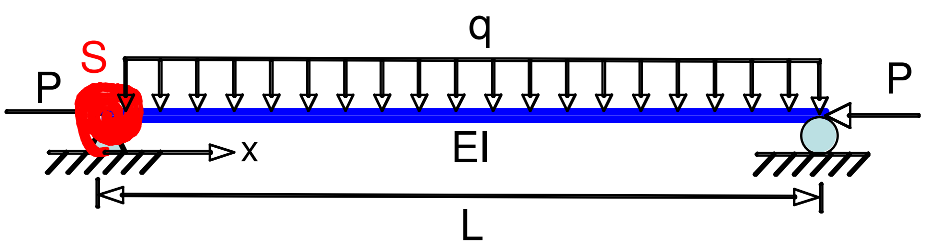
CE 595 HW #3
LYC

**Given:**




**<u>Step 1</u> – Assume an approximate displacement function. Select from the trigonometric family of functions, so that it automatically satisfies the forced boundary conditions (only). To begin with select a one-parameter family approximation.**

Displacement function = y (x)

Assume Solution $y^{*} = a_1 \mathrm{Sin}\left(\frac{\pi x}{L}\right)$

First and second order derivative of y* with respect to x

```
>> syms a1 x L S E I P q
>> y = a1*sin(pi*x/L);

>> diff1_y= diff(y,x)

diff1_y =

a1*cos(pi*x/L)*pi/L

>> diff2_y=diff(y,x,2)

diff2_y =

-a1*sin(pi*x/L)*pi^2/L^2
```

**<u>Step 2</u> – Calculate the strain energy stored in the beam-column. Do not forget to include the contribution of the rotational spring S. The strain energy stored in the spring will use a point value (x=0) for the rotation**

Strain Energy $U = \frac{EI}{2}\int_0^L (y^{''})^2 dx + \frac{1}{2}S(y^{'})^2\Big|_0$

```
>> U=simplify((E*I)/2*int(diff2_y^2,x,0,L)+1/2*S*(pi/L*a1)^2)

U =

1/4*pi^2*a1^2*(E*I*pi^2+2*S*L)/L^3
```

**<u>Step 3</u> – Calculate the work done by the external forces. Remember to include the contribution of the distributed loading**

Work Done $W_e = \frac{P}{2}\int_0^L (y^{'})^2 dx + q\int_0^L y dx$

```
>> We = simplify(P/2*int(diff_y^2,x,0,L)+q*int(y,x,0,L))

We =

1/4*a1*(P*pi^3*a1+8*q*L^2)/pi/L
```

**<u>Step 4</u> – Write the expression for the total potential energy of the system.**

```
>> PI=simplify(U-We)

PI =
```

1/4*a1*(pi^5*a1*E*I+2*pi^3*a1*S*L-L^2*P*pi^3*a1-8*L^4*q)/L^3/pi

**Step 5 – Minimize the potential energy with respect to the parameter. Determine the value of the parameter, and get the complete displacement function. Assume that the spring stiffness S = 3EI/L and the axial force P = 6EI/L$^2$.**

$\frac{\partial \Pi}{\partial a_1} = 0$ gives

```
>> new_a1 = solve(diff(PI,a1),a1);
>> new_a1 = subs(new_a1,{S,P},{3*E*I/L,6*E*I/L^2})

new_a1 =

4*L^4*q/pi^5/E/I


>> new_y=subs(y,{a1},{new_a1})

new_y =

4*L^4*q/pi^5/E/I*sin(pi*x/L)
```

**Step 6 – Calculate the Bending moment diagram for the problem using the calculated displacement function.**

```
>> M=E*I*diff(new_y,x,2)

M =

-4*L^2*q/pi^3*sin(pi*x/L)
```

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**<u>Step 7</u> – If the distributed loading q is assumed to be zero, this will become a buckling problem. In this problem, find $P=P_{cr}$ from your solution in Step 5. Assume that the parameter cannot be equal to zero. This will give you an eigenvalue problem to calculate $P_{cr}$.**

```
>> syms Pcr
>> new_PI = simplify(subs(PI,{q,P},{0,Pcr}),a1)

new_PI =

1/4*pi^2*(E*I*pi^2+2*S*L-Pcr*L^2)*a1^2/L^3


>> new_term=diff(new_PI,a1)

new_term =

1/2*pi^2*(E*I*pi^2+2*S*L-Pcr*L^2)*a1/L^3


>> Pcr = solve(new_term,Pcr)

Pcr =

(E*I*pi^2+2*S*L)/L^2


>> new_Pcr = subs(Pcr,{S,P},{3*E*I/L,6*E*I/L^2})

new_Pcr =
```

**(E*I*pi^2+6*E*I)/L^2**

**15.8696*E*I/L^2**

**<u>Step 8</u> – Go back to the beginning of the problem, and use a two-parameter family approximation. If you developed Steps 1-7 using mathematical software, then this will be easy. Find the solutions for Steps 5, 6, and 7 again using the two-parameter approximation.**

### 1. Assume Deflection:

```
>> y = a1*sin(pi*x/L)+a2*sin(2*pi*x/L);
>> diff1_y= diff(y,x)

diff1_y =

a1*cos(pi*x/L)*pi/L+2*a2*cos(2*pi*x/L)*pi/L

>> diff2_y=simplify(diff(y,x,2))

diff2_y =

-pi^2*(a1*sin(pi*x/L)+4*a2*sin(2*pi*x/L))/L^2

>> spring_Energy = subs(1/2*S*diff(y,x)^2, {x,S},{0,3*E*I/L});
spring_Energy =

3/2*E*I/L*(a1*pi/L+2*a2*pi/L)^2
```

### 2. Strain Energy stored in the system.

```
>> U=simplify((E*I)/2*int(diff2_y^2,x,0,L)+spring_Energy)

U =

1/4*E*I*pi^2*(pi^2*a1^2+16*pi^2*a2^2+6*a1^2+24*a1*a2+24*a2^2)/L^3
```

### 3. Work done by external forces

```
>> We = P/2*int(diff1_y^2,x,0,L)+q*int(y,x,0,L);
```

### 4,5 . Total potential energy and find the value of the parameter and displacement function

```
>> PI = U-We;
>> term1=diff(PI,a1);
>> term2 = diff(PI,a2);
 >> solution = solve(term1, term2, a1, a2)

solution =

    a1: [1x1 sym]
    a2: [1x1 sym]

>> new_a1 = subs(solution.a1,{P},{6*E*I/L^2})

new_a1 =
```

**16*E*I/pi/(4*E^2*I^2*pi^4-36*E^2*I^2)*L^4*q**

```
>> new_a2 = subs(solution.a2,{P},{6*E*I/L^2})

new_a2 =
```

**-12/pi^3/(4*E^2*I^2*pi^4-36*E^2*I^2)*E*I*L^4*q**

```
>> new_y = subs(y,{a1,a2},{new_a1,new_a2})
```

**new_y =**

**1/E/I*L^4*q*(4*sin(pi*x/L)*pi^2-3*sin(2*pi*x/L))/pi^3/(-9+pi^4)** ← displacement function

### 6. Bending moment

```
>> M=E*I*diff(new_y,'x',2)

M =
```

**L^4*q*(-4*sin(pi*x/L)*pi^4/L^2+12*sin(2*pi*x/L)*pi^2/L^2)/pi^3/(-9+pi^4)**

### 7. Find critical buckling load, Pcr

```
>> new_PI = subs(PI,{q},{0});
>> new_term1=collect(diff(new_PI,a1),a1)

new_term1 =

(1/4*E*I*pi^2*(2*pi^2+12)/L^3-1/2*P*pi^2/L)*a1+6*E*I*pi^2*a2/L^3

>> new_term2=collect(diff(new_PI,a2),a2)

new_term2 =

(1/4*E*I*pi^2*(32*pi^2+48)/L^3-2*P*pi^2/L)*a2+6*E*I*pi^2*a1/L^3
```

**We can write the simultaneous equations in the format of matrix as follows:**

**[ 2 x 2 matrix]** $\begin{Bmatrix} a1 \\ a2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$

**Since a1,a2≠0 (i.e., to avoid trivial solutions), we have to solve the eigen-value problem in order to get Pcr.**

**Find the determinant of [ 2 x 2 matrix] and solve it for P**

```
matrix =

[(1/4*E*I*pi^2*(2*pi^2+12)/L^3-1/2*P*pi^2/L),                    6*E*I*pi^2/L^3]
[                  6*E*I*pi^2/L^3,            (1/4*E*I*pi^2*(32*pi^2+48)/L^3-2*P*pi^2/L)]


>> Pcr = simplify(solve(det(matrix),P))

Pcr =
```

**14.7*E/L^2**

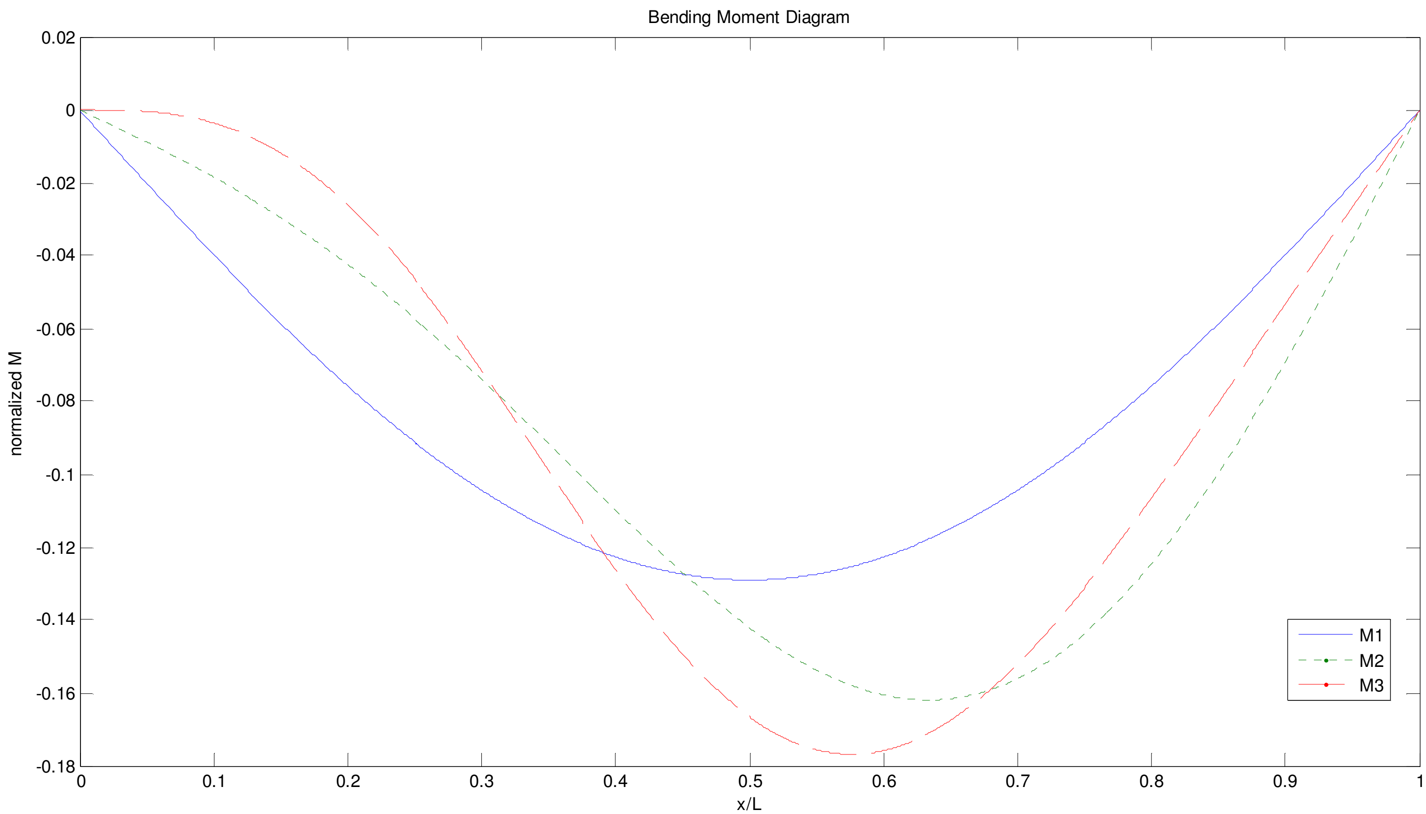
**<u>Step 9</u> – If you can do it easily, then for bonus fun and points solve the problem again using a three-parameter family approximation (using math software only). This is optional (not required).**

```
>> syms a1 a2 a3 x L q P E I S
>> y = a1*sin(pi*x/L)+a2*sin(2*pi*x/L)+a3*sin(3*pi*x/L);
>> diff1_y= diff(y,x);
>> diff2_y=diff(y,x,2);
>> spring_Energy = 1/2*S*diff(y,x)^2;
>> spring_Energy = subs(spring_Energy,{x,S},{0,3*E*I/L});
>> U=simplify((E*I)/2*int(diff2_y^2,x,0,L)+spring_Energy);
>> We = P/2*int(diff1_y^2,x,0,L)+q*int(y,x,0,L);
>> PI = U-We;
>> term1=diff(PI,a1);
>> term2=diff(PI,a2);
>> term3=diff(PI,a3);
>> solution = solve(term1, term2,term3, a1, a2,a3)
```

```
solution =

    a1: [1x1 sym]
    a2: [1x1 sym]
    a3: [1x1 sym]

>> new_a1 = subs(solution.a1,{S,P},{3*E*I/L,6*E*I/L^2});
>> new_a2 = subs(solution.a2,{S,P},{3*E*I/L,6*E*I/L^2});
>> new_a3 = subs(solution.a3,{S,P},{3*E*I/L,6*E*I/L^2});
>> new_y=simplify(subs(y,{a1,a2,a3},{new_a1,new_a2,new_a3}))

new_y =

2/243*L^4*q/E/I*(486*sin(pi*x/L)*pi^4-36*sin(pi*x/L)*pi^2-432*sin(pi*x/L)-
369*sin(2*pi*x/L)*pi^2+270*sin(2*pi*x/L)+2*sin(3*pi*x/L)*pi^4-
108*sin(3*pi*x/L)*pi^2+144*sin(3*pi*x/L))/pi^3/(pi^6-14*pi^2+12)

>> M=E*I*diff(new_y,'x',2);

>> % buckling load P=Pcr
>> new_PI = subs(PI,{q},{0});
>> new_term1=collect(diff(new_PI,a1),a1)

new_term1 =

(1/4*E*I*pi^2*(2*pi^2+12)/L^3-1/2*P*pi^2/L)*a1+1/4*E*I*pi^2*(24*a2+36*a3)/L^3

 >> new_term2=collect(diff(new_PI,a2),a2)

new_term2 =

(1/4*E*I*pi^2*(32*pi^2+48)/L^3-2*P*pi^2/L)*a2+1/4*E*I*pi^2*(24*a1+72*a3)/L^3
```

```
>> new_term3=collect(diff(new_PI,a3),a3)

new_term3 =

(1/4*E*I*pi^2*(162*pi^2+108)/L^3-9/2*P*pi^2/L)*a3+1/4*E*I*pi^2*(36*a1+72*a2)/L^3


>> % eigenvalue problem

matrix =

[  78.3134*E*I/L^3-4.9348*P/L    59.2176*E*I/L^3      88.8264*E*I/L^3   ]
[59.2176*E*I/L^3    897.7080*E*I/L^3-19.7392*P/L    177.6529*E*I/L^3  ]
[88.8264*E*I/L^3    177.6529*E*I/L^3          4212*E*I/L^3-44.4132*P/L   ]



>> new_Pcr = simplify(solve(det(matrix),Pcr))

new_Pcr =
```

**14.415*E/L^2**




Moment is normalized by setting q=1 and L=1.