1) Let $X$ be the random variable equal to the number of molecules in a small volume $V$ for a gas in equilibrium. We obtained in class that $X$ has Poisson distribution:

$$f(k) = P(X = k) = \frac{(nV)^k \exp(-nV)}{k!}, \quad k = 0, 1, 2, \ldots$$

where $n$ is the number density.

a) Plot the probability density function $f(k)$ for different values of $nV$, for example, $nV = 0.5, 1, 2$ and so on. What is the value of $nV$ for which the Gaussian distribution

$$g(k) = \frac{\exp(-\frac{(k-nV)^2}{2nV})}{\sqrt{2\pi nV}}$$

is a good approximation of $f(k)$?

b) For pressure of 1 atm and room temperature, find the size of volume $V$ such that with the probability of 95% the number of ideal gas molecules in this volume is within 5% of the average value $nV$. This is the size for which molecular fluctuations become important for standard conditions.

c) Assuming the exponential variation of density in Earth atmosphere:

$$n = n_0 \exp\left(-\frac{mgh}{kT}\right)$$

where $n_0 = 2.7 \times 10^{25}$ $1/m^3$ is the number density at sea level; $h$ is the altitude in meters, $m$ is the molecular mass (assume molecular weight of air is 30), $k$ is the Boltzmann constant, and $T$ is the temperature. Assume constant temperature of 273 K.

Find the altitude at which molecular fluctuations in $1 cm^3$ become significant, i.e. the standard deviation of the number of molecules in $V = 1 cm^3$ is equal to 5% of the average value $nV$.

2) If $t$ is the time between collisions for a molecule of ideal gas in equilibrium, then the probability density of $t$ is equal to (prove that as a **bonus** problem)

$$f(t) = \frac{1}{\tau} \exp\left(-\frac{t}{\tau}\right)$$

where $\tau$ is the mean time between collisions.

a) What is the most probable time between collisions?

b) What percentage of molecules do not collide during the time interval (i) $\frac{\tau}{2}$; (ii) $2\tau$; (iii) $5\tau$?