

## 1.7 REVIEW OF THE EQUATIONS OF MOTION

At this point it is useful to review the airplane equations of motion as derived so far. There are three sets of equations:

1. The force equations obtained by substituting Eqns (1.50) into Eqns (1.19).
2. The moment equations as stated in Eqns (1.25). Note that the effect of spinning rotors (Eqns (1.30) has been omitted!
3. The kinematic equations as expressed by Eqns (1.45).

These three sets are repeated here for convenience:

### 1. For the force equations in the airplane body-fixed axis system XYZ:

$$\text{Force along X: } m(\dot{U} - VR + WQ) = -mg \sin \Theta + F_{A_x} + F_{T_x} \quad (1.51a)$$

$$\text{Force along Y: } m(\dot{V} + UR - WP) = mg \sin \Phi \cos \Theta + F_{A_y} + F_{T_y} \quad (1.51b)$$

$$\text{Force along Z: } m(\dot{W} - UQ + VP) = mg \cos \Phi \cos \Theta + F_{A_z} + F_{T_z} \quad (1.51c)$$

### 2. For the moment equations in the airplane body-fixed axis system XYZ:

$$\text{Rolling moment about X: } I_{xx}\dot{P} - I_{xz}\dot{R} - I_{xz}PQ + (I_{zz} - I_{yy})RQ = L_A + L_T \quad (1.52a)$$

$$\text{Pitching moment about Y: } I_{yy}\dot{Q} + (I_{xx} - I_{zz})PR + I_{xz}(P^2 - R^2) = M_A + M_T \quad (1.52b)$$

$$\text{Yawing moment about Z: } I_{zz}\dot{R} - I_{xz}\dot{P} + (I_{yy} - I_{xx})PQ + I_{xz}QR = N_A + N_T \quad (1.52c)$$

### 3. For the kinematic equations:

$$\text{Roll rate about X: } P = \dot{\Phi} - \dot{\Psi} \sin \Theta \quad \dot{\Phi} = \quad (1.53a)$$

$$\text{Pitch rate about Y: } Q = \dot{\Theta} \cos \Phi + \dot{\Psi} \cos \Theta \sin \Phi \quad \dot{\Theta} = \quad (1.53b)$$

$$\text{Yaw rate about Z: } R = \dot{\Psi} \cos \Theta \cos \Phi - \dot{\Theta} \sin \Phi \quad \dot{\Psi} = \quad (1.53c)$$

By substituting Eqn (1.35) into Eqn (1.34) followed by substitution of Eqn (1.34) into Eqn (1.33) the following relation is obtained between the earth axes velocity components and the body axes velocity components of the airplane:

$$\begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{Bmatrix} = \begin{bmatrix} \cos \Psi & -\sin \Psi & 0 \\ \sin \Psi & \cos \Psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \Theta & 0 & \sin \Theta \\ 0 & 1 & 0 \\ -\sin \Theta & 0 & \cos \Theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \Phi & -\sin \Phi \\ 0 & \sin \Phi & \cos \Phi \end{bmatrix} \begin{Bmatrix} U \\ V \\ W \end{Bmatrix} \quad (1.36)$$

THE UNIVERSITY OF CHICAGO

Department of Chemistry

Chicago, Illinois

June 15, 1954

Dear Mr. [Name]:

I have your letter of June 10, 1954, regarding the [subject].

The information you requested is being reviewed.

I will contact you again once a decision has been reached.

Very truly yours,

[Signature]

[Name]

Director of [Department]

University of Chicago

Chicago, Illinois

Enclosed are [documents]

Thank you for your interest.

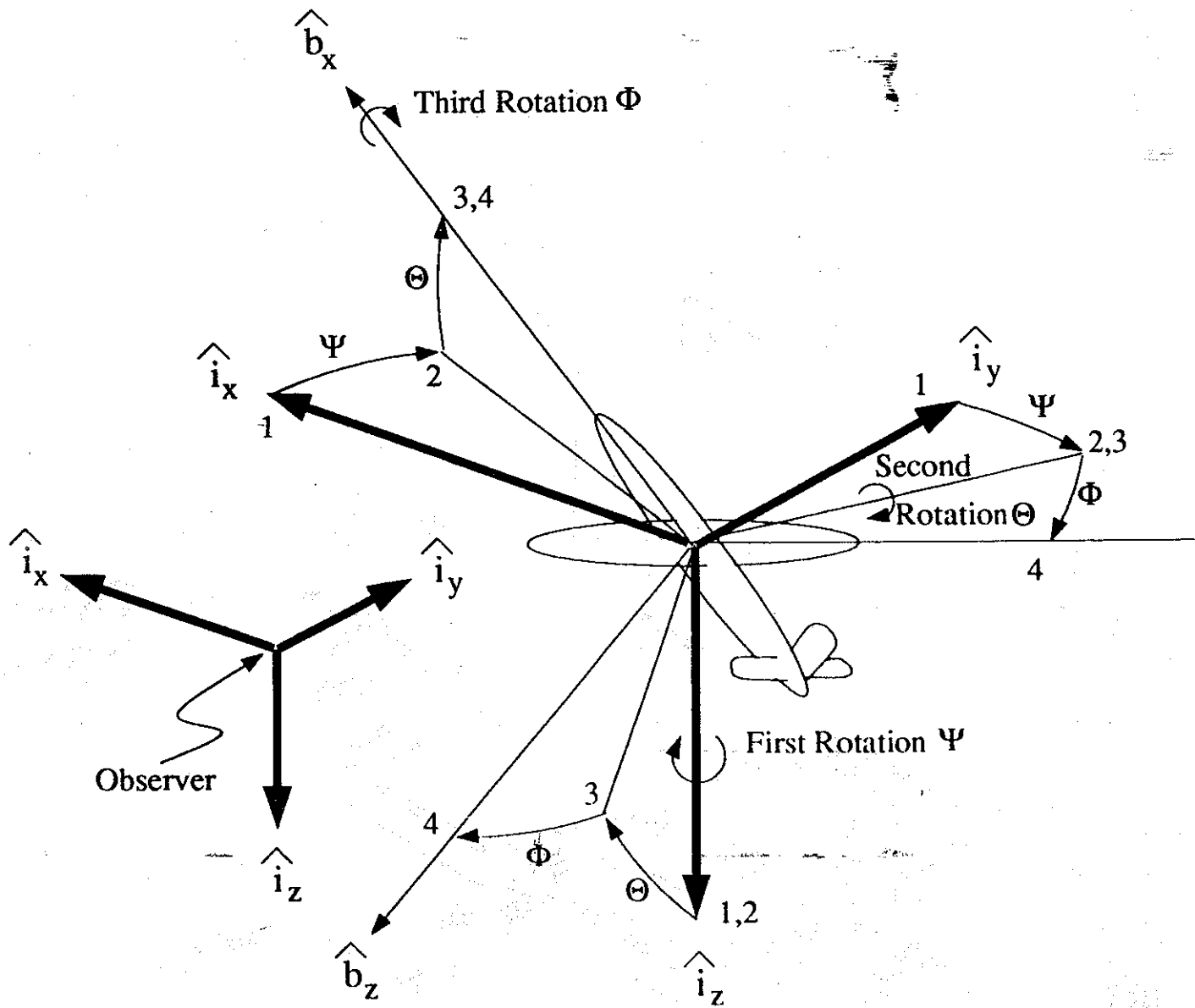


FIGURE 3 Definition of fixed wing aircraft and helicopter Euler angles.

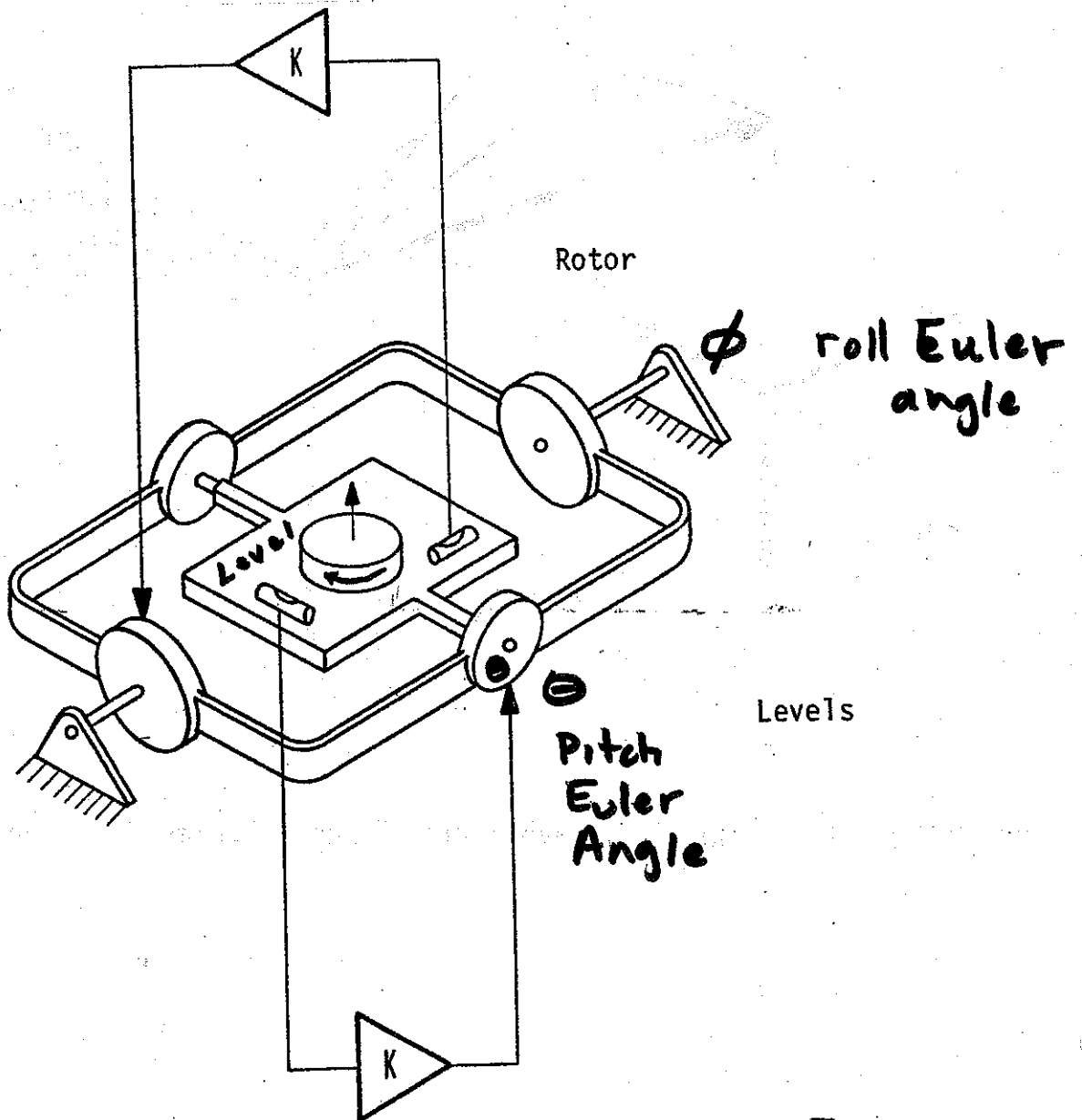


Fig. 6.1.1 Vertical Gyro Principle

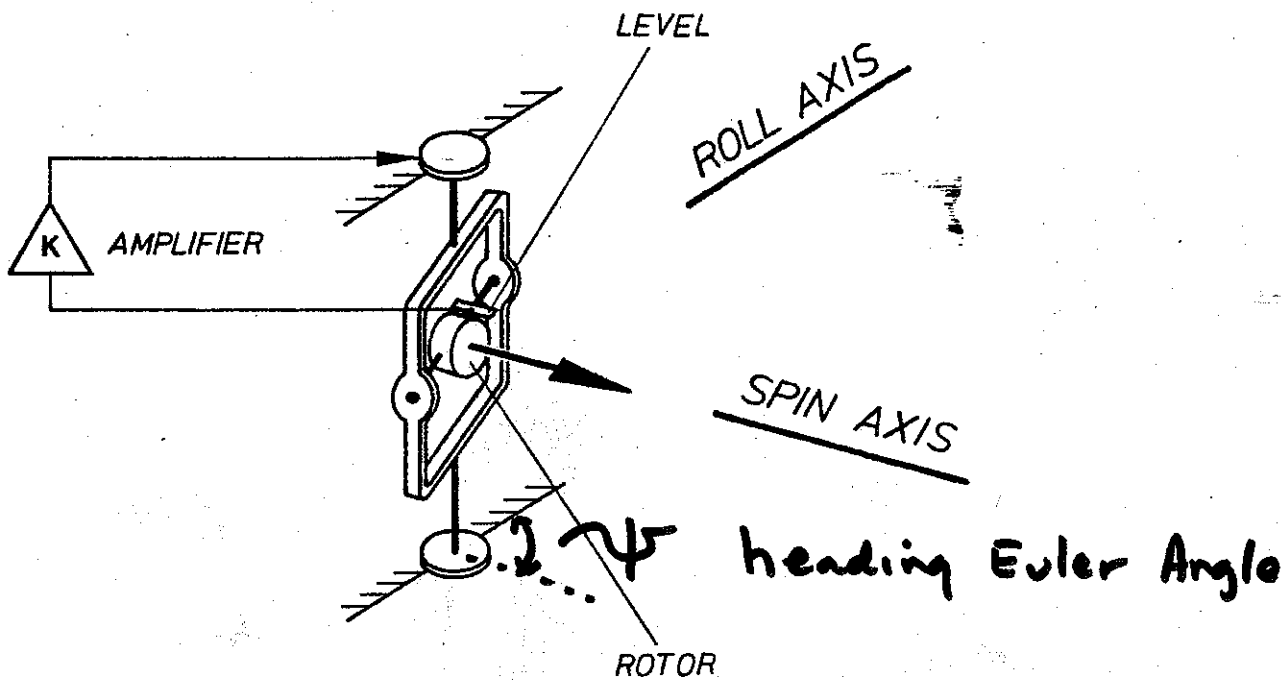


Fig. 6.2.1 Directional Gyro Principle

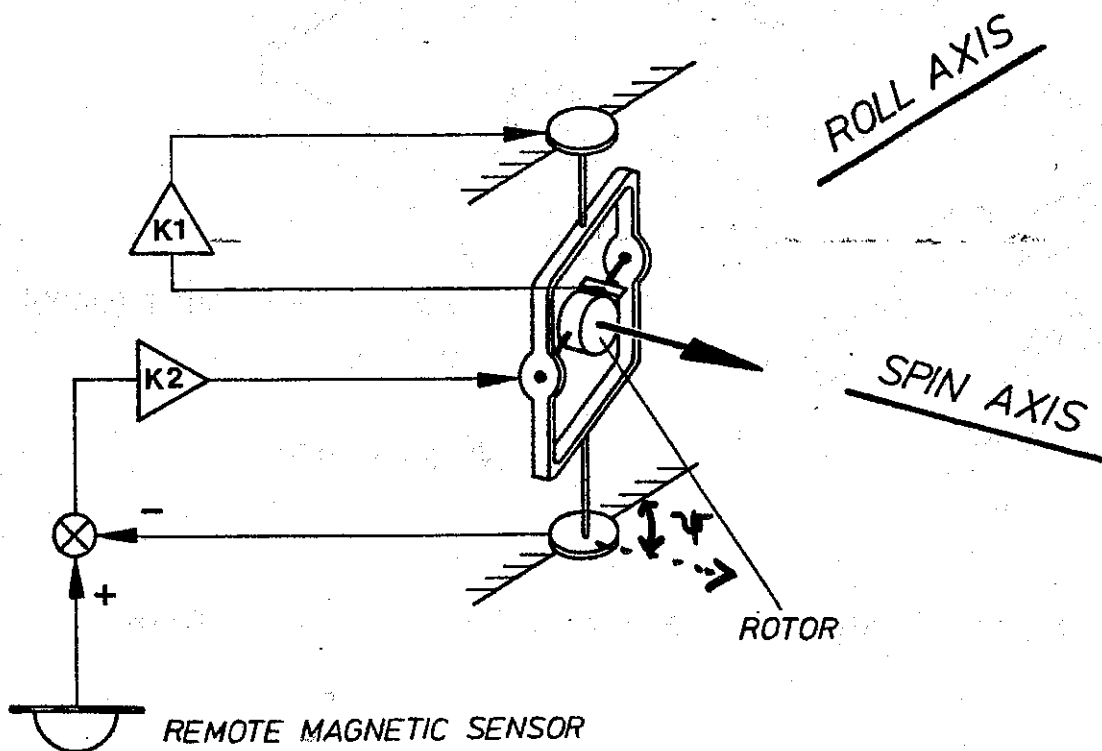


Fig. 6.2.2 Directional Gyro in the Slaved Mode

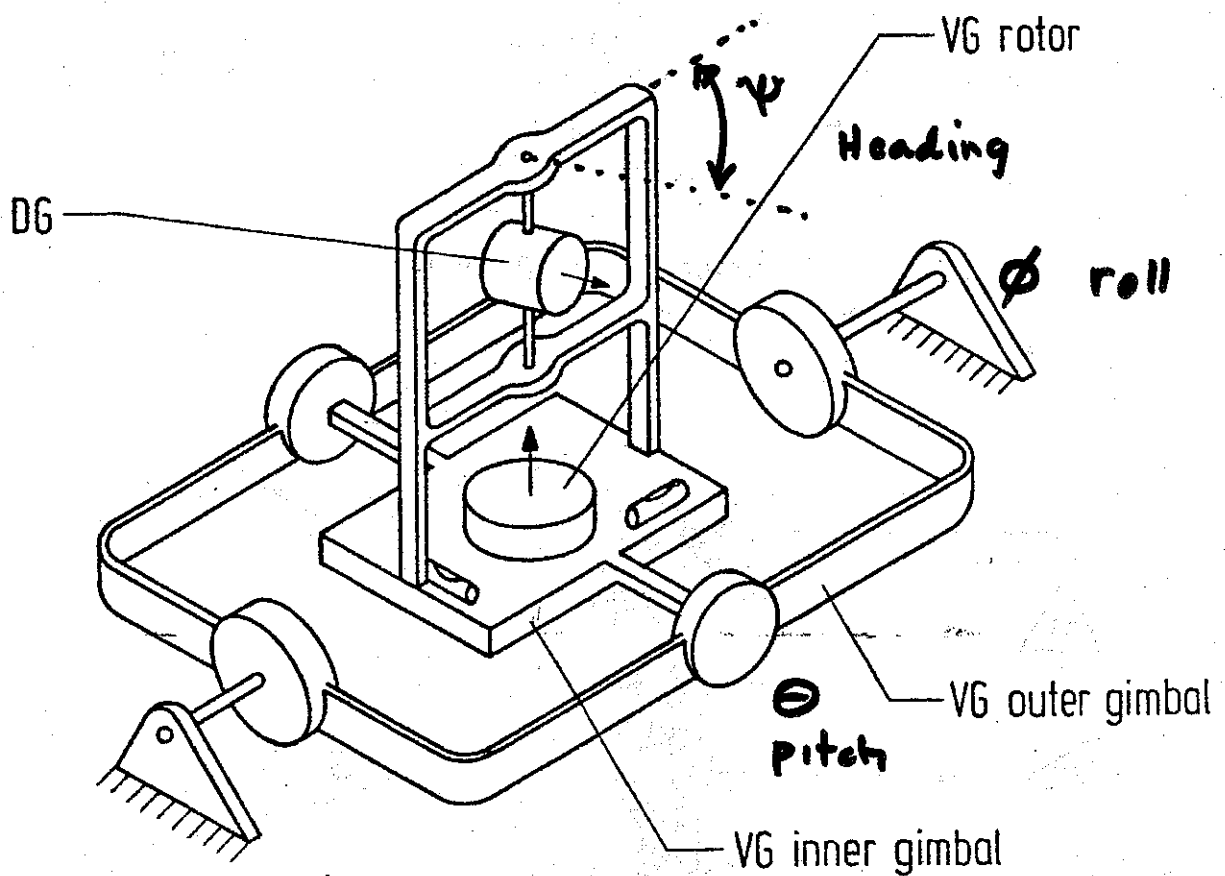


Fig. 6.3.1 Attitude and Heading Reference System Principle