

$$U = \frac{1}{2} EI \int_0^L (u'')^2 dx + \frac{1}{2} K \times [u'(0)]^2$$

$$W_e = \frac{1}{2} P \times \int_0^L (u')^2 dx + \int_0^L q \cdot u dx$$

$$\pi = U - W_e$$

$$= \frac{1}{2} EI \int_0^L (u'')^2 dx + \frac{1}{2} K \times [u'(0)]^2 - \frac{1}{2} P \times \int_0^L (u')^2 dx - \int_0^L q u dx$$

$$\delta \pi = 0$$

$$\therefore \int_0^L \left[\frac{EI}{2} (u'')^2 - \frac{P}{2} (u')^2 - q u \right] dx + \frac{1}{2} K \times [u'(0)]^2$$

$$\therefore \int_0^L -q \times \delta u dx - \int_0^L \frac{P}{2} \times 2(u') \times \delta u' + \frac{K}{2} \times 2 u'(0) \delta u'(0) + \int_0^L EI u'' \times \delta u'' dx$$

$$\therefore \int_0^L -q \delta u dx - [P u' \times \delta u]_0^L + \int_0^L P u'' \times \delta u dx + K u' \times \delta u' \Big|_{x=0} + [EI u'' \times \delta u']_0^L$$

$$- [EI u''' \cdot \delta u]_0^L + \int_0^L EI u'' \cdot \delta u dx = 0$$

$$\therefore \int_0^L (EI u'' + P u'' - q) \delta u dx - [(P u' + EI u'') \delta u]_0^L + EI u' \times \delta u' \Big|_L$$

$$+ (K u' - EI u'') \cdot \delta u' \Big|_0 = 0$$

$$\therefore \boxed{EI u'' + P u'' - q = 0} \rightarrow \text{governing d.e.}$$

$$(Pu' + EIu''') \delta u \Big|_0^L = 0$$

$$\therefore Pu' + EIu''' = 0 \quad \text{or} \quad \delta u = 0 \quad @ \quad x=0, L$$

Since $\delta u = 0$ @ $x=0$; $\boxed{Pu' + EIu''' = 0 \quad @ \quad x=L}$

$$EIu'' \times \delta u' \Big|_L = 0$$

Since $\delta u' \Big|_L = 0 \longrightarrow \checkmark$ satisfied.

$$(Ku' - EIu'') \cdot \delta u' \Big|_0 = 0$$

$$\therefore \boxed{Ku' - EIu'' = 0}$$

or $\delta u' = 0$ $\swarrow$ x

Boundary conditions are

@ $x=0$

$$\boxed{\begin{array}{l} \delta u = 0 \\ Ku' - EIu'' = 0 \end{array}}$$

@ $x=L$

$$\boxed{\begin{array}{l} \delta u' = 0 \\ Pu' + EIu''' = 0 \end{array}}$$