

$$U = \frac{1}{2} EI \int_0^L (u'')^2 dx + \frac{1}{2} K \times [u'(0)]^2$$

$$We = \frac{1}{2} P \times \int_0^L (u')^2 dx + \int_0^L q \cdot u dx$$

$$\Pi = U - We$$

$$= \frac{1}{2} EI \int_0^L (u'')^2 dx + \frac{1}{2} K \times [u'(0)]^2 - \frac{1}{2} P \times \int_0^L (u')^2 dx - \int_0^L q \cdot u dx$$

$$\delta \Pi = 0$$

$$\therefore \int_0^L \left[ \frac{EI}{2} (u'')^2 - \frac{P}{2} (u')^2 - qu \right] dx + \frac{1}{2} K \times [u'(0)]^2$$

$$\therefore \int_0^L -q \cdot \delta u dx - \int_0^L P \times q(u) \times \delta u' + \frac{K}{2} \times 2u'(0) \delta u(0) + \int_0^L EI u'' \times \delta u'' dx$$

$$\therefore \int_0^L -q \delta u dx - \left[ Pu' \times \delta u \right]_0^L + \int_0^L P u'' \times \delta u dx + \left[ Ku' \times \delta u' \right]_{x=0} + \left[ EI u'' \times \delta u'' \right]_0^L$$

$$- \left[ EI u''' \cdot \delta u \right]_0^L + \int_0^L EI u'' \cdot \delta u dx = 0$$

$$\therefore \int_0^L (EI u'' + Pu'' - q) \delta u dx - \left[ (Pu' + EI u'') \delta u \right]_0^L + \left. \left[ EI u'' \times \delta u' \right]_0^L + \left. \left[ (Ku' - EI u'') \cdot \delta u' \right]_0^L \right. = 0$$

$$\therefore \boxed{EI u'' + Pu'' - q = 0} \rightarrow \text{governing d.e.}$$

$$(Pu' + EIu''') \delta u \Big|_0^L = 0$$

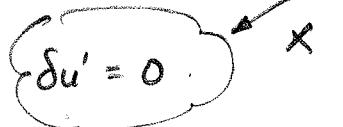
$$\therefore Pu' + EIu''' = 0 \quad \text{or} \quad \delta u = 0 \quad @ x=0, L$$

Since  $\delta u = 0 @ x=0$ ;  $\boxed{Pu' + EIu''' = 0 @ x=L}$

$$EIu'' \times \delta u' \Big|_L = 0$$

Since  $\delta u' \Big|_L = 0 \longrightarrow \checkmark \text{ satisfied}$

$$(Ku' - EIu'') \cdot \delta u' \Big|_0 = 0$$

$\therefore \boxed{Ku' - EIu'' = 0}$  or  $\delta u' = 0$  

Boundary conditions are

@  $x=0$

$$\boxed{\begin{array}{l} \delta u = 0 \\ Ku' - EIu'' = 0 \end{array}}$$

@  $x=L$

$$\boxed{\begin{array}{l} \delta u' = 0 \\ Pu' + EIu''' = 0 \end{array}}$$