1.7 REVIEW OF THE EQUATIONS OF MOTION

At this point it is useful to review the airplane equations of motion as derived so far. There are three sets of equations:

1. The force equations obtained by substituting Eqns (1.50) into Eqns (1.19).

2. The moment equations as stated in Eqns (1.25). Note that the effect of spinning rotors (Eqns (1.30) has been omitted!

3. The kinematic equations as expressed by Eqns (1.45).

These three sets are repeated here for convenience:

1. For the force equations in the airplane body–fixed axis system XYZ:

$$\text{Force along X:}\quad m(\dot{U} - VR + WQ) = -mg\sin\Theta + F_{A_x} + F_{T_x} \tag{1.51a}$$

$$\text{Force along Y:}\quad m(\dot{V} + UR - WP) = mg\sin\Phi\cos\Theta + F_{A_y} + F_{T_y} \tag{1.51b}$$

$$\text{Force along Z:}\quad m(\dot{W} - UQ + VP) = mg\cos\Phi\cos\Theta + F_{A_z} + F_{T_z} \tag{1.51c}$$

2. For the moment equations in the airplane body–fixed axis system XYZ:

$$\text{Rolling moment about X:}\quad I_{xx}\dot{P} - I_{xz}\dot{R} - I_{xz}PQ + (I_{zz} - I_{yy})RQ = L_A + L_T \tag{1.52a}$$

$$\text{Pitching moment about Y:}\quad I_{yy}\dot{Q} + (I_{xx} - I_{zz})PR + I_{xz}(P^2 - R^2) = M_A + M_T \tag{1.52b}$$

$$\text{Yawing moment about Z:}\quad I_{zz}\dot{R} - I_{xz}\dot{P} + (I_{yy} - I_{xx})PQ + I_{xz}QR = N_A + N_T \tag{1.52c}$$

3. For the kinematic equations:

$$\text{Roll rate about X:}\quad P = \dot{\Phi} - \dot{\Psi}\sin\Theta \tag{1.53a}$$

$$\text{Pitch rate about Y:}\quad Q = \dot{\Theta}\cos\Phi + \dot{\Psi}\cos\Theta\sin\Phi \tag{1.53b}$$

$$\text{Yaw rate about Z:}\quad R = \dot{\Psi}\cos\Theta\cos\Phi - \dot{\Theta}\sin\Phi \tag{1.53c}$$

Equations (1.51) and (1.52) are referred to as the general airplane equations of motion. That is a rather generous description since many assumptions have been made in their derivation. Many of these assumptions serve to reduce the 'generality' of these equations!

Equations (1.51) and (1.52) are as yet incomplete: the aerodynamic and thrust forces and moments in their right hand sides must still be expressed in terms of the motion variables. That will be done in Chapter 3.

From a mathematical viewpoint the equations (1.51), (1.52) and (1.53) form a set of nine differential equations in nine variables: the velocity components U, V and W, the angular rate com-

Table 2–1 Laplace Transform Pairs

	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!}t^{n-1}e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
14	$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$

Table 2–1 (*Continued*)

18	$\frac{1}{a^2}(1 - e^{-at} - ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
21	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
22	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin \omega_n \sqrt{1-\zeta^2}\, t$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
23	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_n \sqrt{1-\zeta^2}\, t - \phi)$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
24	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_n \sqrt{1-\zeta^2}\, t + \phi)$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$
25	$1 - \cos \omega t$	$\frac{\omega^2}{s(s^2 + \omega^2)}$
26	$\omega t - \sin \omega t$	$\frac{\omega^3}{s^2(s^2 + \omega^2)}$
27	$\sin \omega t - \omega t \cos \omega t$	$\frac{2\omega^3}{(s^2 + \omega^2)^2}$
28	$\frac{1}{2\omega} t \sin \omega t$	$\frac{s}{(s^2 + \omega^2)^2}$
29	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
30	$\frac{1}{\omega_2^2 - \omega_1^2}(\cos \omega_1 t - \cos \omega_2 t) \quad (\omega_1^2 \neq \omega_2^2)$	$\frac{s}{(s^2 + \omega_1^2)(s^2 + \omega_2^2)}$
31	$\frac{1}{2\omega}(\sin \omega t + \omega t \cos \omega t)$	$\frac{s^2}{(s^2 + \omega^2)^2}$