# A&AE 421 Homework #1
# Solution of a simple nonlinear differential equation using linearization
# Due: Monday 8/27/07

Read the handout sheet entitled "Linearization of Nonlinear Equations."

Given the nonlinear differential equation

$$\dot{\mathbf{x}} = \mathbf{cos(x)}. \qquad (1)$$

Define $\Delta x$ as a small perturbation variable and
$x_1$ as the steady state (reference or trim) variable
where $x = x_1 + \Delta x$.

When x is a function of time, we denote this as x(t). Since $x_1$ is a constant we do not use the explicit time dependent notation for it. Therefore, we can write for the time dependent case $x(t) = x_1 + \Delta x(t)$ where we show the explicit time dependence of the small perturbation variable $\Delta x(t)$.

a) Find constant values of $x_1$ that satisfy the steady state (reference or trim) differential equation $\dot{\mathbf{x}}_1 = \mathbf{cos(x_1) = 0}$. Note there are an infinite number of constants, $x_1$, that satisfy this condition.

b) Linearize the nonlinear differential equation (1) about $x_1$. Express your answer in terms of $\Delta x$ and $x_1$ (not in terms of x).

c) For your linear differential equation from part b, find the small perturbation time response, $\Delta x(t)$, to initial condition $\Delta x(0) = x(0)-x_1$. Hint: use Laplace transforms, i.e., $\Delta x(t) = L^{-1}\Delta x(s)$.

d) For what values of $x_1$ does $\Delta x(t = \infty)=0$?

e) For what values of $x_1$ does $\Delta x(t = \infty)= \infty$ ?