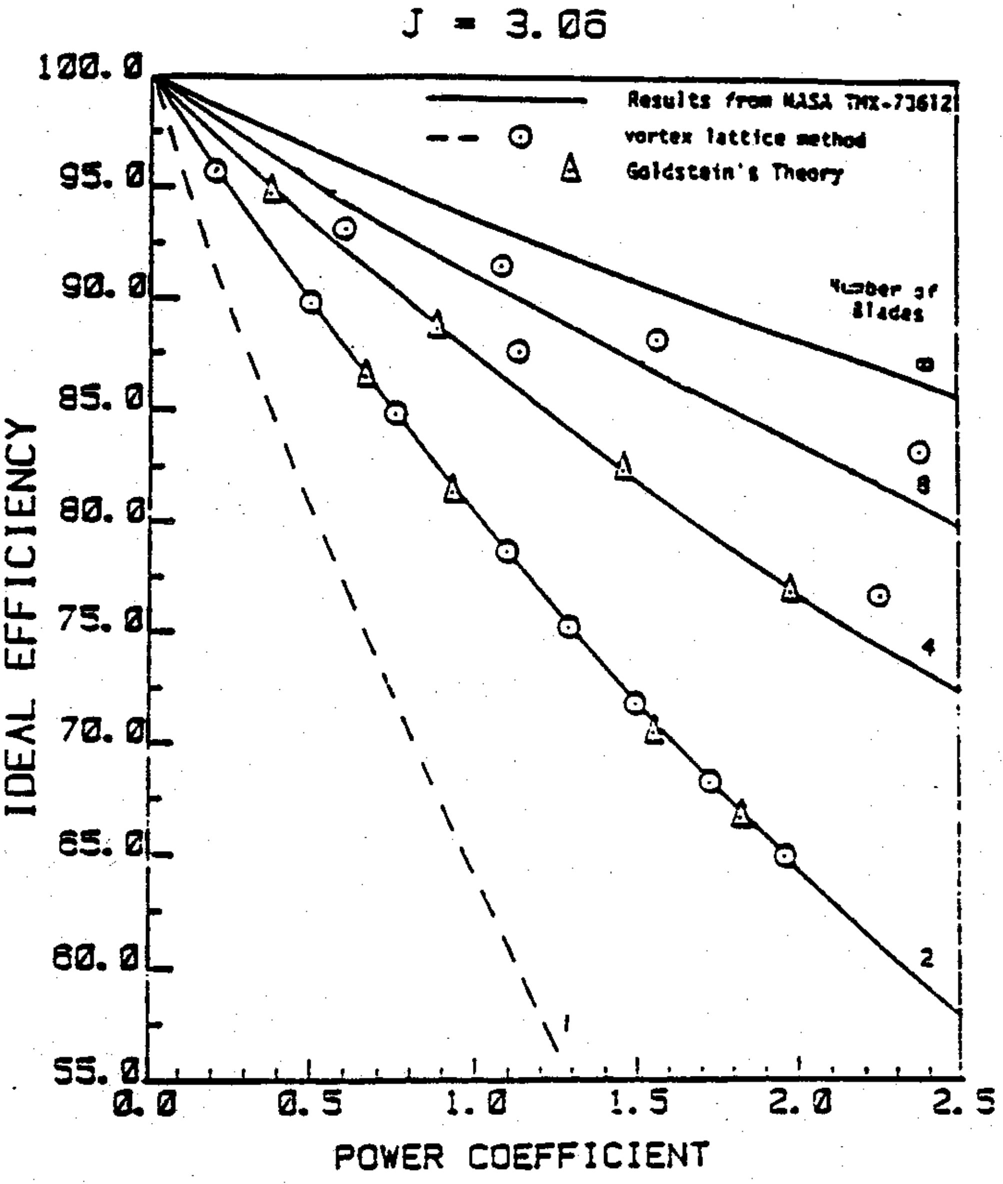


Figure 2 Ideal efficiency versus power coefficient of propeller SR-2

COMPUTATION

OF

PROPELLER PERFORMANCE

MOMENTUM ANALYSIS

GOLDSTIEN/LOCK

POTENTIAL FLOW

- LIFTING LINE
- VORTEX LATTICE
- LIFTING SURFACE
- PANEL METHODS

TRANSONIC SMALL DISTURBANCE

EULER

NAVIER STOKES

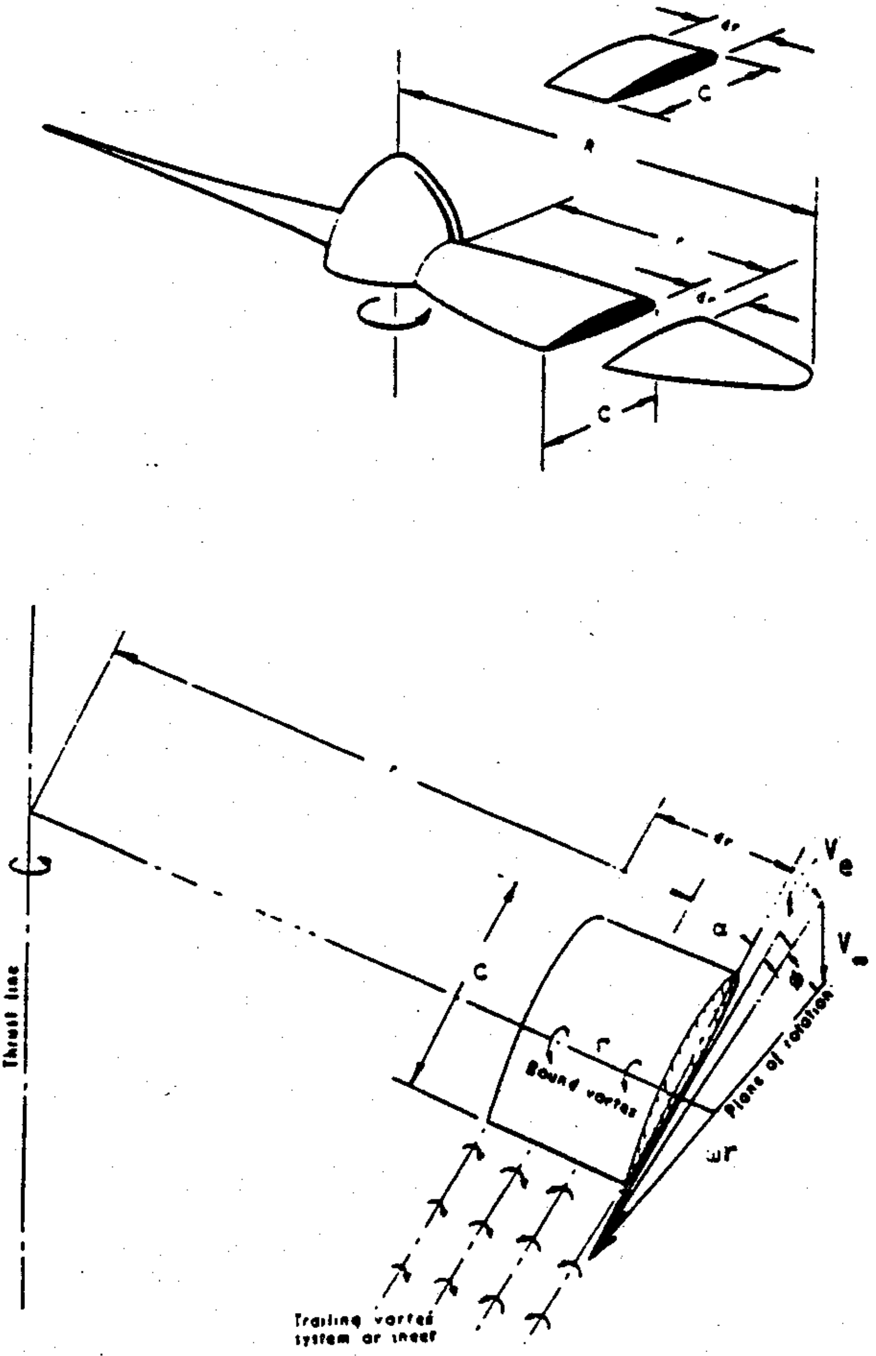


Figure 3. The blade element (Dommasch)

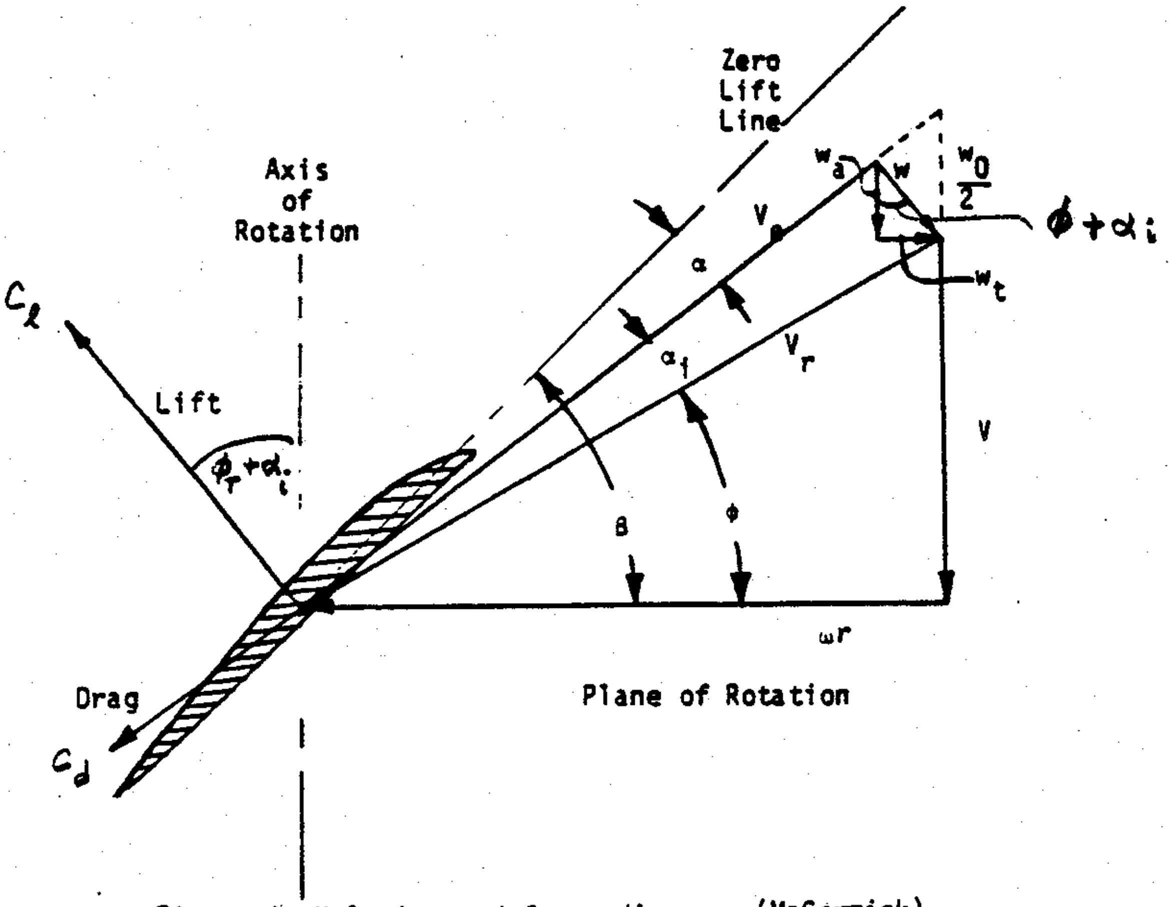


Figure 4. Velocity and force diagram (McCormick)

ϕ = inflow angle

BLADE ELEMENT THEORY

Both the Goldstein theory and the lifting line theory divide the propeller into a number of radial segments (strips) in order to calculate the forces at any radial station. (See Fig. 3) The velocity and force diagram at a radial location, r, is shown in Fig. 4.

The thrust and power on an element, dr , at r are

$$dT = B \rho \frac{V_e^2}{2} c[C_\ell \cos(\phi + \alpha_i) - C_d \sin(\phi + \alpha_i)]dr$$

$$dP = \omega r B \rho \frac{V_e^2}{2} c[C_\ell \sin(\phi + \alpha_i) + C_d \cos(\phi + \alpha_i)]dr$$

B = number of blades

The total thrust and power are obtained by integrating from the hub to the tip. In dimensionless variables (McCormick 1979)

$$C_T = \frac{T}{\rho n^2 D^4} = \frac{\pi}{8} \int_{x_h}^{1} (J^2 + \pi^2 x^2)\, \sigma\, [C_\ell \cos(\phi + \alpha_i) - C_d \sin(\phi + \alpha_i)]dx \tag{1a}$$

$$C_P = \frac{P}{\rho n^3 D^5} = \frac{\pi}{8} \int_{x_h}^{1} \pi x (J^2 + \pi^2 x^2)\, \sigma\, [C_\ell \sin(\phi + \alpha_i) + C_d \cos(\phi + \alpha_i)]dx \tag{1b}$$

where $x = r/R$ $\qquad X_h = r_{hub}$

$J = \frac{V_\infty}{nD}$ - Advance Ratio

$\frac{BcR}{\pi R^2} = \frac{Bc}{\pi R}$ $\qquad \sigma(x) = \frac{Bc(x)}{\pi R}$ - Solidity $= \frac{\text{blade area}}{\text{disk area}}$

Specification of the geometry gives $\beta(x)$ and specification of the flight conditions gives J and ϕ. (Since $\phi = \tan^{-1} J/\pi x$) The difficulty in determining the thrust and power is finding the induced angle of attack, α_i , and the lift and drag coefficients C_ℓ , C_d.

In the inviscid theories ($C_d = 0$), the lift is related to the circulation, Γ , through the Kutta Joukowski theorem

$$\vec{L} = \rho \vec{V} \times \vec{\Gamma} \qquad\qquad C_\ell = \frac{2\Gamma}{V_e c}$$

For the elemental section, dr , the lift is perpendicular to the resultant velocity V_e .

The circulation distribution and induced angle of attack distribution are determined from a bound vortex model of the blades and a vortex model of the wake.

Once the lift coefficient distribution is obtained from the inviscid model, the drag coefficient, C_d , can be found from experimental airfoil section data that relates C_ℓ to drag coefficient C_d, thickness t/c , Mach number and Reynolds. An alternate method is to determine the induced angle of attack from the inviscid vortex model and therefore the local angle of attack of the blade element. Experimental data is then used for both C_ℓ and C_d.

WAKE MODEL

One of the most crucial issues in the model of the propeller is the specification of the vortex wake shed by the propeller blade. The actual wake of a propeller blade is a viscous vortex sheet in which the outboard region rolls up to form a helical tip vortex as shown in Fig 5 . The helical vortex is itself unstable (Windwall) and sinusoidal perturbations develope, as can be seen in the smoke flow visualization picture of Fig. 6.

Extensive investigations in the helicopter industry have developed wake models for hovering rotors, rotors in axial motion and rotors in forward motion. The first two are applicable to propellers. Models by Landgrebe and Gray (See Bramwell) conclude that the wake consists of a strong tip vortex and an inner vortex sheet of opposite sense as shown in Figs. 7 , 8 . The outer part of the sheet moves faster than both the inner part of the sheet and the tip vortex causing a progressive distortion of the vortex sheet.

Two general methods are used to model the wake : the prescribed wake analysis and the free wake analysis. In the prescribed wake anlalysis the position of the vorticity in the wake is specified based on an approximate model of the wake or based on actual experimental measurements of the wake position. The most common model for propellers is to assume that the wake lies on a helical plane and that no roll-up or wake contraction occurs. This is analogous to the plane wake model used in studying the aerodynamics of finite wings.

The free wake analysis begins by assuming an initial location for the positions of a number of short straight line vortex filaments representing the wake. The strengths of the filaments are determined from the gradient of the bound circulation. The vortex filaments then move due to the induced velocities from the other filaments and the bound vortex. By iterating, the positions change until a converged solution is obtained indicating that the positions of the vortex filaments are steady in their own induced flowfield.

The effects of rollup and contraction are taken into account at the expense of considerable computer cost. In addition, the influence of spinners and nacelles on the wake position can be obtained by combining a free wake analysis with a panel model of the interfering bodies.

The complexity of the vortex wake is illustrated in Fig. 9 , which shows a free wake analysis of the tip region of an aircraft wing. The same effect would occur at the tip of a propeller.

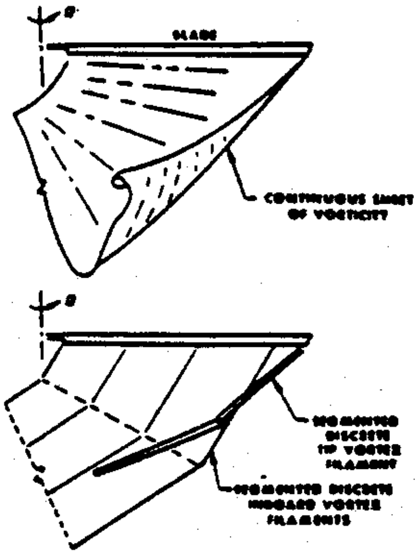


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Figure 7. Vortex Wake (Gray)

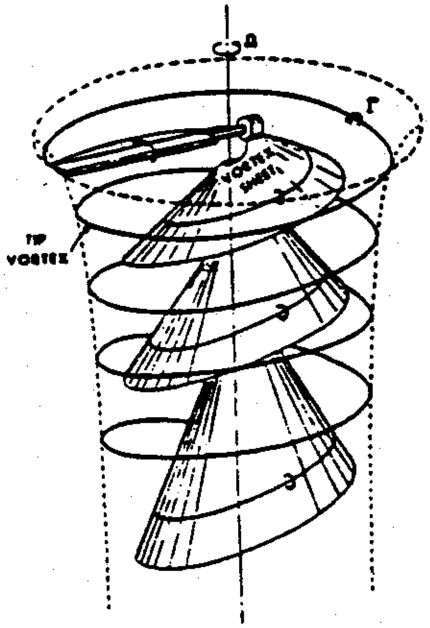

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Figure 8. Hovering rotor wake structure (Landgrebe)

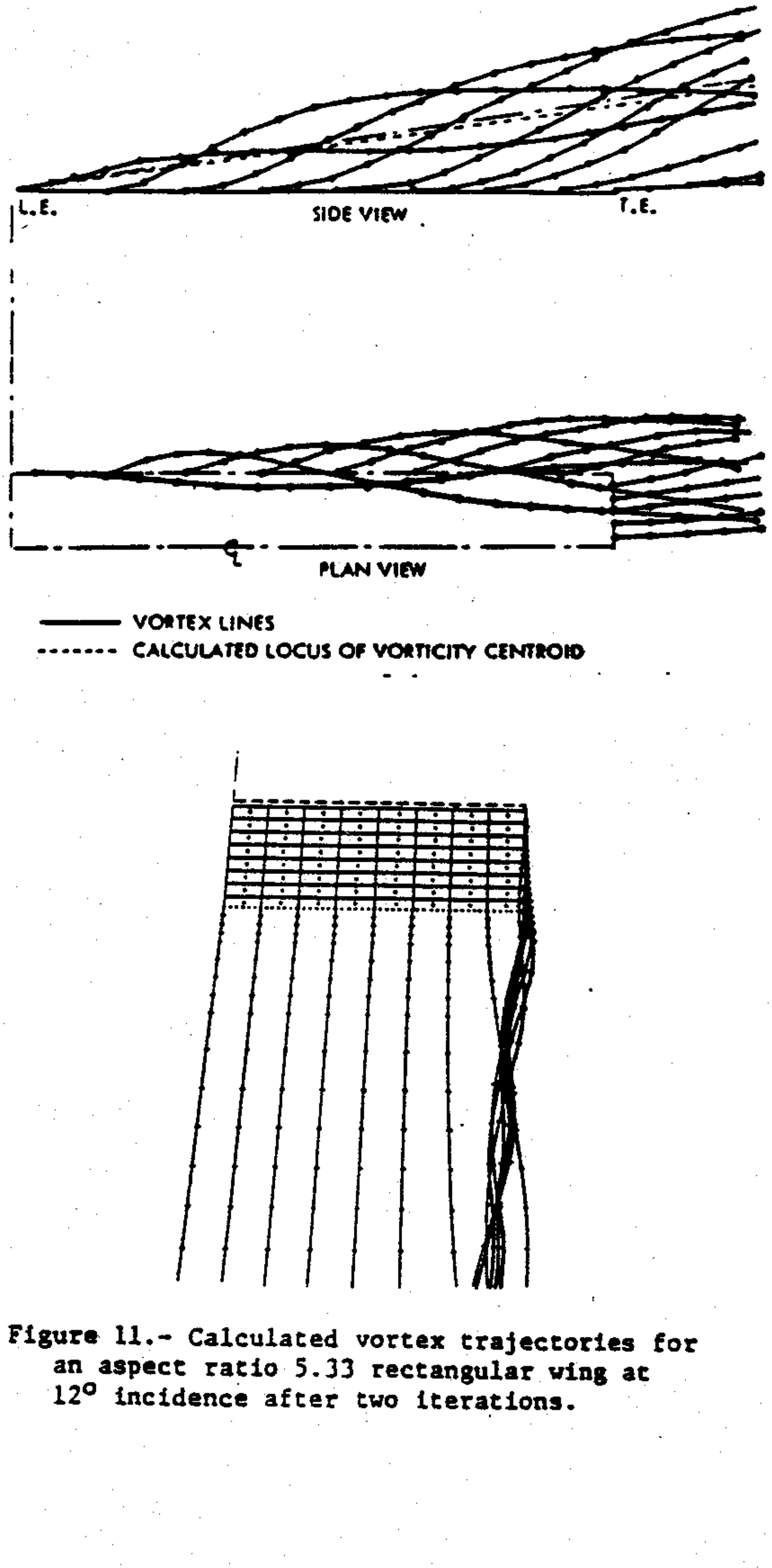

Figure 11.- Calculated vortex trajectories for an aspect ratio 5.33 rectangular wing at 12° incidence after two iterations.

GOLDSTEIN'S THEORY

In 1929, Goldstein published a paper "On the Vortex Theory of the Screw Propeller" which presented an exact solution for the problem of a propeller with a finite number of blades operating with light loading. He models each blade with a single bound vortex having a circulation, $\Gamma(r)$. From each point on the bound vortex, a trailing vortex of strength, $-d\Gamma/dr$ is shed. The trailing vorticies are assumed to lie on a rigid helical wake prescribed by the forward velocity and rotational velocity of the propeller. The equation for this surface for a single blade is

$$\theta - \frac{\omega z}{V_\infty} = 0 \qquad 0 \leq r \leq R$$

where r, θ, and z are cylindrical polar coordinates. Additional blades are added by constructing similar surfaces with the proper orientation. (i.e. for a two bladed propeller two helical surfaces are defined at 0 and π). The condition of the rigid helical wake implies that the distribution $\Gamma(r)$ along the blade is such that at a given thrust the energy loss is minimum. (The Betz condition.)

By moving far downstream, the problem solved by Goldstein reduces to solving for the potential flow about an infinite rigid helix. That is solving Laplace's equation

$$\nabla^2 \phi = 0$$

for the velocity potential ϕ, subject to the boundary conditions of no flow through the helical wake and vanishing velocities as $r \to \infty$.

The solution to the potential flow problem is expressed in a semi-infinite series of modified Bessel functions and is generally used in tabluar or graphical form. In a computer analysis the appropriate functions are interpolated from the tabular data or curve fits to the graphical data are used.

Goldstein gives numerical values in terms of the function K which is related to the circulation by

$$\Gamma = \frac{2\pi V_\infty w_0}{B\omega} K$$

where w_0 is the axial translation velocity of the infinitely long rigid helix.

The induced velocities in the plane of rotation, will be half of the induced velocities in the far downstream wake because they are only influenced by a semi-infinite helical vortex.

For convenience, the circulation is expressed in terms of the Goldstein kappa function

$$\kappa = K/\cos^2(\phi + \alpha_i)$$

At the propeller plane

$$\Gamma = \frac{4\pi r}{B} \kappa \overbrace{w \sin(\phi + \alpha_i)}^{\omega_t} = \frac{4\pi r}{B} \kappa w_t \quad .$$

The function κ depends on the number of blades B, the angle $\phi + \alpha_i$ and the dimensionless radius $x = r/R$. A table of κ factors for a two bladed propeller is given in Table I and graphs are shown in Fig. 10 The circulation is related to the lift coefficient by

$$\Gamma = 1/2 \; c \; C_\ell \; V_e$$

Therefore,

$$\sigma C_\ell \frac{V_e}{\omega R} = 8x \kappa \frac{w_t}{\omega R} \qquad 2$$

where

$$\sigma(x) = \frac{Bc(x)}{\pi R}$$

tangential component of induced velocity

↓

guess Wt & iterate

a linear lift curve with slope a_0 is assumed, then

$$C_\ell = a_0 \alpha = a_0(\beta - \phi - \alpha_i) = a_0(\beta - \tan^{-1} \frac{w_t}{w_a}) \qquad 3$$

From the geometry of Fig. 4, the axial induced velocity w_a is

$$\frac{w_a}{\omega R} = \frac{1}{2}\left[-\frac{V_\infty}{\omega R} + \sqrt{(\frac{V_\infty}{\omega R})^2 + 4 \frac{w_t}{\omega R} (x - \frac{w_t}{\omega R})} \right] \qquad 4$$

and the velocity V_e is

$$\frac{V_e}{\omega R} = [(\frac{V_\infty}{\omega R} + \frac{w_a}{\omega R})^2 + (x - \frac{w_t}{\omega R})^2]^{1/2} \qquad 5$$

NOT omega

Equations 2,3,4,5 and an interpolation routine for κ can be solved interatively for $w_t/\omega R$. The value of the lift coefficient , C_ℓ , and induced angle of attack , α_i , are then determined. The power and thrust are then found from Eq. 1, with appropriate values for C_d from experimental data.

An approximation was developed by Prandtl in which he approximated the helical vortex by a series of two-dimensional sheets, on the assumption that the radius of curvature of the outer parts was large enough so they could be considered as a set of infinite straight strips as shown in Fig. 11. The solution of the two dimensional flow about the vortex sheets gives an approximation

$$\kappa \approx F = \frac{2}{\pi} \cos^{-1} \quad \exp\left[- \frac{B(1-x)}{2 \sin\phi} \right]$$

The Prandtl approximation becomes increasingly valid as the advance ratio, J, becomes small and the number of blades becomes large.

Strictly speaking the Goldstein method is only applicable to propellers operating with the ideal circulation distribution. However, Lock assumed that the kappa factors, κ , would apply to other load distribution with reasonable accuracy so that off-design conditions could be investigated.

The effects of heavy loading and wake contraction were added to the Goldstein analysis by Theodorsen. He showed that the K(x) functions from Goldstein theory are valid for heavy loading if they are applied to a helix moving with a forward velocity $V_\infty - w$ rather than at V_∞ .

r/R

Values of κ for the indicated values of x

$\sin(\phi+\alpha_i)$	0.3	0.45	0.6	0.7	0.75	0.8	0.85	0.9	0.95
0.05	1.000	1.000	1.000	0.999	0.997	0.994	0.985	0.950	0.780
0.1	1.000	0.998	0.997	0.988	0.971	0.937	0.877	0.773	0.586
0.2	0.994	0.991	0.961	0.901	0.852	0.784	0.694	0.378	0.415
0.3	0.978	0.959	0.874	0.774	0.709	0.634	0.548	0.444	0.308
0.4	0.958	0.906	0.783	0.663	0.595	0.520	0.442	0.351	0.243
0.5	0.944	0.848	0.690	0.564	0.501	0.434	0.367	0.289	0.199
0.6	0.930	0.784	0.608	0.492	0.435	0.376	0.316	0.249	0.171
0.7	0.922	0.722	0.547	0.441	0.387	0.333	0.278	0.218	0.149
0.8	0.916	0.677	0.502	0.398	0.347	0.297	0.247	0.193	0.131
0.9	0.933	0.657	0.464	0.360	0.311	0.265	0.220	0.172	0.117
1.0	1.012	0.633	0.425	0.325	0.279	0.238	0.197	0.134	0.103

Table I. Golstein κ Factors for 2-bladed Propeller (Dommasch)

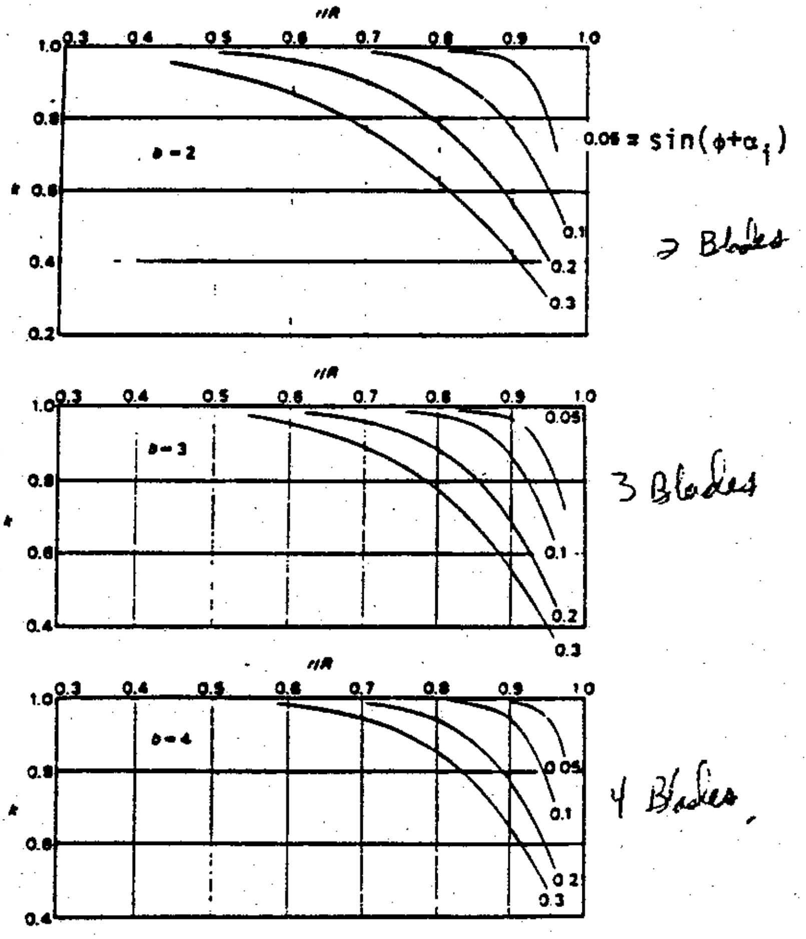

Figure 10. Goldstein κ Factors (Bramwell)

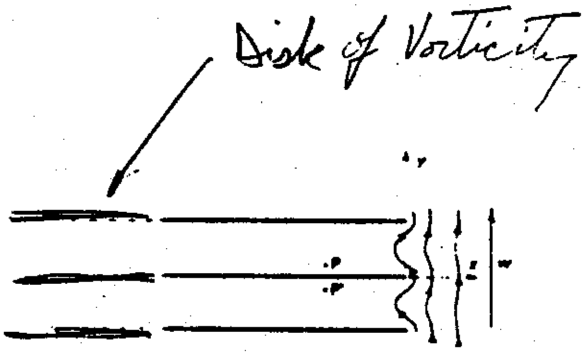

Figure 11. Flow about vortex sheets (Bramwell)

Prandtl's approx. for low J high # B.

LIFTING LINE THEORY

As the name implies, each propeller blade is modelled by a single lifting line vortex, generally at the quarter chord of the airfoil section. Lifting line theory has been applied to propellers by Lerbs, Baskin et al, Sullivan, Chang and Egolf et al.

The propeller is considered to be a thin blade of arbitrary plane form, rotating with a constant angular velocity about a common axis in an unbounded fluid. The presence of solid boundaries, including the hub, is ignored.

Since the propeller blade is assumed sufficiently thin, its presence in the fluid can be represented by the distribution of horseshoe vortices lying in the chord plane of each blade. There are M vortex lattices assumed along the blade radius, each having a constant circulation (See Fig. 2.1) . Steady flow conditions are assumed, so that the circulation is only a function of location along the radius.

The vortex wake of the propeller is assumed to be a constant pitch and diameter helix, since the induced velocity is small compared with the resultant velocity of the propeller, and there is no force existing in the radial direction on a helical surface. This assumption, which is conincident with Goldstein's helical vortex model, dispenses with the essential difficulty in lifiting line theory that the induced velocity and the geometry of the trailing helical vortex sheet are mutually dependent.

The bound vortex is at the 1/4 chord point, and the tangent condition is required to be satisfied by the velocities at the 3/4 chord point of each lattice. This rule was first proposed by Pistolesi , as agreeing with the fact that the sectional lift coefficient determined at the 1/4 chord point is consistent with that obtained by the thin airfoild theory. Induced velocities defined at each 3/4 chord point by any vortex line is obtained by integration according to Biot-Savart law . A set of simultaneous linear equations, relating the vortex strength and the boundary conditions, are obtained by summing up all the induced effects at each control point of the lattice elements. Vortex strengths can be found by solving the simultaneous equations.

Propeller Analysis Code

There is a propeller analysis code at Purdue created by Professor Sullivan and his students. To obtain the FORTRAN code that performs propeller analysis log into ecn and do the following:

```
cp /home/roger/g/aerodyn/AAE515/propeller/gold.f   gold.f
```

To get a sample data deck type:

```
cp /home/roger/g/aerodyn/AAE515/propeller/prop.dat  prop.dat
```

To compile the program

```
f77   gold.f
mv   a.out   gold
```

To run program gold type

```
gold
```

For questions read the FORTRAN code or see Professor Sullivan.

Useful refereences:

Barnes W. McCormick, Aerodynamics, Aeronautics, and Flight Mechanics, John Wiley and Sons, 1979, Chapter 6.

Daniel Dommasch, Sidney Sherry and Thoman Connolly, Airplanr Aerodynamics, Pitnam Publishing Corporation, New York, 1967, Chapter 7.

Martin Simons, Model Aircraft Aerodynamics, Argus Books, Herts, England, 1994, Chapter 14.

Sample Output of program gold
Using Propidat as the input file.

```
'Session Name:roger

     NAME OF INPUT DATA FILE -  prop.dat

                    INPUT DATA


    =      0 OF RADIAL STATIONS =    15  PRINT PARAMETER
        X           COR         BETA          TOC          CD
      0.300000     0.101000    50.799999     0.370000     0.000000
      0.350000     0.122000    40.799999     0.240000     0.000000
      0.400000     0.138000    34.599998     0.175000     0.000000
      0.450000     0.150000    30.400000     0.140000     0.000000
      0.500000     0.152000    27.299999     0.119000     0.000000
      0.550000     0.150000    25.000000     0.109000     0.000000
      0.600000     0.146000    23.200001     0.102000     0.000000
      0.650000     0.140000    21.700001     0.097000     0.000000
      0.700000     0.132000    20.400000     0.095000     0.000000
      0.750000     0.124000    19.100000     0.090000     0.000000
      0.800000     0.111000    18.000000     0.087000     0.000000
      0.850000     0.102000    17.200001     0.085000     0.000000
      0.900000     0.088000    16.400000     0.082000     0.000000
      0.950000     0.074000    15.600000     0.081000     0.000000
      1.000000     0.064000    15.100000     0.080000     0.000000
1
prop.dat
        J        # BLADE      VELOCITY       DIA        DENSITY      BETAIN        BETOUT
      0.740741    3.000000    300.000000   108.000000    0.002378    15.000000    15.000000


     X      BETA     PHI     KAPPA    ALPHAI       WT/WR        WA/WR        VE/WR        CL          DCTDX        DCPDX

    0.300   0.887   0.666   0.980   0.6153E-01   0.1561E-01   0.1752E-01   0.3808E+00   0.9988E+00   0.4059E-01   0.3408E-01
    0.350   0.712   0.593   0.953   0.3784E-01   0.9415E-02   0.1290E-01   0.4217E+00   0.5115E+00   0.3322E-01   0.2667E-01
    0.400   0.604   0.533   0.946   0.2429E-01   0.5962E-02   0.9574E-02   0.4642E+00   0.2950E+00   0.2757E-01   0.2157E-01
    0.450   0.531   0.483   0.945   0.1705E-01   0.4149E-02   0.7600E-02   0.5080E+00   0.1941E+00   0.2441E-01   0.1884E-01
    0.500   0.476   0.441   0.930   0.1285E-01   0.3113E-02   0.6387E-02   0.5528E+00   0.1443E+00   0.2230E-01   0.1707E-01
    0.550   0.436   0.405   0.921   0.1111E-01   0.2687E-02   0.6080E-02   0.5984E+00   0.1270E+00   0.2310E-01   0.1764E-01
    0.600   0.405   0.374   0.915   0.1057E-01   0.2559E-02   0.6315E-02   0.6446E+00   0.1251E+00   0.2603E-01   0.1988E-01
    0.650   0.379   0.348   0.889   0.1049E-01   0.2544E-02   0.6790E-02   0.6914E+00   0.1273E+00   0.2953E-01   0.2259E-01
    0.700   0.356   0.325   0.868   0.1032E-01   0.2506E-02   0.7195E-02   0.7386E+00   0.1309E+00   0.3294E-01   0.2523E-01
    0.750   0.333   0.305   0.834   0.9342E-02   0.2268E-02   0.6985E-02   0.7862E+00   0.1220E+00   0.3291E-01   0.2518E-01
    0.800   0.314   0.287   0.784   0.8627E-02   0.2094E-02   0.6884E-02   0.8340E+00   0.1188E+00   0.3249E-01   0.2483E-01
    0.850   0.300   0.271   0.714   0.9275E-02   0.2260E-02   0.7863E-02   0.8821E+00   0.1277E+00   0.3605E-01   0.2767E-01
    0.900   0.286   0.256   0.614   0.9386E-02   0.2292E-02   0.8426E-02   0.9303E+00   0.1295E+00   0.3524E-01   0.2711E-01
    0.950   0.272   0.243   0.459   0.9775E-02   0.2395E-02   0.9263E-02   0.9788E+00   0.1207E+00   0.3067E-01   0.2367E-01
    1.000   0.264   0.232   0.000   0.3198E-01   0.8559E-02   0.3172E-01   0.1027E+01   0.0000E+00   0.0000E+00   0.0000E+00


   POWER        CT        CP       EFF    VIN       RPM      THRUST   POWER   # BLADES   BETA 3/4  THRUST/
```

```
     0.7407    0.0203    0.0157    0.9565   300.0000 2700.0000  642.5375  366.4038    3.0000    15.0000    1.7536
1
prop.dat
       J        # BLADE       VELOCITY       DIA          DENSITY       BETAIN         BETOUT
    0.296296      3.000000    120.000000   108.000000     0.002378     15.000000     15.000000

     X       BETA     PHI     KAPPA    ALPHAI        WT/WR         WA/WR         VE/WR          CL           DCTDX          DCPDX

    0.300    0.887    0.305   0.976    0.2008E+00    0.3038E-01    0.5489E-01    0.3082E+00    0.2395E+01    0.7747E-01    0.4040E-01
    0.350    0.712    0.263   0.975    0.1732E+00    0.2640E-01    0.5661E-01    0.3571E+00    0.1732E+01    0.9313E-01    0.4776E-01
    0.400    0.604    0.232   0.976    0.1534E+00    0.2357E-01    0.5819E-01    0.4061E+00    0.1376E+01    0.1100E+00    0.5600E-01
    0.450    0.531    0.207   0.979    0.1388E+00    0.2154E-01    0.5985E-01    0.4554E+00    0.1164E+01    0.1285E+00    0.6536E-01
    0.500    0.476    0.186   0.974    0.1252E+00    0.1948E-01    0.6047E-01    0.5048E+00    0.1036E+01    0.1436E+00    0.7265E-01
    0.550    0.436    0.170   0.971    0.1141E+00    0.1780E-01    0.6100E-01    0.5544E+00    0.9574E+00    0.1589E+00    0.8012E-01
    0.600    0.405    0.156   0.969    0.1049E+00    0.1640E-01    0.6144E-01    0.6040E+00    0.9055E+00    0.1744E+00    0.8773E-01
    0.650    0.379    0.144   0.956    0.9706E-01    0.1520E-01    0.6181E-01    0.6537E+00    0.8644E+00    0.1876E+00    0.9423E-01
    0.700    0.356    0.134   0.949    0.8925E-01    0.1393E-01    0.6140E-01    0.7035E+00    0.8348E+00    0.1984E+00    0.9903E-01
    0.750    0.333    0.125   0.930    0.8185E-01    0.1270E-01    0.6048E-01    0.7534E+00    0.7942E+00    0.2038E+00    0.1008E+00
    0.800    0.314    0.117   0.893    0.7456E-01    0.1144E-01    0.5890E-01    0.8033E+00    0.7681E+00    0.2010E+00    0.9815E-01
    0.850    0.300    0.111   0.837    0.7078E-01    0.1090E-01    0.5949E-01    0.8531E+00    0.7471E+00    0.2029E+00    0.9932E-01
    0.900    0.286    0.104   0.747    0.6650E-01    0.1023E-01    0.5926E-01    0.9029E+00    0.7246E+00    0.1904E+00    0.9293E-01
    0.950    0.272    0.099   0.576    0.6607E-01    0.1035E-01    0.6218E-01    0.9526E+00    0.6738E+00    0.1659E+00    0.8246E-01
    1.000    0.264    0.094   0.000    0.1695E+00    0.4414E-01    0.1636E+00    0.9900E+00    0.0000E+00    0.0000E+00    0.0000E+00

   POWER          CT         CP        EFF     VIN         RPM        THRUST    POWER     # BLADES    BETA 3/4   THRUST/

     0.2963     0.1099     0.0548     0.5936  120.0000 2700.0000 3471.1646 1275.8163    3.0000    15.0000     2.7207
1
prop.dat
       J        # BLADE       VELOCITY       DIA          DENSITY       BETAIN         BETOUT
    1.901235      3.000000    770.000000   108.000000     0.002378     15.000000     35.000000

     X       BETA     PHI     KAPPA    ALPHAI        WT/WR         WA/WR         VE/WR          CL           DCTDX          DCPDX

    0.300    1.236    1.111   1.069    0.2585E-01    0.1584E-01    0.7346E-02    0.6752E+00    0.6236E+00    0.4476E-01    0.9093E-01
    0.350    1.061    1.046   0.975    0.3467E-02    0.2103E-02    0.1207E-02    0.6991E+00    0.7049E-01    0.7741E-02    0.1484E-01
    0.400    0.953    0.987   0.904   -0.8676E-02   -0.5220E-02   -0.3516E-02    0.7254E+00   -0.1580E+00   -0.2372E-01   -0.4426E-01
    0.450    0.880    0.931   0.851   -0.1399E-01   -0.8375E-02   -0.6412E-02    0.7541E+00   -0.2375E+00   -0.4559E-01   -0.8418E-01
    0.500    0.826    0.880   0.801   -0.1490E-01   -0.8904E-02   -0.7583E-02    0.7849E+00   -0.2504E+00   -0.5629E-01   -0.1038E+00
    0.550    0.785    0.833   0.755   -0.1291E-01   -0.7719E-02   -0.7199E-02    0.8177E+00   -0.2189E+00   -0.5543E-01   -0.1027E+00
    0.600    0.754    0.790   0.712   -0.9539E-02   -0.5719E-02   -0.5779E-02    0.8522E+00   -0.1646E+00   -0.4590E-01   -0.8562E-01
    0.650    0.728    0.750   0.664   -0.5794E-02   -0.3485E-02   -0.3786E-02    0.8881E+00   -0.1013E+00   -0.3046E-01   -0.5725E-01
    0.700    0.705    0.713   0.616   -0.2031E-02   -0.1222E-02   -0.1420E-02    0.9253E+00   -0.3611E-01   -0.1145E-01   -0.2167E-01
    0.750    0.682    0.679   0.564    0.9013E-03    0.5471E-03    0.6768E-03    0.9637E+00    0.1615E-01    0.5355E-02    0.1020E-01
    0.800    0.663    0.648   0.504    0.3955E-02    0.2406E-02    0.3155E-02    0.1003E+01    0.7304E-01    0.2401E-01    0.4602E-01
    0.850    0.649    0.619   0.438    0.7919E-02    0.4846E-02    0.6693E-02    0.1043E+01    0.1421E+00    0.4730E-01    0.9144E-01
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    0.900   0.635   0.592   0.359    0.1146E-01   0.7056E-02   0.1024E-01   0.1084E+01   0.2001E+00   0.6312E-01   0.1230E+00
    0.950   0.621   0.567   0.255    0.1589E-01   0.9853E-02   0.1494E-01   0.1126E+01   0.2402E+00   0.6968E-01   0.1372E+00
    1.000   0.613   0.544   0.000    0.6838E-01   0.4592E-01   0.6534E-01   0.1166E+01   0.0000E+00   0.0000E+00   0.0000E+00

   POWER        CT        CP        EFF     VIN       RPM       THRUST    POWER    # BLADES   BETA 3/4  THRUST/

    1.9012   -0.0015   -0.0016    1.7728  770.0000 2700.0000  -46.2200  -36.4997    3.0000   35.0000    1.2663
1
prop.dat
       J        # BLADE      VELOCITY      DIA        DENSITY      BETAIN       BETOUT
    1.111111    3.000000   450.000000  108.000000    0.002378    15.000000    35.000000

     X      BETA    PHI     KAPPA    ALPHAI       WT/WR        WA/WR        VE/WR        CL           DCTDX        DCPDX

    0.300   1.236   0.867   1.018    0.8579E-01   0.3240E-01   0.2301E-01   0.4621E+00   0.1775E+01   0.8267E-01   0.1097E+00
    0.350   1.061   0.791   0.953    0.7169E-01   0.2706E-01   0.2319E-01   0.4963E+00   0.1249E+01   0.9087E-01   0.1166E+00
    0.400   0.953   0.724   0.913    0.6525E-01   0.2471E-01   0.2452E-01   0.5328E+00   0.1028E+01   0.1055E+00   0.1336E+00
    0.450   0.880   0.666   0.890    0.6326E-01   0.2411E-01   0.2698E-01   0.5712E+00   0.9441E+00   0.1280E+00   0.1618E+00
    0.500   0.826   0.616   0.853    0.6266E-01   0.2407E-01   0.2986E-01   0.6112E+00   0.9250E+00   0.1520E+00   0.1924E+00
    0.550   0.785   0.571   0.821    0.6324E-01   0.2450E-01   0.3328E-01   0.6526E+00   0.9467E+00   0.1810E+00   0.2302E+00
    0.600   0.754   0.533   0.796    0.6409E-01   0.2507E-01   0.3690E-01   0.6951E+00   0.9880E+00   0.2142E+00   0.2743E+00
    0.650   0.728   0.498   0.757    0.6542E-01   0.2585E-01   0.4089E-01   0.7384E+00   0.1031E+01   0.2472E+00   0.3191E+00
    0.700   0.705   0.468   0.718    0.6611E-01   0.2637E-01   0.4460E-01   0.7826E+00   0.1075E+01   0.2782E+00   0.3617E+00
    0.750   0.682   0.441   0.671    0.6659E-01   0.2680E-01   0.4823E-01   0.8274E+00   0.1101E+01   0.3036E+00   0.3975E+00
    0.800   0.663   0.416   0.613    0.6608E-01   0.2679E-01   0.5117E-01   0.8728E+00   0.1136E+01   0.3164E+00   0.4164E+00
    0.850   0.649   0.394   0.541    0.6907E-01   0.2840E-01   0.5683E-01   0.9185E+00   0.1168E+01   0.3343E+00   0.4460E+00
    0.900   0.635   0.374   0.447    0.7172E-01   0.2990E-01   0.6251E-01   0.9645E+00   0.1188E+01   0.3265E+00   0.4416E+00
    0.950   0.621   0.356   0.318    0.8009E-01   0.3429E-01   0.7350E-01   0.1010E+01   0.1161E+01   0.2962E+00   0.4124E+00
    1.000   0.613   0.340   0.000    0.2727E+00   0.1642E+00   0.2337E+00   0.1022E+01   0.0000E+00   0.0000E+00   0.0000E+00

   POWER        CT        CP        EFF     VIN       RPM       THRUST    POWER    # BLADES   BETA 3/4  THRUST/

    1.1111    0.1508    0.1979    0.8464  450.0000 2700.0000 4763.2266 4604.6177    3.0000   35.0000    1.0344
1
prop.dat
       J        # BLADE      VELOCITY      DIA        DENSITY      BETAIN       BETOUT
    2.716049    3.000000  1100.000000  108.000000    0.002378    15.000000    45.000000

     X      BETA    PHI     KAPPA    ALPHAI       WT/WR        WA/WR        VE/WR        CL           DCTDX        DCPDX

    0.300   1.410   1.237   1.126    0.3296E-01   0.2880E-01   0.8943E-02   0.9146E+00   0.8824E+00   0.8191E-01   0.2486E+00
    0.350   1.236   1.186   1.011    0.1078E-01   0.9354E-02   0.3670E-02   0.9327E+00   0.2436E+00   0.3495E-01   0.9794E-01
    0.400   1.127   1.137   0.911   -0.2390E-02  -0.2059E-02  -0.9586E-03   0.9526E+00  -0.4776E-01  -0.9344E-02  -0.2522E-01
    0.450   1.054   1.091   0.834   -0.9308E-02  -0.8007E-02  -0.4263E-02   0.9746E+00  -0.1722E+00  -0.4268E-01  -0.1133E+00
    0.500   1.000   1.046   0.771   -0.1188E-01  -0.1019E-01  -0.6059E-02   0.9986E+00  -0.2169E+00  -0.6219E-01  -0.1644E+00
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    0.550   0.960   1.004   0.712   -0.1131E-01   -0.9707E-02   -0.6331E-02   0.1025E+01   -0.2072E+00   -0.6597E-01   -0.1748E+00
    0.600   0.928   0.964   0.655   -0.9064E-02   -0.7787E-02   -0.5509E-02   0.1052E+01   -0.1668E+00   -0.5764E-01   -0.1536E+00
    0.650   0.902   0.926   0.599   -0.6024E-02   -0.5183E-02   -0.3946E-02   0.1082E+01   -0.1117E+00   -0.4101E-01   -0.1100E+00
    0.700   0.880   0.890   0.546   -0.2643E-02   -0.2280E-02   -0.1856E-02   0.1112E+01   -0.4970E-01   -0.1897E-01   -0.5124E-01
    0.750   0.857   0.856   0.492    0.2092E-03    0.1458E-03    0.1264E-03   0.1145E+01    0.3187E-02    0.1255E-02    0.3411E-02
    0.800   0.838   0.824   0.433    0.3343E-02    0.2898E-02    0.2664E-02   0.1178E+01    0.6437E-01    0.2483E-01    0.6789E-01
    0.850   0.824   0.794   0.370    0.7568E-02    0.6592E-02    0.6383E-02   0.1212E+01    0.1403E+00    0.5417E-01    0.1494E+00
    0.900   0.810   0.765   0.299    0.1155E-01    0.1010E-01    0.1028E-01   0.1248E+01    0.2071E+00    0.7492E-01    0.2082E+00
    0.950   0.796   0.738   0.208    0.1673E-01    0.1473E-01    0.1565E-01   0.1284E+01    0.2563E+00    0.8434E-01    0.2369E+00
    1.000   0.787   0.713   0.000    0.7425E-01    0.6946E-01    0.6922E-01   0.1318E+01    0.0000E+00    0.0000E+00    0.0000E+00


  POWER        CT        CP       EFF    VIN        RPM       THRUST    POWER    # BLADES   BETA 3/4  THRUST/

    2.7160     0.0009    0.0048    0.5006 1100.0000 2700.0000   27.8184  111.1414    3.0000    45.0000    0.2503
1
prop.dat
       J        # BLADE       VELOCITY      DIA         DENSITY      BETAIN       BETOUT
    1.901235     3.000000    770.000000   108.000000    0.002378    15.000000    45.000000


    X       BETA     PHI     KAPPA    ALPHAI        WT/WR         WA/WR         VE/WR         CL            DCTDX         DCPDX

   0.300   1.410   1.111   1.085    0.6036E-01    0.3753E-01    0.1586E-01    0.6742E+00    0.1503E+01    0.9981E-01    0.2226E+00
   0.350   1.236   1.046   0.980    0.4373E-01    0.2710E-01    0.1413E-01    0.6984E+00    0.9141E+00    0.9327E-01    0.1967E+00
   0.400   1.127   0.987   0.906    0.3529E-01    0.2183E-01    0.1335E-01    0.7250E+00    0.6624E+00    0.9287E-01    0.1909E+00
   0.450   1.054   0.931   0.845    0.3249E-01    0.2012E-01    0.1397E-01    0.7538E+00    0.5671E+00    0.1021E+00    0.2079E+00
   0.500   1.000   0.880   0.790    0.3203E-01    0.1988E-01    0.1538E-01    0.7846E+00    0.5513E+00    0.1170E+00    0.2375E+00
   0.550   0.960   0.833   0.741    0.3370E-01    0.2100E-01    0.1783E-01    0.8173E+00    0.5849E+00    0.1405E+00    0.2860E+00
   0.600   0.928   0.790   0.695    0.3650E-01    0.2287E-01    0.2108E-01    0.8516E+00    0.6428E+00    0.1709E+00    0.3496E+00
   0.650   0.902   0.750   0.645    0.3979E-01    0.2509E-01    0.2488E-01    0.8874E+00    0.7089E+00    0.2040E+00    0.4200E+00
   0.700   0.880   0.713   0.595    0.4289E-01    0.2721E-01    0.2888E-01    0.9245E+00    0.7782E+00    0.2369E+00    0.4910E+00
   0.750   0.857   0.679   0.543    0.4559E-01    0.2911E-01    0.3288E-01    0.9627E+00    0.8320E+00    0.2655E+00    0.5538E+00
   0.800   0.838   0.648   0.484    0.4771E-01    0.3065E-01    0.3673E-01    0.1002E+01    0.8947E+00    0.2840E+00    0.5955E+00
   0.850   0.824   0.619   0.417    0.5273E-01    0.3421E-01    0.4306E-01    0.1042E+01    0.9571E+00    0.3080E+00    0.6535E+00
   0.900   0.810   0.592   0.339    0.5735E-01    0.3759E-01    0.4951E-01    0.1083E+01    0.1008E+01    0.3077E+00    0.6605E+00
   0.950   0.796   0.567   0.239    0.6672E-01    0.4448E-01    0.6051E-01    0.1124E+01    0.1017E+01    0.2849E+00    0.6250E+00
   1.000   0.787   0.544   0.000    0.2429E+00    0.1991E+00    0.1985E+00    0.1135E+01    0.0000E+00    0.0000E+00    0.0000E+00

  POWER        CT        CP       EFF    VIN        RPM       THRUST    POWER    # BLADES   BETA 3/4  THRUST/

    1.9012     0.1329    0.2790    0.9056  770.0000 2700.0000 4198.2573 6489.9839    3.0000    45.0000    0.6469
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Session Name:roger

roger.ecn.purdue.edu%  more prop.dat
15,0,0
.30,.101,50.8,.37
.35,.122,40.8,.24
.40,.138,34.6,.175
.45,.150,30.4,.14
.50,.152,27.3,.119
.55,.150,25.0,.109
.60,.146,23.2,.102
.65,.140,21.7,.097
.70,.132,20.4,.095
.75,.124,19.1,.090
.80,.111,18.0,.087
.85,.102,17.2,.085
.90,.088,16.4,.082
.95,.074,15.6,.081
1.0,.064,15.1,.080
2700.,3.,300.,108.,.002378,15.,15.
2700.,3.,120.,108.,.002378,15.,15.
2700.,3.,770.,108.,.002378,15.,35.
2700.,3.,450.,108.,.002378,15.,35.
2700.,3.,1100.,108.,.002378,15.,45.
2700.,3.,770.,108.,.002378,15.,45.
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Sample input file to program gold

gold.f FORTRAN Listing

```
C       REVISED 12/12/85   JPS
C   AERODYNAMIC PROPELLER ANALYSIS
C   METHOD- STRIP THEORY WITH GOLDSTEIN INDUCTION FACTOR
C   SEE NOTES FOR DERIVATION AND EQUATIONS
C
C   INPUT FORMAT
C   CARD #1
C   M,IPRINT        (FORMAT(3I10)
C   CARD#2,M+1
C   X,COR,BETA,TOC,CD   (FORMAT(7F13.6))
C   CARD #M+2
C   RPM,B,VIN,DIA,RHO     (FORMAT(7F13.6))
C
C
C   VARIABLE DEFINITIONS
C         M - NUMBER OF RADIAL STATIONS
C         IPRINT - PRINT PARAMETER ( =0,FULL OUTPUT ;=1,
SUMMARY OUTPUT ONLY)
C         X - DIMENSIONLESS RADIAL POSITION,R/R,(MUST BE
.3,.35,.4,.45--.95,1.)
C         COR - CHORD OVER RADIUS, C/R
C         BETA - TWIST DISTRIBUTION- ANGLE PLANE OF ROTATION TO
ZERO LIFT LINE
C         TOC - THICKNESS OVER CHORD, T/C
C         CD - DRAG COEFFCIENT
C         RPM - REVOLUTIONS PER MINUTE
C         B - NUMBER OF BLADES
C         VIN - VELOCITY AT INFINITY ( FLIGHT SPEED )
(FEET/SECOND)
C         DIA - DIAMETER (inches)
C         RHO - DENSITY OF AIR (SLUGS PER CUBIC FOOT) (.002378
AT STP)
C         BETAIN- BETA 3/4 OF INPUT TWIST DISTRIBUTION
C         BETOUT- BETA 3/4 DESIRED FOR CALCULATION
C
C
        PROGRAM GOLD
        DIMENSION DCTDX(20),DCPDX(20),GUESS(20),ERROR(20)
        DIMENSION
X(20),COR(20),BETA(20),TOC(20),CD(20),CL(20)
        DIMENSION BETAI(20),HEADER(20)
        character*64 filnam,fnam2
        character IY
        REAL J,KAPPA,KAPPA2,KAPPA3,KAPPA4
        INTEGER OUTPUT
        DATA INPUT,OUTPUT,PI,DEG2RA/12,11,3.14159,.017453/
C
        open(11,file='gold.out')
C
C   GET NAME AND OPEN FILE FOR INPUT
C
        WRITE(*,5000)
5000    FORMAT(2X,'FILE NAME OF INPUT DATA ?  ',/)
        READ(*,5001)FILNAM
```

```
 5001    FORMAT(A)
         open(INPUT,file=FILNAM)
         WRITE(OUTPUT,5002)FILNAM
 5002    FORMAT(5X,'NAME OF INPUT DATA FILE -  ',A)
         WRITE(*,5800)
 5800    FORMAT(5X,' INPUT COMMENTS FOR HEADER OF PRINTED
OUTPUT
     1   -80 CHARS MAX')
         READ(*,5801)(HEADER(I), I=1,20)
 5801    FORMAT(20A4)
         WRITE(OUTPUT,5802)(HEADER(I), I=1,20)
 5802    FORMAT(5X,20A4)
C
C   GET NAME AND OPEN DISC FILE FOR OUTPUT
C
         WRITE(*,5010)
 5010    FORMAT(5X,'DO YOU WANT OUTPUT WRITTEN TO DISC ? (y OR
n)')
         READ(*,5011)IY
 5011    FORMAT(A)
         IF(IY.Ne.'y')gO TO 5012
         WRITE(*,5013)
 5013    FORMAT(5X,'INPUT FILE NAME FOR OUTPUT',/)
         IFILE=13
         read(*,5001)fnam2
         open(IFILE,file=fnam2,status='new')
 5012    CONTINUE
C
C
         READ (INPUT,1000)M,IPRINT
 1000    FORMAT(2I10)
         WRITE(OUTPUT,6000)
 6000    FORMAT(20X,'INPUT DATA',//)
         WRITE(OUTPUT,2000)M,IPRINT
 2000    FORMAT(5X,'NUMBER OF RADIAL STATIONS = ',I5,'  PRINT
PARAMETER
     1   =  ',I5)
         WRITE(OUTPUT,5003)
5003
FORMAT(8X,'X',10X,'COR',11X,'BETA',12X,'TOC',10X,'CD')
         DO 10 I=1,M
         READ(INPUT,1100)X(I),COR(I),BETAI(I),TOC(I),CD(I)
         WRITE(OUTPUT,2100)X(I),COR(I),BETAI(I),TOC(I),CD(I)
10       CONTINUE
1100     FORMAT(7F13.6)
2100     FORMAT(1X,7F13.6)
         IF(IPRINT.EQ.1)WRITE(OUTPUT,6200)
20
READ(INPUT,1100,END=500)RPM,B,VIN,DIA,RHO,BETAIN,BETOUT
C
C   ROTATE BLADE TO DESIRED BETA 3/4 (BETOUT) FROM
C   THE BETA 3/4 OF THE INPUT TWIST DISTRIBUTION (BETAIN)
C
         DO 23 I=1,M
```

```
 23      BETA(I)=( BETAI(I) +BETOUT - BETAIN)*DEG2RA
 C
 C
         IB=IFIX(B+.5)
         RPS=RPM/60.
         J=VIN/RPS/DIA*12.
         VINWR=J/PI
         IF (IPRINT.EQ.1)GO TO 21
        WRITE(OUTPUT,5803)(HEADER(I), I=1,20)
 5803    FORMAT('1',20A4)
        WRITE(OUTPUT,5001)FILNAM
        WRITE(OUTPUT,5004)
 5004   FORMAT(8X,'J',7X,'# BLADE',7X,'VELOCITY',7X,'DIA',9X,
     1  'DENSITY',7X,'BETAIN',10X,'BETOUT')
       WRITE(OUTPUT,2100)J,B,VIN,DIA,RHO,BETAIN,BETOUT
        WRITE(OUTPUT,6100)
 6100    FORMAT(//,5X,'X      BETA     PHI     KAPPA
ALPHAI',9X,
     1
'WT/WR',11X,'WA/WR',8X,'VE/WR',10X,'CL',10X,'DCTDX',10X,'DCPD
X'
     2  ,/)
21      CONTINUE
C
C   BEGIN CALCULATION
C
        DO 30 I=1,M
        PHI=ATAN(VINWR/X(I))
C
C  SEE FIG. 4 FOR ANGLE DEFINITIONS
C  SIGMA - SOLIDITY
C
        SIGMA=B*COR(I)/PI
C
C   LAST STATION AT TIP
        IF (I.EQ.15) GO TO 65
C  BEGIN ITERATION
        ITER=1
C
C  FIRST GUESS SET TO AVOID SQRT OF NEGATIVE NUMBER
       GUESS(ITER)=(X(I)-SQRT(X(I)**2+VINWR**2))/2 + .00001
40      WTWR=GUESS(ITER)
C  EQUATION #4
c        print *,X(I),WTWR,ITER,kappa
       ARG=VINWR**2+4.*WTWR*(X(I)-WTWR)
        IF(ARG)95,95,96
95      WAWR=WTWR*10
        GO TO 97
96     WAWR=.5*(-VINWR+SQRT(VINWR**2+4.*WTWR*(X(I)-WTWR)))
97      CONTINUE
C  EQUATION #5
       VEWR=SQRT((VINWR+WAWR)**2+(X(I)-WTWR)**2)
C
       ALPHAI=ATAN(WTWR/WAWR)-PHI
```

```
C  EQUATION #3
         CL(I)=2*PI*(BETA(I)-ALPHAI-PHI)
         PHIALP=PHI+ALPHAI
C OBTAIN GOLDSTEIN KAPPA FACTOR FROM SUBFUNCTION
KAPPA2,KAPPA3,KAPPA4
         GO TO (41,42,43) IB-1
41       KAPPA=KAPPA2(X(I),PHIALP)
c        print *,kappa
         GO TO 44
42       KAPPA=KAPPA3(X(I),PHIALP)
         GO TO 44
43       KAPPA=KAPPA4(X(I),PHIALP)
44       CONTINUE
C  EQUATION #2
         ERROR(ITER)=SIGMA*CL(I)*VEWR-8.*X(I)*KAPPA*WTWR
         IF(ABS(ERROR(ITER))-.00001)60,60,50
50       ITER=ITER+1
         IF(ITER.GE.20)GO TO 60
C NO MORE THAN 20 ITERATIONS ( NORMALLY 4-6)
         IF(ITER.GT.2)GO TO 55
C SECOND GUESS ONLY
         GUESS(ITER)=GUESS(ITER-1)+.01
         GO TO 40
55       CONTINUE
C  NEWTON'S METHOD FOR GUESS> 2
         SLOPE=(ERROR(ITER-2)-ERROR(ITER-1))/(GUESS(ITER-2)-
GUESS(ITER-1))
C
         BINTER=ERROR(ITER-2)-SLOPE*GUESS(ITER-2)
         GUESS(ITER)=-BINTER/SLOPE
         GO TO 40
C
C   AT X=1.0 CL=0.0
C
65       CL(I)=0.0
         ALPHAI=BETA(I)-PHI
         VRWR=SQRT(VINWR**2+1)
         WAWR=VRWR*SIN(ALPHAI)*COS(ALPHAI+PHI)
         WTWR=VRWR*SIN(ALPHAI)*SIN(PHI+ALPHAI)
         VEWR=SQRT((VINWR+WAWR)**2+(X(I)-WTWR)**2)
         KAPPA=0.0
60       CONTINUE
         WRITE(*,2200)ITER
2200     FORMAT(1X,'THE NUMBER OF ITERATIONS =  ',I10)
         C1=PI*SIGMA*(J**2+(PI*X(I))**2)/8.
         COS1=COS(PHI+ALPHAI)
         SIN1=SIN(PHI+ALPHAI)
C  EQUATION #1A  (INTEGRAND ONLY)
         DCTDX(I)=C1*(CL(I)*COS1-CD(I)*SIN1)
C  EQUATION #1B  (INTEGRAND ONLY)
         DCPDX(I)=C1*PI*X(I)*(CL(I)*SIN1+CD(I)*COS1)
         IF (IPRINT.EQ.1)GO TO 30
C  OUTPUT RADIAL DISTRIBUTION INFORMATION (SKIP IF IPRINT=1)
```

```
WRITE(OUTPUT,2300)X(I),BETA(I),PHI,KAPPA,ALPHAI,WTWR,WAWR,VEW
R
     1   ,CL(I),DCTDX(I),DCPDX(I)
C  WRITE OUTPUT TO DISC  IF FILE OPENED
        IF(IY.NE.'y')GO TO 30

WRITE(IFILE,2350)X(I),BETA(I),PHI,KAPPA,ALPHAI,WTWR,WAWR,VEWR
     1   ,CL(I),DCTDX(I),DCPDX(I)
2300    FORMAT(1X,4(2X,F6.3),7(3X,E11.4))
2350    FORMAT(12E11.4)
30      CONTINUE
C
C  END OF MAIN CALUCLATION LOOP
C
        IF(IPRINT.EQ.0)WRITE(OUTPUT,6200)
6200
FORMAT(//,7X,'J',9X,'CT',9X,'CP',7X,'EFF',4X,'VIN',8X,'RPM',
     1  7X,'THRUST',4X,'POWER',4X,'# BLADES',4X,'BETA
3/4',2X,'THRUST/
     2  POWER',/)
C
C  INTERGRATE USING TRAPEZOID RULE FOR TOTL POWER AND THRUS
C
        CTSUM=0.0
        CPSUM=0.0
        DO 70 I=2,M-1
        CPSUM=CPSUM+DCPDX(I)
        CTSUM=CTSUM+DCTDX(I)
70      CONTINUE
        STEP=X(2)-X(1)
        CP=STEP*(CPSUM+(DCPDX(1)+DCPDX(M))/2.)
        CT=STEP*(CTSUM+(DCTDX(1)+DCTDX(M))/2.)
        ETA=CT*J/CP
        C10=RHO*(RPS)**2*(DIA/12.)**4
        THRUST=CT*C10
        POWER=CP*C10*RPS*DIA/550./12.
        IF (POWER)73,72,73
72      TOHP=0.0
        GO TO 74
73      TOHP=THRUST/POWER
74      CONTINUE
        write(20,2401)j
        write(21,2401)ct
        write(22,2401)cp
        write(23,2401)eta
2401    format(f10.6)

WRITE(OUTPUT,2400)J,CT,CP,ETA,VIN,RPM,THRUST,POWER,B,BETOUT,T
OHP
        IF (IY.NE.'y')GO TO 71

WRITE(IFILE,2350)J,CT,CP,ETA,VIN,RPM,THRUST,POWER,B,BETOUT,TO
HP
```

```
2400    FORMAT(1X,12F10.4,/)
71      CONTINUE
        GO TO 20
500     STOP
        END
        FUNCTION KAPPA2(X,PHIALP)
C
C   RETURNS THE GOLDSTEIN KAPPA FACTOR FOR TWO BLADED
PROPELLERS
C
        REAL KAPPA2
        DIMENSION F(11,14)
        DATA F/
     1
1.00,1.00,.994,.978,.958,.944,.930,.922,.916,.935,1.02,
     2
1.00,1.00,.993,.961,.940,.912,.880,.855,.837,.843,.891,
     3
1.00,.999,.992,.955,.923,.880,.833,.788,.757,.750,.762,
     4
1.00,.998,.991,.959,.906,.848,.784,.722,.677,.657,.633,
     5
1.00,.998,.981,.930,.865,.794,.725,.663,.618,.592,.563,
     6
1.00,.997,.971,.902,.824,.742,.666,.605,.560,.528,.494,
     7
1.00,.997,.961,.874,.783,.690,.608,.547,.502,.464,.425,
     8
.999,.992,.931,.824,.723,.627,.550,.494,.448,.412,.375,
     9
.999,.988,.901,.774,.663,.564,.492,.441,.398,.360,.325,
     1
.997,.971,.852,.709,.595,.501,.435,.387,.347,.311,.279,
     2
.994,.937,.784,.634,.520,.434,.376,.333,.297,.265,.238,
     3
.985,.877,.694,.548,.442,.367,.316,.278,.247,.220,.197,
     4
.950,.773,.578,.444,.351,.289,.249,.218,.193,.172,.154,
     5
.780,.586,.415,.308,.243,.199,.171,.149,.131,.117,.105
     6  /
C
C
        KAPPA2=0.0
        IRAD=nint((100*X-25.)/5.)
c       print *,' irad = ',irad
        IF (IRAD.EQ.15)GO TO 5
        SINPHI=SIN(PHIALP)
        IF (SINPHI-.1)10,10,20
C
C  SINPHI LESS THAN .1
C
10      IF (SINPHI-.05)11,11,12
```

```
11      KAPPA2=F(1,IRAD)
        GO TO 5
12      KAPPA2=(F(2,IRAD)-F(1,IRAD))*(SINPHI-.05)/.05 +
F(1,IRAD)
        GO TO 5
20      ISINPH=IFIX(SINPHI*10.)+1.
        KAPPA2=(F(ISINPH+1,IRAD)-F(ISINPH,IRAD))*(SINPHI-
FLOAT(
     1  ISINPH-1)/10.)/.1 + F(ISINPH,IRAD)
5       CONTINUE
        RETURN
        END

        FUNCTION KAPPA3(X,PHIALP)
C
C   RETURNS THE GOLDSTEIN KAPPA FACTOR FOR TWO BLADED
PROPELLERS
C
        REAL KAPPA3
        DIMENSION F(11,14)
        DATA F/
     1
1.00,1.00,.997,.992,.984,.975,.973,.983,1.01,1.06,1.18,
     2
1.00,1.00,.996,.990,.978,.963,.952,.949,.956,.984,1.07,
     3
1.00,.999,.996,.989,.972,.952,.930,.914,.903,.908,.950,
     4
1.00,.999,.995,.987,.966,.940,.909,.880,.849,.831,.832,
     5
1.00,.999,.993,.976,.945,.905,.863,.825,.786,.761,.751,
     6
1.00,.999,.990,.966,.923,.871,.817,.769,.724,.692,.669,
     7
1.00,.999,.988,.955,.902,.836,.771,.714,.661,.622,.588,
     8
1.00,.999,.976,.924,.856,.781,.711,.650,.597,.558,.523,
     9
1.00,.998,.964,.892,.809,.725,.650,.586,.533,.494,.457,
     1
.999,.994,.935,.843,.746,.658,.582,.520,.470,.429,.393,
     2
.998,.980,.884,.774,.670,.581,.508,.450,.404,.367,.335,
     3
.990,.948,.810,.684,.581,.496,.429,.377,.337,.304,.278,
     4
.973,.872,.693,.566,.471,.396,.341,.299,.265,.239,.218,
     5
.863,.692,.512,.406,.331,.275,.236,.203,.182,.164,.149
     6  /
C
C
        KAPPA3=0.0
        IRAD=nint((100*X-25.)/5.)
```

```
        IF (IRAD.EQ.15)GO TO 5
        SINPHI=SIN(PHIALP)
        IF (SINPHI-.1)10,10,20
C
C  SINPHI LESS THAN .1
C
10      IF (SINPHI-.05)11,11,12
11      KAPPA3=F(1,IRAD)
        GO TO 5
12      KAPPA3=(F(2,IRAD)-F(1,IRAD))*(SINPHI-.05)/.05 +
F(1,IRAD)
        GO TO 5
20      ISINPH=IFIX(SINPHI*10.)+1.
        KAPPA3=(F(ISINPH+1,IRAD)-F(ISINPH,IRAD))*(SINPHI-
FLOAT(
     1  ISINPH-1)/10.)/.1 + F(ISINPH,IRAD)
5       CONTINUE
        RETURN
        END


        FUNCTION KAPPA4(X,PHIALP)
C
C   RETURNS THE GOLDSTEIN KAPPA FACTOR FOR TWO BLADED
PROPELLERS
C
        REAL KAPPA4
        DIMENSION F(11,14)
        DATA F/
     1
1.00,1.00,.998,.996,.991,.985,.986,.999,1.03,1.10,1.25,
     2
1.00,1.00,.998,.995,.989,.980,.975,.979,.998,1.04,1.14,
     3
1.00,1.00,.997,.995,.986,.976,.965,.959,.961,.979,1.03,
     4
1.00,1.00,.997,.994,.984,.971,.954,.939,.924,.917,.923,
     5
1.00,1.00,.996,.991,.973,.949,.921,.894,.868,.849,.842,
     6
1.00,1.00,.996,.987,.961,.927,.888,.848,.812,.781,.762,
     7
1.00,1.00,.995,.984,.950,.905,.855,.803,.756,.713,.681,
     8
1.00,1.00,.992,.965,.917,.859,.800,.743,.692,.647,.612,
     9
1.00,1.00,.989,.945,.883,.812,.745,.682,.627,.581,.543,
     1
1.00,.999,.973,.909,.830,.750,.678,.615,.562,.516,.477,
     2
.999,.993,.940,.852,.759,.674,.601,.541,.490,.447,.412,
     3
.998,.985,.882,.774,.671,.585,.517,.459,.413,.375,.345,
```

```
     4
.995,.943,.777,.651,.554,.476,.414,.369,.329,.298,.272,
     5
.945,.770,.590,.476,.396,.334,.290,.255,.228,.205,.187
     6  /
C
C
        KAPPA4=0.0
        IRAD=nint((100*X-25.)/5.)
        IF (IRAD.EQ.15)GO TO 5
        SINPHI=SIN(PHIALP)
        IF (SINPHI-.1)10,10,20
C
C  SINPHI LESS THAN .1
C
10      IF (SINPHI-.05)11,11,12
11      KAPPA4=F(1,IRAD)
        GO TO 5
12      KAPPA4=(F(2,IRAD)-F(1,IRAD))*(SINPHI-.05)/.05 +
F(1,IRAD)
        GO TO 5
20      ISINPH=IFIX(SINPHI*10.)+1.
       KAPPA4=(F(ISINPH+1,IRAD)-F(ISINPH,IRAD))*(SINPHI-
FLOAT(
     1  ISINPH-1)/10.)/.1 + F(ISINPH,IRAD)
5       CONTINUE
        RETURN
        END
```

```
% Matlab Code to perform propeller analysis in a manner
similar to gold.f

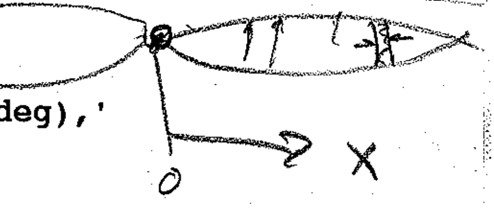
echo off
format compact
clear
disp(' ')
disp('Start new run')
disp('Propeller Analysis using Goldsteins Classical Vortex
Theory')
disp('  This code works for two bladed propellers only.')
disp(' ')

% Input constants
Din=12;        % propeller diameter (inches)
Pin=6;        % pitch of the propeller (inches)
RPM=4000;      % RPM of propeller
Vmph=0 ;     % airspeed in mph
rho=.002378; % air density (slug/ft**2) (sea level)
%rho=.002065; % air density (slug/ft**2) at 4750 feet

% Pin gives geometric angle to the flat part of the
%   rear of the propeller
aoldeg=-6;     % angle of zero lift of the propeller (degrees)
               %  measured from mean chord line (typically
negative)
beta0deg=.5; % angle from flat part of the prop to mean
chord line
a=2*pi;        % lift curve slope of propeller
Cd0=.00655;    % 2-d minimum drag coefficient
k=.01;         % Cd = Cd0+k*Cl*Cl
B=2 ;          % number of blades (2 for standard type
propeller)


% input nondimensional properties at each radial location
% cR=c/R, x=r/R
x=[.3,.35,.4,.45,.5,.55,.6,.65,.7,.75,.8,.85,.9,.95,1.];
cR=.09*ones(size(x));
% END OF INPUTS

% Display inputs section
disp(['Diameter Din= ',num2str(Din),' inches'])
disp(['Pitch Pin= ',num2str(Pin),' inches'])
disp(['Angle of zero lift aoldeg= ',num2str(aoldeg),'
degrees'])
disp(['Angle Flat side to mean chord beta0deg=
',num2str(beta0deg),' degrees'])
disp(['RPM= ',num2str(RPM),' rpm'])
disp(['2-d Lift curve slope a= ',num2str(a),' per rad'])
disp(['2-d Min drag coef Cd0= ',num2str(Cd0)])
disp(['k from 2-d drag polar= ',num2str(k)])
disp(['Number of blades B= ',num2str(B)])
disp(['air density rho= ',num2str(rho),' slug/ft^3'])
```



```
disp(['Airspeed Vmph= ',num2str(Vmph),' mph'])


% derived constants
V=Vmph*88/60; % airspeed in ft/sec
D=Din/12;     % Diameter in feet
R=D/2;        % Radius in feet
n=RPM/60;     % propeller frequency (rev/sec) or (hz)
omega=2*pi*n; % frequency of revolution of the propeller
(rad/sec)
lamda=V/(omega*R);
r2d=180/pi;
Vt=omega*R;   % tip velocity (ft/sec)
J=V/(n*D);    % advance ratio

% Output scalar constants
disp(['Airspeed V= ',num2str(V),' ft/sec'])
disp(['Propeller Diameter D= ',num2str(D),' feet'])
disp(['Propeller Radius R= ',num2str(R),' feet'])
disp(['propeller RPS n= ',num2str(n),' hertz'])
disp(['omega= ',num2str(omega),' rad/sec'])
disp(['lamda= ',num2str(lamda)])
disp(['r2d= ',num2str(r2d),' deg/rad'])
disp(['Tip speed Vt= ',num2str(Vt),' ft/sec'])
disp(['Advance Ratio J= ',num2str(J)])
disp(' ')

%derived section constants
c=R*cR;    % chord in feet
cin=c*12;  % chord in inches
disp(' ')
disp('x in r/R and is nondimensional, cR=c/R, cin in the
chord in inches')
echo on
disp([x',      cR',       cin'])
echo off
disp(' ')

beta1=atan(((Pin/Din)/pi)./x);
beta=beta1+(beta0deg-aoldeg)/r2d;
sigma=B*c/(pi*R);
r=x*R;
Vr=Vt*sqrt(x.*x+lamda*lamda);
phi=atan(lamda./x);
WtVt=.02*ones(size(c));  %initial guess
nr=length(c);
nr1=nr-1;
aiold=zeros(size(c));
for ii=1:40
     WaVt(1:nr1)=.5*(-
lamda+sqrt(lamda*lamda+4*WtVt(1:nr1).*(x(1:nr1)-
WtVt(1:nr1))));
     ai(1:nr1)=atan(WtVt(1:nr1)./WaVt(1:nr1))-phi(1:nr1);
```

```
     %ai(1:nr1)=atan((V+WaVt(1:nr1)*Vt)./(omega*r(1:nr1)-
WtVt(1:nr1)*Vt))-phi(1:nr1);
     e=sum(abs(ai(1:nr1)-aiold(1:nr1)));
     iter=['Loop index= ',num2str(ii),'  error=
',num2str(e)];
     disp(iter)
     if e<.0001 ; break; end
     aiold(1:nr1)=ai(1:nr1);
     Cl(1:nr1)=a*(beta(1:nr1)-ai(1:nr1)-phi(1:nr1));
     VeVt(1:nr1)=sqrt((lamda+WaVt(1:nr1)).^2+(x(1:nr1)-
WtVt(1:nr1)).^2);
     gamma(1:nr1)=.5*c(1:nr1).*Cl(1:nr1).*VeVt(1:nr1)*Vt;
     sinphialp(1:nr1)=sin(phi(1:nr1)+ai(1:nr1));
     kappa(1:nr1)=kappa2(x(1:nr1),sinphialp(1:nr1));
     WtVt(1:nr1)=B*gamma(1:nr1)./(4*pi*Vt*r(1:nr1).*kappa(1:
nr1));
end
Cl(nr)=0;
ai(nr)=beta(nr)-phi(nr);
VrVt=sqrt(lamda*lamda+1);
WaVt(nr)=VrVt*sin(ai(nr))*cos(ai(nr)+phi(nr));
WtVt(nr)=VrVt*sin(ai(nr))*sin(ai(nr)+phi(nr));
VeVt(nr)=sqrt((lamda+WaVt(nr))^2+(x(nr)-WtVt(nr))^2);
kappa(nr)=0;

Cd=Cd0+k*Cl.*Cl;
ZT=(pi/8)*(J*J+pi*pi*(x.*x)).*sigma;
ZP=pi*ZT.*x;
dCTdx=ZT.*(Cl.*cos(phi+ai)-Cd.*sin(phi+ai));
dCPdx=ZP.*(Cl.*sin(phi+ai)+Cd.*cos(phi+ai));

% Overall propeller performance
CT=trapi(dCTdx,x);
CP=trapi(dCPdx,x);
eta=CT*J/CP;
T=CT*rho*n^2*D^4;
P=CP*rho*n^3*D^5;
HP=P/550;
Pwatt=1.356*P;
torque=P/omega;
Clmax=max(Cl);
Toz=T*16;
PestWatts=1.31*D^4*(Pin/12)*(RPM/1000)^3;
% The above approximate formula works for
% Top Flite, Zinger and Master Airscrews reasonably well.
% For Rev Up props subract .5 in from the pitch.
% For APC props use constant 1.11 instead of 1.31.
% For thin carbon fiber folding props use 1.18 instead of
1.31.
% Ref: Electric Motor Handbook, by Robert J. Boucher,
%    AstroFlight, Inc.

echo on
```

```
dat=[ x',      beta',     phi',      kappa',      ai',
WtVt']
disp(' ')
dat2=[x',      WaVt',     VeVt',     Cl',      dCTdx',
dCPdx']
echo off

disp(' ')
disp(['Advance Ratio J= ',num2str(J)])
disp(['Thrust Coefficient CT= ',num2str(CT)])
disp(['Power Coefficient CP= ',num2str(CP)])
disp(['Propeller efficiency eta= ',num2str(eta)])
disp(['Speed V= ',num2str(V),' ft/sec'])
disp(['RPM= ',num2str(RPM),' rpm'])
disp(['Thrust T= ',num2str(T),' pounds'])
disp(['Thrust Toz= ',num2str(Toz),' ounces'])
disp(['Power used P= ',num2str(P),' ft*lbf/sec'])
disp(['Horsepower used HP= ',num2str(HP),' HP'])
disp(['Power used Pwatt= ',num2str(P),' watts'])
disp(['Torque used Q= ',num2str(torque),' ft*lbf'])
disp(['Clmax= ',num2str(Clmax)])
disp(' ')
disp(['Estimated power used, PestWatts=
',num2str(PestWatts),' watts, Ref: Boucher'])
disp(' ')

subplot(211)
plot(x,dCTdx)
z=axis;
axis([0,1,0,z(4)])
xlabel('nondimensional radial location')
ylabel('dCTdx')
subplot(212)
plot(x,dCPdx)
z=axis;
axis([0,1,0,z(4)])
xlabel('nondimensional radial location')
ylabel('dCPdx')

disp(' For model aircraft propellers this code
underestimates')
disp(' the power required. The underestimation is worse for
RPM>10,000')
disp(' where it may underestimate by a factor of .5')
disp(' For RPM <10,000 the factor is about .75')
```

Appendix

Master Airscrew 9X8

Radius = 4.5

% Radius	Radius(in.)	Chord(in.)	Thickness(in.)	Chord/Radius	Thickness/Chord	Radial Dist.(in.)
0.35	1.58	0.90	0.18	0.57	0.19	9.90
0.50	2.25	0.86	0.14	0.38	0.16	14.14
0.65	2.93	0.79	0.12	0.27	0.15	18.38
0.80	3.60	0.69	0.11	0.19	0.16	22.62
0.95	4.28	0.58	0.08	0.14	0.14	26.86

Hieght	Mean Chord Angle (deg.)	aoi correction	Measured Beta(deg.)	Theoretical Pitch(in.)	Theoretical Beta(deg.)
0.35	23.02	6.00	31.02	8.00	38.95
0.28	19.12	6.00	27.12	8.00	29.50
0.25	18.00	6.00	26.00	8.00	23.52
0.20	16.85	6.00	24.85	8.00	19.48
0.14	13.91	6.00	21.91	8.00	16.59

Table 1

Master Airscrew 9X6

Radius = 4.5

% Radius	Radius(In.)	Chord(In.)	Thickness(In.)	Chord/Radius	Thickness/Chord	Radial Dist.(In.)
0.35	1.58	0.85	0.16	0.54	0.19	9.90
0.50	2.25	0.83	0.15	0.37	0.18	14.14
0.65	2.93	0.74	0.12	0.25	0.17	18.38
0.80	3.60	0.65	0.10	0.18	0.15	22.62
0.95	4.28	0.55	0.08	0.13	0.14	26.86

Hieght	Mean Chord Angle (deg.)	aol correction	Measured Beta(deg.)	Theoretical Pitch(In.)	Theoretical Beta(deg.)
0.33	22.84	6.00	28.84	6.00	31.23
0.27	18.74	6.00	24.74	6.00	23.00
0.20	15.28	6.00	21.28	6.00	18.08
0.16	13.80	6.00	19.80	6.00	14.86
0.12	12.07	6.00	18.07	6.00	12.59

Table 2

Master Airscrew 9X5

Radius = 4.5

% Radius	Radius(In.)	Chord(In.)	Thickness(In.)	Chord/Radius	Thickness/Chord	Radial Dist.(In.)
0.35	1.58	0.83	0.16	0.52	0.19	9.90
0.50	2.25	0.82	0.14	0.36	0.16	14.14
0.65	2.93	0.74	0.11	0.25	0.15	18.38
0.80	3.60	0.65	0.09	0.18	0.13	22.62
0.95	4.28	0.53	0.06	0.12	0.11	26.86

Hieght	Mean Chord Angle (deg.)	aoi correction	Measured Beta(deg.)	Theoretical Pitch(In.)	Theoretical Beta(deg.)
0.31	22.07	6.00	28.07	5.00	26.81
0.25	17.38	6.00	23.38	5.00	19.48
0.19	14.98	6.00	20.98	5.00	15.22
0.15	13.34	6.00	19.34	5.00	12.46
0.10	10.33	6.00	16.33	5.00	10.54

Table 3

Master Airscrew 10X5

Radius = 5.00

% Radius	Radius(in.)	Chord(in.)	Thickness(in.)	Chord/Radius	Thickness/Chord	Radial Dist.(in.)
0.35	1.75	0.85	0.16	0.49	0.19	11.00
0.50	2.50	0.83	0.15	0.33	0.18	15.71
0.65	3.25	0.74	0.12	0.23	0.17	20.42
0.80	4.00	0.65	0.10	0.16	0.15	25.13
0.95	4.75	0.55	0.08	0.12	0.14	29.85

Hieght	Mean Chord Angle (deg.)	aoi correction	Measured Beta(deg.)	Theoretical Pitch(in.)	Theoretical Beta(deg.)
0.31	22.07	6.00	28.07	5.00	24.45
0.25	17.38	6.00	23.38	5.00	17.66
0.19	14.98	6.00	20.98	5.00	13.76
0.15	13.34	6.00	19.34	5.00	11.25
0.10	10.33	6.00	16.33	5.00	9.51

Table 4

Master Airscrew 8X5

Radius = 4.00

% Radius	Radius(In.)	Chord(In.)	Thickness(In.)	Chord/Radius	Thickness/Chord	Radial Dist.(In.)
0.35	1.40	0.83	0.16	0.59	0.19	8.80
0.50	2.00	0.82	0.14	0.41	0.16	12.57
0.65	2.60	0.74	0.11	0.28	0.15	16.34
0.80	3.20	0.65	0.09	0.20	0.13	20.11
0.95	3.80	0.53	0.06	0.14	0.11	23.88

Hieght	Mean Chord Angle (deg.)	aol correction	Measured Beta(deg.)	Theoretical Pitch(in.)	Theoretical Beta(deg.)
0.31	22.07	6.00	28.07	5.00	29.61
0.25	17.38	6.00	23.38	5.00	21.70
0.19	14.98	6.00	20.98	5.00	17.02
0.15	13.34	6.00	19.34	5.00	13.97
0.10	10.33	6.00	16.33	5.00	11.83

Table ~~6~~ 5