1. Shape function matrix w.r.t. the generalized coordinates:

$$\mathbf{N_s}(\mathbf{x},\mathbf{y}) := \begin{pmatrix} 1 & \mathbf{x} & \mathbf{y} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \mathbf{x} & \mathbf{y} \end{pmatrix}$$ **Answer**

2. Shape function matrix w.r.t nodal displacement:

Interpolation approach

$$\mathbf{u} := \begin{pmatrix} \mathbf{u1} \\ \mathbf{v1} \\ \mathbf{u2} \\ \mathbf{v2} \\ \mathbf{u3} \\ \mathbf{v3} \end{pmatrix} \qquad \mathbf{a} := \begin{pmatrix} \mathbf{a1} \\ \mathbf{a2} \\ \mathbf{a3} \\ \mathbf{a4} \\ \mathbf{a5} \\ \mathbf{a6} \end{pmatrix}$$

$\mathbf{x1} := 12$ $\quad\mathbf{x2} := 18$ $\quad\mathbf{x3} := 12$

$\mathbf{y1} := 0$ $\quad\mathbf{y2} := 0$ $\quad\mathbf{y3} := 2$

$$\mathbf{A} := \begin{pmatrix} 1 & \mathbf{x1} & \mathbf{y1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \mathbf{x1} & \mathbf{y1} \\ 1 & \mathbf{x2} & \mathbf{y2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \mathbf{x2} & \mathbf{y2} \\ 1 & \mathbf{x3} & \mathbf{y3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \mathbf{x3} & \mathbf{y3} \end{pmatrix} \qquad \mathbf{A}^{-1} \rightarrow \begin{pmatrix} 3 & 0 & -2 & 0 & 0 & 0 \\ -\frac{1}{6} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 3 & 0 & -2 & 0 & 0 \\ 0 & -\frac{1}{6} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\mathbf{N_{nodal}}(\mathbf{x},\mathbf{y}) := \mathbf{N_s}(\mathbf{x},\mathbf{y})\cdot\mathbf{A}^{-1} \rightarrow \begin{pmatrix} 3-\frac{\mathbf{y}}{2}-\frac{\mathbf{x}}{6} & 0 & \frac{\mathbf{x}}{6}-2 & 0 & \frac{\mathbf{y}}{2} & 0 \\ 0 & 3-\frac{\mathbf{y}}{2}-\frac{\mathbf{x}}{6} & 0 & \frac{\mathbf{x}}{6}-2 & 0 & \frac{\mathbf{y}}{2} \end{pmatrix}$$ **Answer**

3. Strain displacement matrix w.r.t. generalized coordinates:

$$\mathbf{B} := \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Answer

4. Strain displacement matrix w.r.t. nodal displacement:

$$\mathbf{B1}(\mathbf{x}, \mathbf{y}) := \mathbf{B} \cdot \mathbf{A}^{-1} \rightarrow \begin{pmatrix} -\frac{1}{6} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{2} & 0 \end{pmatrix}$$

Answer

5. Plane stress elasticity matrix:

$\mathbf{E} := 29000 \cdot \mathbf{ksi}$ $\qquad \boldsymbol{\nu} := 0.3$ $\qquad \mathbf{h} := 0.25 \cdot \mathbf{in}$

$$\mathbf{D} := \frac{\mathbf{E} \cdot \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix}}{1 - \nu^2} = \begin{pmatrix} 3.187 \times 10^4 & 9.56 \times 10^3 & 0 \\ 9.56 \times 10^3 & 3.187 \times 10^4 & 0 \\ 0 & 0 & 1.115 \times 10^4 \end{pmatrix} \cdot \mathbf{ksi}$$

6. Stress-displacement matrix [DB]

$$\mathbf{DB} := \begin{pmatrix} 3.187 \times 10^4 & 9.56 \times 10^3 & 0 \\ 9.56 \times 10^3 & 3.187 \times 10^4 & 0 \\ 0 & 0 & 1.115 \times 10^4 \end{pmatrix} \mathbf{ksi} \cdot \begin{pmatrix} -\frac{1}{6} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{2} & 0 \end{pmatrix}$$

$$\mathbf{DB} = \begin{pmatrix} -5.312\times10^{3} & -4.78\times10^{3} & 5.312\times10^{3} & 0 & 0 & 4.78\times10^{3} \\ -1.593\times10^{3} & -1.593\times10^{4} & 1.593\times10^{3} & 0 & 0 & 1.593\times10^{4} \\ -5.575\times10^{3} & -1.858\times10^{3} & 0 & 1.858\times10^{3} & 5.575\times10^{3} & 0 \end{pmatrix}\cdot\mathbf{ksi}$$

Answer

7. Generalized element stiffness matrix

$$\mathbf{Area} := \frac{1\cdot(\mathbf{x2}-\mathbf{x1})\cdot(\mathbf{y3}-\mathbf{y1})}{2}$$

$$\mathbf{Area} = 6$$

$$\mathbf{D_m} := \begin{pmatrix} 3.187\times10^{4} & 9.56\times10^{3} & 0 \\ 9.56\times10^{3} & 3.187\times10^{4} & 0 \\ 0 & 0 & 1.115\times10^{4} \end{pmatrix}\mathbf{ksi}$$

$$\mathbf{K_{bar}} := \mathbf{Area}\,\mathbf{B^T}\cdot\mathbf{D_m}\cdot\mathbf{B}\cdot\mathbf{in}$$ per unit thickness

$$\mathbf{K_{bar}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.912\times10^{5} & 0 & 0 & 0 & 5.736\times10^{4} \\ 0 & 0 & 6.69\times10^{4} & 0 & 6.69\times10^{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6.69\times10^{4} & 0 & 6.69\times10^{4} & 0 \\ 0 & 5.736\times10^{4} & 0 & 0 & 0 & 1.912\times10^{5} \end{pmatrix}\cdot\frac{\mathbf{kip}}{\mathbf{in}}$$

Answer

If h = 0.25 in,

$$\mathbf{K_{bar1}} := \mathbf{h}\cdot\mathbf{Area}\cdot\left(\mathbf{B^T}\cdot\mathbf{D_m}\cdot\mathbf{B}\right)$$

$$K_{bar1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.78\times10^4 & 0 & 0 & 0 & 1.434\times10^4 \\ 0 & 0 & 1.672\times10^4 & 0 & 1.672\times10^4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.672\times10^4 & 0 & 1.672\times10^4 & 0 \\ 0 & 1.434\times10^4 & 0 & 0 & 0 & 4.78\times10^4 \end{pmatrix} \cdot \frac{kip}{in}$$

Answer

7. Generalized element stiffness matrix:

$$K := \left(A^{-1}\right)^T \cdot \frac{K_{bar}}{in} \cdot A^{-1} \cdot in$$ Per unit thickness

Using h = 0.25 in $K_1 := \left(A^{-1}\right)^T \cdot K_{bar1} \cdot A^{-1}$

$$K = \begin{pmatrix} 2.204\times10^4 & 1.036\times10^4 & -5.312\times10^3 & -5.575\times10^3 & -1.673\times10^4 & -4.78\times10^3 \\ 1.036\times10^4 & 4.966\times10^4 & -4.78\times10^3 & -1.858\times10^3 & -5.575\times10^3 & -4.78\times10^4 \\ -5.312\times10^3 & -4.78\times10^3 & 5.312\times10^3 & 0 & 0 & 4.78\times10^3 \\ -5.575\times10^3 & -1.858\times10^3 & 0 & 1.858\times10^3 & 5.575\times10^3 & 0 \\ -1.673\times10^4 & -5.575\times10^3 & 0 & 5.575\times10^3 & 1.673\times10^4 & 0 \\ -4.78\times10^3 & -4.78\times10^4 & 4.78\times10^3 & 0 & 0 & 4.78\times10^4 \end{pmatrix} \cdot \frac{kip}{in}$$

Answer

$$K_1 = \begin{pmatrix} 5.509\times10^3 & 2.589\times10^3 & -1.328\times10^3 & -1.394\times10^3 & -4.181\times10^3 & -1.195\times10^3 \\ 2.589\times10^3 & 1.242\times10^4 & -1.195\times10^3 & -464.583 & -1.394\times10^3 & -1.195\times10^4 \\ -1.328\times10^3 & -1.195\times10^3 & 1.328\times10^3 & 0 & 0 & 1.195\times10^3 \\ -1.394\times10^3 & -464.583 & 0 & 464.583 & 1.394\times10^3 & 0 \\ -4.181\times10^3 & -1.394\times10^3 & 0 & 1.394\times10^3 & 4.181\times10^3 & 0 \\ -1.195\times10^3 & -1.195\times10^4 & 1.195\times10^3 & 0 & 0 & 1.195\times10^4 \end{pmatrix} \cdot \frac{kip}{in}$$

Answer