ECE 616 Homework #0

Homework 0 is your first and only tutorial-style homework, intended to remind or introduce how to use a computer to correctly perform and interpret discrete Fourier transforms (DFTs). DFTs will be a core tool used on homework throughout this course.

The homework consists of 4 problems. Four very similar problems and their detailed solutions are viewable in the Homework section on Brightspace or at engineering.purdue.edu/~amw/ece616/ as web pages. They can also be downloaded as .m files. Especially if you have limited experience with performing DFTs, you should review these solutions carefully before attempting these homework problems. It is recommended to attempt these problems without directly coping code from the solution guides. In all future homework, you will **not** have a coding guide for your problems.

Chapter 1, Section 1.5 (in particular Section 1.5.2) from *Ultrafast Optics* is also highly recommended reading for this homework. MATLAB is encouraged but not required.

Problem 1

See Problem 1 example here online or here on brightspace

part (i) Plot a sine wave with a frequency of 5.5 MHz with amplitude 1 from time = 0s to time = 1.5 us. Plot using 1,000 points.

part (ii) Perform a discrete Fourier transform (DFT) of the plot from part (i). Plot the amplitude of the DFT. Center the DFT around 0 Hz. (This problem is bad!! You will fix it in part (iii))

part (iii) Given that a noise-free sine wave was generated in part (i), it should alarm us that the power spectrum of the DFT from part (ii) is asymmetric and has energy around 0 Hz. Explain why this occurred then propose and implement a fix. Obtain a correct DFT. Center the DFT around 0 Hz.

Problem 2

See Problem 2 example here online or here on brightspace

part (i) Create a Gaussian function centered at t = 0 with peak amplitude of 1 and $t_p = 3us$. I.e. the Gaussian is of the form

$$e^{-(t/t_p)^2}.$$

Plot the function using a time window period T = 120 us, centered around t=0. Use N = 2048 points.

part (ii) Perform a DFT on the Gaussian from part (i). Plot the magnitude and the phase of the Gaussian. Center the DFT around 0 Hz.

part (iii) Perform an inverse DFT on the result from part (ii). Center the returned temporal function around t=0.

part (iv) Create a Gaussian function centered at t = 0 with peak amplitude of 1 and t_p = 25 us. Use 2048 sampling points. Plot the function using a time window period T = 18 ms, centered around t=0. Perform a DFT and plot its magnitude and phase in the frequency domain. Then replace the phase on the DFT with phase$(f) = f/10^3$. Perform an inverse DFT and remark on the changes compared to the initial temporal function.

Problem 3

See Problem 3 example here online or here on brightspace
In this problem, we will explore the importance of sampling rate. Nyquist sampling theorem states that we require a sampling rate of at least $2 \cdot B$, where B is the bandwidth of the original signal. We will consider a sinusoid of frequency f_0. The bandwidth of this signal is f_0 and we require a sampling rate that is at least $2 \cdot f_0$ for meaningfully representing the signal using discrete values. A sampling rate S corresponds to drawing the time samples at integer multiples of $1/S$.

part (i) Plot a sine wave of frequency 50 kHz with amplitude 1 between time = -200 us and time = 200 us. Choose two different sampling rates: (a) a sampling rate S less than $2 \cdot B$ and (b) a sampling rate S greater than $2 \cdot B$.

part (ii) Perform DFT for both cases from part (i). Center the returned spectrum around f = 0. Discuss your observations.

Problem 4

See Problem 4 example here online or here on brightspace
In this problem, we will explore the temporal and spectral representations of a frequency comb. Please refer to Problem #1.7 from the textbook - Ultrafast Optics (Andrew M. Weiner). A frequency comb is a series of discrete, equally spaced frequency lines that are phase-locked to each other. In the time domain, this results in a series of pulses that are separated by the inverse of spacing between the frequency lines.

part (i) Plot the spectrum given by the equation in Problem #1.7. Use the values given in (b) for the constants. Define the frequency samples at steps of 100 MHz and use 1024 samples. Center the spectrum around frequency = 0. Perform the DFT and plot the magnitude of the temporal pulse.

part (ii) Repeat part (i) with frequency samples defined at the steps of 25 MHz. Use $4096 = 1024 \cdot 4$ samples in this part.