AAE 421 MATLAB Code to Compute
Stability and Control Derivatives

Purpose:
Given basic aircraft constants, such as wing area and wing cord, this software computes all stability and control derivatives needed to run several other dynamic response computer programs that compute equations of motion, transfer functions and perform dynamic simulation.

To run the modeling software:
To use this software run under MATLAB the script called RunFirst.m. As written, this script will execute another script called BasicConstants_Cessna182.m. You can change this one script call (see listing below) to something like BasicConstants_YourCraft.m. You will then need to write the BasicConstants_YourCraft.m file that includes the basic constants that apply to your aircraft.

RunFirst.m also calls a script called DerivedConstants.m which computes intermediate variables needed to compute the stability and control derivatives. There is no need to change this script.

RunFirst.m then computes the 67 “constant” needed for the subsequent stability and control software. In doing this RunFirst.m calls a unique function for each stability and control derivative. The naming convention is straightforward. For instance to compute \( C_m \alpha \) the script calls MATLAB function Cm_alpha.m.

Additional documentation is provided in the MATLAB script. Listings of RunFirst.m, BasicConstants_Cessna182.m and DerivedConstants.m are given below.

Copies of all the software are provided in a single compressed file called SandC.zip. All software and documentation can be found at the web site


Script RunFirst.m
% ****************************************************************************
% RunFirst - Runs each of the stability derivatives functions
% and sets up the array called constant that is used by other
% MATLAB programs to perform various dynamic analysis.
% ****************************************************************************
% A&AE 421 Fall 2001 - Purdue University
% Note: This code is provided for a first order approximation of the dynamic
% stability and control derivatives of an airplane.
% Equations/Figures can be found in :
% (Ref.1) Roskam, Jan. "Airplane Flight Dynamics and Automatic Flight
% Controls"
% Published by DARcorporation
% 120 E. Ninth St., Suite 2
% Lawarence, KS 66044
% (Ref.2) Roskam, Jan. "Methods for Estimating Satbility and
% Control Derivatives of Conventional Subsonic Airplanes"
% Published by the Author
% 519 Boulder
% Laurance, Kansas 66044
% Third Printing, 1997.
% (Ref.3) Roskam, Jan. "Airplane Design: Part IV: Preliminary Calculation
clear all
close all

format short g
% The next script gather data needed for the stability and
% control derivative computations.
%BasicConstants_BoilerXP % Use this script to model the Boiler Xpress aircraft
BasicConstants_Cessna182 % Use this script to model the Cessna 182 aircraft
%BasicConstants_YourCraft % Write this script in the same format as above
% to model your aircraft.

DerivedConstants % This script computes some intermediate constants used below

% The 57 "constant" computed below are used by four dynamic and control
% software programs. Specifically, Simulink scripts FlatEarth.mdl and
% E_Earth.mdl
% use the first 547 "constant". These MATLAB programs perform 6-degree-of-
% freedom
% aircraft simulation over a flat earth or an elliptical earth.
% Two other programs do simplified dynamic modeling (compute transfer
% functions)
% for the longitudinal or lateral-directional degrees of freedom. They use all
% 67 of the "constant" defined below.
% These programs are called LatSC.m or LongSC.m. Examples of these programs are
% called
% CessnaLongSC.m and CessnaLatSC.m. This software can be found at the
% following web site.
% http://roger.ecn.purdue.edu/~andrisan/Courses/AAE421_S2001/Docs_Out/DC_Software
% /index.html
%
% Mass related inputs
constant(1)=W;      % W, Weight, pounds (lbf)
constant(2)=32.17405;  % g, Acceleration of gravity, ft/(sec*sec)
constant(3)=constant(1)/constant(2); % mass, slugs
constant(4)=Ixx;    % Ixx, slug*ft*ft
constant(5)=Iyy;   % Iyy, slug*ft*ft
constant(6)=Izz;   % Izz, slug*ft*ft
constant(7)=Ixz;     % Ixz, slug*ft*ft
constant(8)=eta_p;    % propeller efficiency, eta, nondimensional
constant(9)=0; % unassigned

% Derived constants from the inertia data
constant(10)=constant(4)*constant(6)-constant(7)*constant(7); %gamma
constant(11)=((constant(5)-constant(6))*constant(6)-
constant(7)*constant(7))/constant(10);% c1
constant(12)=(constant(4)-constant(5)+constant(6))*constant(7)/constant(10);% c2
constant(13)= constant(6)/constant(10); % c3
constant(14)= constant(7)/constant(10); % c4
constant(15)=(constant(6)-constant(4))/constant(5); % c5
constant(16)= constant(7)/constant(5); % c6
constant(17)= 1/constant(5); % c7
constant(18)= (constant(4)*(constant(4)-constant(6)+
constant(5)+constant(7)))/constant(10); % c8
constant(19)= constant(4)/constant(10); % c9

% aircraft geometry
constant(20)=S_w;    % S, wing area, ft^2
constant(21)=c_w; % cbar, mean geometric chord, ft
class(22)=b_w; % b, wing span, ft
class(23)=0; % phiT, thrust inclination angle, RADIANS
class(24)=0; % dT, thrust offset distance, ft

% Nondimensional Aerodynamic stability and control derivatives

% Drag Polar CD=k(CLstatic-CLdm)^2 + CDm
class(25)=Cd_0; % CDm, CD for minimum drag

class(26)=k; % k
class(27)=0; % CLdm, at the minimum drag point

% Lift Force

class(28)=CL_0(S_w,S_h,M,tc_w,alpha_0,epsilon_t,i_w,i_h,epsilon_0_h,AR_w,Lambda_c4,Lambda_c2,lambda_w,kappa,beta,b_w,d,AR_h,eta_h); % CL0

class(29)=CL_alpha(AR_w,AR_h,Lambda_c2,lambda_w,l_h,b_w,d,eta_h,S_h,S_w,kappa_h,Lambda_c2_h,beta,kappa); % Clalpha

class(30)=CL_de(S_w,S_h,AR_w,ce_ch,eta_ce,eta_ie,beta,kappa_h,lambda_h,Lambda_a_c2_h,tc_h,delta_e,Cl_alpha_h); % ClDeltaE

class(31)=CL_alpha_dot(l_h,b_w,lambda,AR_w,AR_h,Lambda_c4,lambda_c4_h,beta,kappa_h,V_h,eta_h); % Clalpha_dot

class(32)=CL_q(Xw,b_w,c_w,c_h,AR_w,Lambda_c4,Lambda_c2,Lambda_c2_h,Xh,S_h,S_w,eta_h,AR_h,beta,V_h,beta_h,kappa_h); % ClQ

% Side Force

class(33)=0; % CY0

class(34)=Cy_beta(two_r_one,eta_v,beta,AR_v,b_v,Z_h,x_over_c_v,lambda_v,S_v,S_w,Lambda_c4,Lambda_c4_v,Z_w,d,dihedral,wingloc,Z_w1,S_h,S_o); % CyBeta

class(35)=Cy_da(S_w); % CyDeltaA

class(36)=Cy_dr(S_w,b_w,S_h_v,b_v,c_v,x_AC_vh,two_r_one,AR_v,b_v,S_h,S_v,w_d,diheral,wingloc,Z_w1,S_w1,S_o); % CyDeltaR

% Rolling Moment

class(39)=0; % Cl0

class(40)=Cl_beta(CL_wb,theta,theta_h,Lambda_c2,Lambda_c4,Lambda_c4_h,S_w,b_w,AR_w,AR_h,Z_v,lambda_v,M,two_r_one,eta_v,beta,AR_v,b_v,Z_h,x_over_c_v,lambda_v,S_v,S_w,d,diheral,wingloc,Z_w1,S_h,S_o,Cl_hb); % ClBeta

class(41)=Cl_da(S_w,AR_w,ca_cw,eta_ia,eta_oa,beta,kappa,Lambda_c4,lambdaw_cw,Cl_alpha_w); % ClDeltaA

class(42)=Cl_dr(S_w,b_w,S_h_v,b_v,c_v,x_AC_vh,two_r_one,AR_v,b_v,S_h,S_v,w_d,diheral,wingloc,Z_w1,S_w1,S_h,S_o,beta,Cl_alpha,v); % ClDeltaR

% Pitching Moment

class(45)=Cm_0(S_h_slip,S_w,S_h,M,tc_w,epsilon_t,i_w,i_h,epsilon_0_h,AR_w,Lambda_c4,lambdaw_c4,lambda_w,beta,Cm_0_r,Cm_o_t,Lambda_c2_h,kappa_h,AR_h,Xw); % CMO

% Cm0

class(46)=Cm_alpha(AR_w,AR_h,Lambda_c2,lambda_w,l_h,b_w,c_w,d,eta_h,S_h,S_w,kappa_h,Lambda_c2_h,beta,kappa,Xw); % Cmalpha

class(47)=Cm_de(S_w,S_h,AR_w,ce_ch,eta_ce,eta_ie,beta,kappa_h,lambda_h,Lambdaw_a_c2_h,tc_h,delta_e,Cl_alpha_h,V_h); % CMdeltaE

class(48)=Cm_a_dot(l_h,b_w,lambda_w,AR_w,Lambda_c4,M,Cl_alpha,q_bar_h,q_bar,S_h,x,w_c,S_w); % Cmalpha_dot

class(49)=Cm_q(Xw,b_w,c_w,c_h,AR_w,Lambda_c4,Lambda_c2,Xh,S_h,S_w,eta_h,AR_h,beta,V_h,b_beta_h,Cl_alpha,B); % CMQ

% Yawing Moment

class(50)=0; % CMO
constant(51)=Cn_beta(S_w,b_w,alpha,l_v,Z_v,l_f,S_b_s,Rl_f,x_m,hl_fuse,h2_fuse,hmax_fuse,wmax_fuse,two_r_one,eta_v,M,AR_v,b_v,Z_h,x_over_c_v,lambda_v,S_v,Z_w,d,Z_w1,Z_h,Lambda_c4,Lambda_c2_v); % CNbeta
constant(52)=Cn_da(S_w,AR_w,ca_cw,eta_la,eta_ao,beta,kappa,Lambda_c4,lambda_w,tc_w,Cl_alpha_w,Cl); % CNdeltaA
constant(53)=Cn_dr(S_w,b_w,S_h,S_v,b_v,c_v,x_AC_vh,two_r_one,AR_v,l_v,Z_v,eta_o_r,eta_ir,cr_cv,beta,kappa_v,Lambda_c2_v,lambda_v,delta_r,AR_v); % CNdeltaR
constant(54)=Cn_p(c_w,B,theta,adelf,delf,b_w,l_v,b_h,Z_v,two_r_one,eta_v,b_v,Z_h,x_over_c_v,lambda_v,S_v,S_w,Lambda_c4,Lambda_c2,Lambda_c4_h,Z_w,d,lambda_w,wingloc,Z_w1,S_h,S_o,beta,Cl_alpha,AR_w,lambda_h,AR_h,b_f,Xw,Cl,Lambda_c4_v,alpaha); % CNP
constant(55)=.264; % XbarRef, nondimensional
constant(56)=.264; % XbarCG, nondimensional

% Trim conditions. These may or not be used by subsequent programs. Small
% variations in these trim flight conditions are OK.
constant(58)=U; % trim speed, Vt, ft/sec
constant(59)=5000; % Trim altitude, ft
constant(60)=0; % Trim alpha, >>>DEGREES<<<This is not used by CessnaLongSC
% The constants below are used only by CessnaLongSC and CessnaLatSC
constant(61)=0; % CLu=0
constant(62)=0; % CDu=0
constant(63)=.096; % CTxu
constant(64)=0; % Cmu
constant(65)=0; % CmTu
constant(66)=0; % CmAlpha
constant(67)=0; % CDdeltae
% *********************************************
% BasicConstants_Cessna182
% *********************************************
% BasicConstants - Identifies, describes, and assigns all of the
%                  the most basic variables for analyzing the control
%                  and stability of a generic aircraft.
% *********************************************
%
% A&AE 421 Fall 2001 - Purdue University
%
% Note: This code is provided for a first order approximation of the dynamic
%       analysis of an airplane and is not intended for final designs.
%
% Equations/Figures can be found in :
%
% (Ref.1) Roskam, Jan. "Airplane Flight Dynamics and Automatic Flight
%         Controls"
%         Published by DARcorporation
%         120 E. Ninth St., Suite 2
%         Lawarence, KS 66044
%
% (Ref.2) Roskam, Jan. "Methods for Estimating Satbility and
%         Control Derivatives of Conventional Subsonic Airplanes"
%         Published by the Author
%         519 Boulder
%         Laurance, Kansas 66044
%         Third Printing, 1997.
%
% (Ref.3) Roskam, Jan. "Airplane Design: Part IV: Preliminary Calculation
%         of Aerodynamic, Thrust and Power Characteristics"
%         Published by Roskam Aviation and Engineering Corporation
%         Rt4, Box 274
%         Ottawa, Kansas 66067
%         Second Printing, 1990.
%
aircraft='Cessna182, cruise configuration';

adelf = 0;           % Two dimensional lift effectiveness parameter Ref.(2),
alpha = 0;           % Angle of attack [deg]
alpha_0 = -.2;      % Airfoil zero-lift AOA [deg]
AR_h = 3.93;         % Aspect ratio of the horizontal tail
AR_w = 7.45;         % Aspect ratio of the wing
b_f = 20;            % Span of the flap [ft]
b_h = 11.33;         % Span of the horizontal tail [ft]
b_h_oe = 5.8333;     % Elevator outboard position [ft]
b_h_ie = 0.5;       % Elevator inboard position [ft]
b_w = 36;            % Span of the wing [ft]
b_v = 5;             % Vertical tail span measured from fuselage centerline
                      % [ft]
b_v_or = 4.95;       % Outboard position of rudder [ft]
b_v_ir = 0.0;        % Inboard position of rudder [ft]
c_a = 1.0;           % Chord of aileron [ft]
C_bar_D_o = .0270;   % Parasite drag
Cd_0 = 0.027;        % Drag coefficient at zero lift (parasite drag)
c_e = 2.70;          % Elevator flap chord [ft]
cf = 1.2;            % Length of the wing flap [ft]
c_h = 6.3;           % Mean aerodynamic chord of the horizontal tail [ft]
CL = 0.307;          % Lift coefficient (3-D)
CL_hb=.307;          % Lift coefficient of the horizontal tail/body
CL_wb=.307;          % Lift coefficient of the wing/body
\( C_{\text{alpha}_h} = 2\pi; \)  \( \) % 2-D Lift curve slope of wing
\( C_{\text{alpha}_v} = 2\pi; \)  \( \) % 2-D Lift curve slope of vertical tail
\( C_{\text{alpha}} = 6; \)  \( \) % Two-dimensional lift curve slope
\( C_{\text{alpha}_w} = C_{\text{alpha}}; \)  \( \) % Two-dimensional lift curve slope
\( C_{\text{m}_0t} = -2; \)  \( \) % Zero lift pitching moment coefficient of the wing root
\( c_r = 1.52; \)  \( \) % Chord of the rudder [ft]
\( c_w = 4.9; \)  \( \) % Mean aerodynamic chord of the wing [ft]
\( c_v = 3.8; \)  \( \) % Mean aerodynamic chord of the vertical tail [ft]
\( D_p = 4; \)  \( \) % Diameter of propeller [ft]
\( d = 4.5; \)  \( \) % Average diameter of the fuselage [ft]
\( delf = 0; \)  \( \) % Streamwise flap deflection [deg]
\( \delta_e = 0; \)  \( \) % Elevator deflection [deg]
\( \delta_f = 0; \)  \( \) % Streamwise flap deflection [deg]
\( \delta_r = 0; \)  \( \) % Rudder deflection [deg]
\( \text{dihedral} = 0; \)  \( \) % Geometric dihedral angle of the wing [deg]
\( \text{dihedral}_h = 0; \)  \( \) % Geometric dihedral angle of the horizontal tail [deg]
\( e = 0.9; \)  \( \) % Oswald efficiency factor
\( \epsilon_t = -0.421; \)  \( \) % Horizontal tail twist angle [deg]
\( \epsilon_0_h = \epsilon_t; \)  \( \) % Horizontal tail twist angle [deg]
\( \eta_h = 0.85; \)  \( \) % Ratio of dynamic pressure at the tail to that of the free stream
\( \eta_{ia} = 0.6; \)  \( \) % Percent span position of inboard edge of aileron
\( \eta_{oa} = 1.0; \)  \( \) % Percent span position of outboard edge of aileron
\( \eta_p = 0.70; \)  \( \) % Propeller Efficiency
\( \eta_v = 1; \)  \( \) % Ratio of the dynamic pressure at the vertical tail to that of the freestream
\( \text{Gamma} = \frac{2\pi}{180}; \)  \( \) % This is the geometric dihedral angle, positive for dihedral, negative for anhedral [rad]
\( h_{1\text{ fuse}} = 4; \)  \( \) % Height of the fuselage at 1/4 of its length
\( h_{2\text{ fuse}} = 3; \)  \( \) % Height of the fuselage at 3/4 of its length
\( h_h = 1.7; \)  \( \) % Height from chord plane of wing to chord plane of horizontal tail [ft] - Fig 3.1, Ref. 2
\( h_{\text{max fuse}} = 5; \)  \( \) % Maximum height of the fuselage [ft]
\( I_{xx} = 948; \)  \( \) % Inertia about the center of mass about the x-axis
\( I_{yy} = 1346; \)  \( \) % Inertia about the center of mass about the y-axis
\( I_{zz} = 1967; \)  \( \) % Inertia about the center of mass about the z-axis
\( \lambda_h = 2; \)  \( \) % Horizontal tail incidence angle [deg]
\( \lambda_w = -.2; \)  \( \) % Wing incidence angle [deg]
\( k = 0.0554; \)  \( \) % k
\( \lambda = 0; \)  \( \) % Wing sweep angle [deg]
\( \lambda_c4 = 0; \)  \( \) % Sweep angle at the quarter-chord of the wing [deg]
\( \lambda_c2 = 0; \)  \( \) % Sweep angle at the half-chord of the wing [deg]
\( \lambda_{c2,v} = 0.35; \)  \( \) % Sweep angle at the half-chord of the vertical tail [deg]
\( \lambda_{c4,v} = 0.35; \)  \( \) % Sweep angle at the quarter-chord of the vertical tail [deg]
\( \lambda_{c2,h} = 0; \)  \( \) % Sweep angle at the half-chord of the horizontal tail [deg]
\( \lambda_{c4,h} = 0; \)  \( \) % Sweep angle at the quarter-chord of the vertical tail [deg]
\( \lambda_{v} = 0.75; \)  \( \) % Taper ratio of the wing
\( \lambda_{h} = 0.75; \)  \( \) % Horizontal tail taper ratio
\( \lambda_v = 0.5; \)  \( \) % Vertical tail taper ratio
\( \lambda_w = \lambda; \)  \( \) % Taper ratio of the wing
\( l_b = 25; \)  \( \) % This is the total length of the fuselage [ft]
\( l_f = 12; \)  \( \) % The horizontal length of the fuselage [ft] (*******)
\( l_h = 12; \)  \( \) % Distance from c/4 of wing to c/4 of horizontal tail [ft]
\( l_v = 13; \)  \( \) % Horizontal distance from the aircraft CG to the vertical tail aero center [ft]
\( M = 0.2; \)  \( \) % Mach number
\( q_{\text{bar}} = 1; \)  \( \) % Dynamic pressure ratio (free stream)
\( q_{\text{bar},h} = 1; \)  \( \) % Dynamic pressure ratio at the tail
\( R_l = 10^6; \)  \( \) % Fuselage Reynolds' number
\( \rho = 0.00204; \)  \( \) % Air density at 5000ft [slugs/ft^3]
\( S_b_s = 60; \)  \( \) % Body side area [ft^2]
\( S_b = S_b_s \) % Body side area [ft^2]

\( S_h = 48 \) % Area of the horizontal tail [ft^2]

\( S_h_{\text{slip}} = 8 \) % Area of horiz. tail that is covered in prop-wash - See fig.(8.64) Ref.(3) [ft^2]

\( S_o = 6 \) % Take X1/l_b and plug into: 
\[ .378 + .527 \times (X1/l_b) = (Xo/l_b) \]
\( S_o \) is the fuselage x-sectional area at \( Xo \). (ft^2) Ref.(2), Fig. 7.2

\( S_w = 174 \) % Area of the wing [ft^2]

\( S_v = 25 \) % Surface area of vertical tail [ft^2]

\( T = 0 \) % Temperature [\( ^\circ F \)]

\( tc_w = .10 \) % Thickness to chord ratio of the wing

\( tc_h = 0.10 \) % Thickness to chord ratio of horizontal tail

\( \theta = 0 \) % This is the wing twist in degrees, negative for washout [deg]

\( \theta_h = 0 \) % Horizontal tail twist between the root and tip stations, negative for washout [deg]

\( \text{two}_r_{\text{one}} = 2 \) % Fuselage depth in region of vertical tail [ft] Ref.(2), Figure 7.5

\( U = 129.73 \) % Free Stream Velocity [knots]

\( U_1 = 220.1 \) % Cruise flight speed [ft/s]

\( W = 2650 \) % Weight of Airplane [lbf]

\( \text{wingloc} = 1 \) % If the aircraft is a highwing: (wingloc=1), low-wing: (wingloc=0)

\( \text{wmax}_f_{\text{use}} = 5 \) % Maximum fuselage width [ft]

\( X_1 = 12.5 \) % Distance from the front of the fuselage where the x-sectional area decrease (dS_x/dx) is greatest (most negative) [ft] - Ref.(2), Fig. 7.2

\( x_{\text{AC}_v_{\text{h}}} = 1 \) % X distance from LE of vertical tail to AC of horizontal tail [ft]

\( X_h = 18 \) % Distance from airplane cg to the horizontal tail ac [ft]

\( x_m = 5 \) % Center of gravity location from the leading edge [ft]

\( x_{\text{over}_c_{\text{v}}} = .5 \) % X distance from leading edge of vertical fin mean chord to horizontal aero center [ft]

\( X_{\text{ref}} = 6 \) % Arbitrary reference point located on the airplane's axis of symmetry. % Measured as positive aft, starting from the leading edge of the mean aero. chord. [ft]

\( X_w = 0.2 \) % Distance from the airplane cg to wing ac (positive aftward) [ft]

\( Z_h = -2.5 \) % Negative of the horizontal distance from the fuselage centerline to the horizontal tail aero center (negative number) - Ref.(2), Fig. 7.6

\( Z_v = .5 \) % Vertical distance from the aircraft CG to the vertical tail aero center - Ref.(2), Fig. 7.18

\( Z_w = 2 \) % This is the vertical distance from the wing root c/4 to the fuselage centerline, positive downward - Ref.(2), Eq.(7.5)

\( Z_{w1} = -0.5 \) % Distance from body centerline to c/4 of wing root chord, positive for c/4 point below body centerline (ft) - Ref.(2), Fig. 7.1
AR_v = b_h^2/S_v;                % Aspect Ratio of Horizontal Tail
AR_h = b_h^2/S_h;                 % Aspect Ratio of Horizontal Tail
AR_w = b_w^2/S_w;                 % Aspect Ratio of Horizontal Tail
B=sqrt(1-M^2*(cos(Lambda_c4))^2); % Compressibility correction factor
beta = sqrt(1-M^2);               % Compressibility correction factor
Beta = beta;                      % Compressibility correction factor
c_a_cw = c_a/c_w;                 % Ratio of aileron chord to wing chord
Cd = Cd_0 + (CL^2/(pi*AR_w*e));   % Drag Coefficient
ce_ch = c_e/c_h;                  % Elevator flap chord
cr_cv = c_r/c_v;                   % Rudder flap chord
eta_ie = b_h_ie/(b_h/2);          % Percent span position of inboard edge of elevator
eta_ir = b_v_ir/b_v;              % Percent span position of inboard edge of rudder
eta_oe = b_h_oe/(b_h/2);          % Percent span position of outboard edge of elevator
eta_or = b_v_or/b_v;              % Percent span position of outboard edge of rudder
kappa=Cl_alpha/(2*pi);            % Ratio of 2D lift coefficient to 2pi for the wing
kappa_h = Cl_alpha_h/(2*pi);      % Ratio of 2D lift coefficient to 2pi for the horiz. tail
kappa_v = Cl_alpha_v/(2*pi);      % Ratio of 2D lift coefficient to 2pi for the vert. tail
V_h = (Xh*S_h)/(c_h*S_w);         % Horizontal Tail Volume Coefficient