$$U = \frac{1}{2} EI \int_0^L (u'')^2 dx + \frac{1}{2} K \times [u'(0)]^2$$

$$We = \frac{1}{2} P \times \int_0^L (u')^2 dx + \int_0^L q \cdot u \, dx$$

$$\pi = U - We$$

$$= \frac{1}{2} EI \int_0^L (u'')^2 dx + \frac{1}{2} K \times [u'(0)]^2 - \frac{1}{2} P \times \int_0^L (u')^2 dx - \int_0^L q u \, dx$$

$$\delta \pi = 0$$

$$\therefore \int_0^L \left[\frac{EI}{2} (u'')^2 - \frac{P}{2} (u')^2 - q u \right] dx + \frac{1}{2} K \times [u'(0)]^2$$

$$\therefore \int_0^L -q \times \delta u \, dx - \int_0^L \frac{P}{\cancel{2}} \times \cancel{2}(u') \times \delta u' + \frac{K}{2} \times 2 u'(0) \, \delta u'(0) + \int_0^L EI \, u'' \times \delta u'' \, dx$$

$$\therefore \int_0^L -q \, \delta u \, dx - \left[P u' \times \delta u \right]_0^L + \int_0^L P u'' \times \delta u \, dx + K u' \times \delta u' \Big|_{x=0} + \left[EI u'' \times \delta u' \right]_0^L$$

$$- \left[EI u''' \cdot \delta u \right]_0^L + \int_0^L EI \, u^{iv} \cdot \delta u \, dx = 0$$

$$\therefore \int_0^L \left(EI u^{iv} + P u'' - q \right) \delta u \, dx - \left[\left(P u' + EI u''' \right) \delta u \right]_0^L + EI u'' \times \delta u' \Big|_L$$

$$+ \left(K u' - EI u'' \right) \cdot \delta u' \Big|_0 = 0$$

$$\therefore \boxed{EI u^{iv} + P u'' - q = 0} \longrightarrow \text{governing d.e.}$$

$$\left(Pu' + EIu'''\right)\delta u \Big|_0^L = 0$$

$\therefore \; Pu' + EIu''' = 0 \quad$ or $\quad \delta u = 0 \quad @ \; x = 0, L$

Since $\delta u = 0 \quad @ \; x = 0 \; ; \quad \boxed{Pu' + EIu''' = 0 \quad @ \; x = L}$

$$EIu'' \times \delta u' \Big|_L = 0$$

Since $\delta u' \Big|_L = 0 \longrightarrow$ ✓ satisfied

$$\left(Ku' - EIu''\right) \cdot \delta u' \Big|_0 = 0$$

$\therefore \; \boxed{Ku' - EIu'' = 0} \quad$ or $\quad \delta u' = 0 \quad$ ×

Boundary conditions are

$@ \; x = 0$

$$\boxed{\begin{array}{l} \delta u = 0 \\ Ku' - EIu'' = 0 \end{array}}$$

$@ \; x = L$

$$\boxed{\begin{array}{l} \delta u' = 0 \\ Pu' + EIu''' = 0 \end{array}}$$