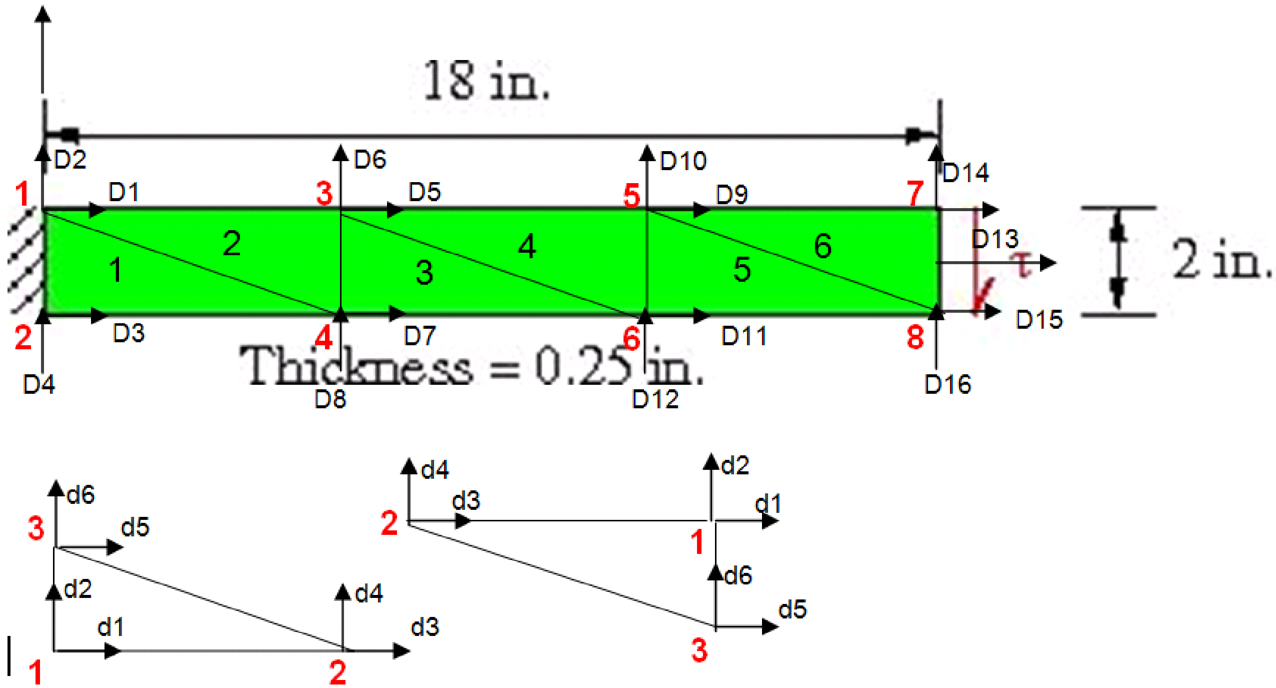
CE595: Homework No. 6



Given: Cantilevered beam with dimensions shown; rigidly fixed at $x = 0$; applied traction at $x = 18$. $E = 27{,}000$ ksi; $\nu = 0.25$. Additional loading due to self-weight of steel $= 0.003$ k/in^3.

Required: Using CST plane stress elements, find the approximate deflection of the free end at $y = 0$.

E.B.
CE 595
Homework No. 6
Solution

$ORIGIN \equiv 1$

Formulate shape functions:

$$ustar_i\left(x, y, a_1, a_2, a_3\right) := a_1 + a_2 \cdot x + a_3 \cdot y$$

$$vstar_i\left(x, y, a_4, a_5, a_6\right) := a_4 + a_5 \cdot x + a_6 \cdot y$$

$$Nbar_i(x, y) := \begin{pmatrix} 1 & x & y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x & y \end{pmatrix}$$

Element 1:

$x1_1 := 0 \quad y1_1 := -1 \qquad x1_2 := 6 \quad y1_2 := -1 \qquad x1_3 := 0 \quad y1_3 := 1$ inches

$$A1 := \begin{pmatrix} 1 & x1_1 & y1_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x1_1 & y1_1 \\ 1 & x1_2 & y1_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x1_2 & y1_2 \\ 1 & x1_3 & y1_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x1_3 & y1_3 \end{pmatrix}$$

$$N1(x, y) := Nbar_i(x, y) \cdot A1^{-1}$$

$$N1(x, y) \rightarrow \begin{pmatrix} \frac{1}{2} - \frac{y}{2} - \frac{x}{6} & 0 & \frac{x}{6} & 0 & \frac{y}{2} + \frac{1}{2} & 0 \\ 0 & \frac{1}{2} - \frac{y}{2} - \frac{x}{6} & 0 & \frac{x}{6} & 0 & \frac{y}{2} + \frac{1}{2} \end{pmatrix}$$

$N1_1(x, y) := N1(x, y)_{1,1} \qquad N1_2(x, y) := N1(x, y)_{1,3} \qquad N1_3(x, y) := N1(x, y)_{1,5}$

Element 2:

$x2_1 := 6$ $y2_1 := 1$ $x2_2 := 0$ $y2_2 := 1$ $x2_3 := 6$ $y2_3 := -1$ inches

$$A2 := \begin{pmatrix} 1 & x2_1 & y2_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x2_1 & y2_1 \\ 1 & x2_2 & y2_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x2_2 & y2_2 \\ 1 & x2_3 & y2_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x2_3 & y2_3 \end{pmatrix}$$

$$N2(x,y) := Nbar_i(x,y) \cdot A2^{-1}$$

$$N2(x,y) \rightarrow \begin{pmatrix} \frac{x}{6} + \frac{y}{2} - \frac{1}{2} & 0 & 1 - \frac{x}{6} & 0 & \frac{1}{2} - \frac{y}{2} & 0 \\ 0 & \frac{x}{6} + \frac{y}{2} - \frac{1}{2} & 0 & 1 - \frac{x}{6} & 0 & \frac{1}{2} - \frac{y}{2} \end{pmatrix}$$

$N2_1(x,y) := N2(x,y)_{1,1}$ $N2_2(x,y) := N2(x,y)_{1,3}$ $N2_3(x,y) := N2(x,y)_{1,5}$

Element 3:

$x3_1 := 6$ $y3_1 := -1$ $x3_2 := 12$ $y3_2 := -1$ $x3_3 := 6$ $y3_3 := 1$ inches

$$A3 := \begin{pmatrix} 1 & x3_1 & y3_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x3_1 & y3_1 \\ 1 & x3_2 & y3_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x3_2 & y3_2 \\ 1 & x3_3 & y3_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x3_3 & y3_3 \end{pmatrix}$$

$$N3(x,y) := Nbar_i(x,y) \cdot A3^{-1}$$

$$N3(x,y) \rightarrow \begin{pmatrix} \frac{3}{2} - \frac{y}{2} - \frac{x}{6} & 0 & \frac{x}{6} - 1 & 0 & \frac{y}{2} + \frac{1}{2} & 0 \\ 0 & \frac{3}{2} - \frac{y}{2} - \frac{x}{6} & 0 & \frac{x}{6} - 1 & 0 & \frac{y}{2} + \frac{1}{2} \end{pmatrix}$$

$N3_1(x,y) := N3(x,y)_{1,1}$ $\quad N3_2(x,y) := N3(x,y)_{1,3}$ $\quad N3_3(x,y) := N3(x,y)_{1,5}$

Element 4:

$x4_1 := 12 \quad y4_1 := 1 \qquad x4_2 := 6 \quad y4_2 := 1 \qquad x4_3 := 12 \quad y4_3 := -1$ inches

$$A4 := \begin{pmatrix} 1 & x4_1 & y4_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x4_1 & y4_1 \\ 1 & x4_2 & y4_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x4_2 & y4_2 \\ 1 & x4_3 & y4_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x4_3 & y4_3 \end{pmatrix}$$

$N4(x,y) := Nbar_i(x,y) \cdot A4^{-1}$

$$N4(x,y) \rightarrow \begin{pmatrix} \frac{x}{6} + \frac{y}{2} - \frac{3}{2} & 0 & 2 - \frac{x}{6} & 0 & \frac{1}{2} - \frac{y}{2} & 0 \\ 0 & \frac{x}{6} + \frac{y}{2} - \frac{3}{2} & 0 & 2 - \frac{x}{6} & 0 & \frac{1}{2} - \frac{y}{2} \end{pmatrix}$$

$N4_1(x,y) := N4(x,y)_{1,1}$ $\quad N4_2(x,y) := N4(x,y)_{1,3}$ $\quad N4_3(x,y) := N4(x,y)_{1,5}$

Element 5:

$x5_1 := 12 \quad y5_1 := -1 \qquad x5_2 := 18 \quad y5_2 := -1 \qquad x5_3 := 12 \quad y5_3 := 1$ inches

$$A5 := \begin{pmatrix} 1 & x5_1 & y5_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x5_1 & y5_1 \\ 1 & x5_2 & y5_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x5_2 & y5_2 \\ 1 & x5_3 & y5_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x5_3 & y5_3 \end{pmatrix}$$

$$N5(x,y) := Nbar_i(x,y)\cdot A5^{-1}$$

$$N5(x,y) \rightarrow \begin{pmatrix} \frac{5}{2} - \frac{y}{2} - \frac{x}{6} & 0 & \frac{x}{6} - 2 & 0 & \frac{y}{2} + \frac{1}{2} & 0 \\ 0 & \frac{5}{2} - \frac{y}{2} - \frac{x}{6} & 0 & \frac{x}{6} - 2 & 0 & \frac{y}{2} + \frac{1}{2} \end{pmatrix}$$

$N5_1(x,y) := N5(x,y)_{1,1}$ $\quad$ $N5_2(x,y) := N5(x,y)_{1,3}$ $\quad$ $N5_3(x,y) := N5(x,y)_{1,5}$

Element 6:

$x6_1 := 18$ $\quad y6_1 := 1$ $\quad$ $x6_2 := 12$ $\quad y6_2 := 1$ $\quad$ $x6_3 := 18$ $\quad y6_3 := -1$ $\quad$ inches

$$A6 := \begin{pmatrix} 1 & x6_1 & y6_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x6_1 & y6_1 \\ 1 & x6_2 & y6_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x6_2 & y6_2 \\ 1 & x6_3 & y6_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x6_3 & y6_3 \end{pmatrix}$$

$$N6(x,y) := Nbar_i(x,y)\cdot A6^{-1}$$

$$N6(x,y) \rightarrow \begin{pmatrix} \frac{x}{6} + \frac{y}{2} - \frac{5}{2} & 0 & 3 - \frac{x}{6} & 0 & \frac{1}{2} - \frac{y}{2} & 0 \\ 0 & \frac{x}{6} + \frac{y}{2} - \frac{5}{2} & 0 & 3 - \frac{x}{6} & 0 & \frac{1}{2} - \frac{y}{2} \end{pmatrix}$$

$N6_1(x,y) := N6(x,y)_{1,1}$ $\quad$ $N6_2(x,y) := N6(x,y)_{1,3}$ $\quad$ $N6_3(x,y) := N6(x,y)_{1,5}$

Formulate B matrix for each element:

Elements 1,3,5:

$$\mathrm{Bodd}(x,y) := \begin{bmatrix} \left(\frac{d}{dx}N1_1(x,y)\right) + 0\cdot 0 & \left(\frac{d}{dx}0\right) + \left(0\cdot N1_1(x,y)\right) & \left(\frac{d}{dx}N1_2(x,y)\right) + (0\cdot 0) & \left(\frac{d}{dx}0\right) + \left(0\cdot N1_2(x,y)\right) & \left(\frac{d}{dx}N1_3(x,y)\right) + (0\cdot 0) & \left(\frac{d}{dx}0\right) + \left(0\cdot N1_3(x,y)\right) \\ \left(0\cdot N1_1(x,y)\right) + \left(\frac{d}{dy}0\right) & (0\cdot 0) + \left(\frac{d}{dy}N1_1(x,y)\right) & \left(0\cdot N1_2(x,y)\right) + \left(\frac{d}{dy}0\right) & (0\cdot 0) + \left(\frac{d}{dy}N1_2(x,y)\right) & \left(0\cdot N1_3(x,y)\right) + \left(\frac{d}{dy}0\right) & (0\cdot 0) + \left(\frac{d}{dy}N1_3(x,y)\right) \\ \left(\frac{d}{dy}N1_1(x,y)\right) + \left(\frac{d}{dx}0\right) & \left(\frac{d}{dy}0\right) + \left(\frac{d}{dx}N1_1(x,y)\right) & \left(\frac{d}{dy}N1_2(x,y)\right) + \left(\frac{d}{dx}0\right) & \left(\frac{d}{dy}0\right) + \left(\frac{d}{dx}N1_2(x,y)\right) & \left(\frac{d}{dy}N1_3(x,y)\right) + \left(\frac{d}{dx}0\right) & \left(\frac{d}{dy}0\right) + \left(\frac{d}{dx}N1_3(x,y)\right) \end{bmatrix}$$

$$\mathrm{Bodd}(x,y) \rightarrow \begin{pmatrix} -\frac{1}{6} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{2} & 0 \end{pmatrix} \qquad \mathrm{Bodd} := \begin{pmatrix} -\frac{1}{6} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{2} & 0 \end{pmatrix}$$

Elements 2,4,6:

$$\mathrm{Beven}(x,y) := \begin{bmatrix} \left(\frac{d}{dx}N2_1(x,y)\right) + 0\cdot 0 & \left(\frac{d}{dx}0\right) + \left(0\cdot N2_1(x,y)\right) & \left(\frac{d}{dx}N2_2(x,y)\right) + (0\cdot 0) & \left(\frac{d}{dx}0\right) + \left(0\cdot N2_2(x,y)\right) & \left(\frac{d}{dx}N2_3(x,y)\right) + (0\cdot 0) & \left(\frac{d}{dx}0\right) + \left(0\cdot N2_3(x,y)\right) \\ \left(0\cdot N2_1(x,y)\right) + \left(\frac{d}{dy}0\right) & (0\cdot 0) + \left(\frac{d}{dy}N2_1(x,y)\right) & \left(0\cdot N2_2(x,y)\right) + \left(\frac{d}{dy}0\right) & (0\cdot 0) + \left(\frac{d}{dy}N2_2(x,y)\right) & \left(0\cdot N2_3(x,y)\right) + \left(\frac{d}{dy}0\right) & (0\cdot 0) + \left(\frac{d}{dy}N2_3(x,y)\right) \\ \left(\frac{d}{dy}N2_1(x,y)\right) + \left(\frac{d}{dx}0\right) & \left(\frac{d}{dy}0\right) + \left(\frac{d}{dx}N2_1(x,y)\right) & \left(\frac{d}{dy}N2_2(x,y)\right) + \left(\frac{d}{dx}0\right) & \left(\frac{d}{dy}0\right) + \left(\frac{d}{dx}N2_2(x,y)\right) & \left(\frac{d}{dy}N2_3(x,y)\right) + \left(\frac{d}{dx}0\right) & \left(\frac{d}{dy}0\right) + \left(\frac{d}{dx}N2_3(x,y)\right) \end{bmatrix}$$

$$\mathrm{Beven}(x,y) \rightarrow \begin{pmatrix} \frac{1}{6} & 0 & -\frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{6} & 0 & -\frac{1}{6} & -\frac{1}{2} & 0 \end{pmatrix} \qquad \mathrm{Beven} := \begin{pmatrix} \frac{1}{6} & 0 & -\frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{6} & 0 & -\frac{1}{6} & -\frac{1}{2} & 0 \end{pmatrix}$$

Find the plane stress elasticity matrix:

$E := 27000 \cdot 10^3$ lb/in^2

$\nu := 0.25$

$t := 0.25$ in

$A_{elem} := 0.5 \cdot 6 \cdot 2 = 6$ in^2

For plane stress, $\sigma_z = 0$.

The elasticity matrix depends on Hooke's Law:

$\sigma_{xx} = (E/1\text{-}v^2)*(\varepsilon_{xx} + v\varepsilon_{yy})$

$\sigma_{yy} = (E/1\text{-}v^2)*(v\varepsilon_{xx} + \varepsilon_{yy})$

$\sigma_{xy} = (E/1\text{-}v^2)*(1\text{-}v)\varepsilon_{xy}$

Therefore:

$$C := \left(\frac{E}{1 - \nu^2}\right) \cdot \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1 - \nu)}{2} \end{bmatrix}$$

$$C = \begin{pmatrix} 2.88 \times 10^7 & 7.2 \times 10^6 & 0 \\ 7.2 \times 10^6 & 2.88 \times 10^7 & 0 \\ 0 & 0 & 1.08 \times 10^7 \end{pmatrix} \quad \text{lb/in}^2$$

Find element stiffness matrices:

Odd Elements:

$k_{odd} := (t \cdot A_{elem}) \cdot (Bodd^T \cdot C \cdot Bodd)$

$$k_{odd} = \begin{pmatrix} 5.25 \times 10^6 & 2.25 \times 10^6 & -1.2 \times 10^6 & -1.35 \times 10^6 & -4.05 \times 10^6 & -9 \times 10^5 \\ 2.25 \times 10^6 & 1.13 \times 10^7 & -9 \times 10^5 & -4.5 \times 10^5 & -1.35 \times 10^6 & -1.08 \times 10^7 \\ -1.2 \times 10^6 & -9 \times 10^5 & 1.2 \times 10^6 & 0 & 0 & 9 \times 10^5 \\ -1.35 \times 10^6 & -4.5 \times 10^5 & 0 & 4.5 \times 10^5 & 1.35 \times 10^6 & 0 \\ -4.05 \times 10^6 & -1.35 \times 10^6 & 0 & 1.35 \times 10^6 & 4.05 \times 10^6 & 0 \\ -9 \times 10^5 & -1.08 \times 10^7 & 9 \times 10^5 & 0 & 0 & 1.08 \times 10^7 \end{pmatrix} \quad \text{lb/in}$$

Because Bodd = - Beven, and [k] = $[B]^T$*[C]*[B]*t*A_{elem}

$[k_{odd}] = [Bodd]^T*[C]*[Bodd]*t*A_{elem}$

$[k_{even}] = [Beven]^T*[C]*[Beven]*t*A_{elem} = -[Bodd]^T*[C]*-[Bodd]*t*A_{elem}$

the negatives cancel and $[k_{odd}] = [k_{even}]$, thus all elements have the same stiffness matrix.

$Stiff := k_{odd}$

Find element force vectors:

Body Forces:

$B_x := 0$ $\quad B_y := -3$ $\quad$ lb/in^3

$$Body := \begin{pmatrix} B_x \\ B_y \end{pmatrix}$$

Generalized body force vector Qbar:

Where the diagonal of element 1 is defined by the line y = -(1/3)x + 1 :

$$Q1bar(x,y) := t\cdot\int_0^6\int_0^{1-\frac{x}{3}} \left(Nbar_1(x,y)^T\right)\cdot Body\,dy\,dx$$

$$Q1(x,y) := \left(A1^{-1}\right)^T\cdot Q1bar(x,y)$$

$$Q1(x,y) \rightarrow \begin{pmatrix} 0 \\ -0.375 \\ 0 \\ 0.75 \\ 0 \\ -0.375 \end{pmatrix} \quad \text{lb}$$

Where the diagonal of element 2 is defined by the line y = -(1/3)x + 1 :

$$Q2bar(x,y) := t\cdot\int_{0}^{6}\int_{0}^{1-\frac{x}{3}} \left(Nbar_i(x,y)^T\right)\cdot Body\,dy\,dx$$

$$Q2(x,y) := \left(A2^{-1}\right)^T\cdot Q2bar(x,y)$$

$$Q2(x,y) \rightarrow \begin{pmatrix} 0 \\ 0.375 \\ 0 \\ -0.75 \\ 0 \\ 0.375 \end{pmatrix} \quad lb$$

Where the diagonal of element 3 is defined by the line y = -(1/3)x + 3 :

$$Q3bar(x,y) := t\cdot\int_{6}^{12}\int_{0}^{3-\frac{x}{3}} \left(Nbar_i(x,y)^T\right)\cdot Body\,dy\,dx$$

$$Q3(x,y) := \left(A3^{-1}\right)^T\cdot Q3bar(x,y)$$

$$Q3(x,y) \rightarrow \begin{pmatrix} 0 \\ -0.375 \\ 0 \\ 0.75 \\ 0 \\ -0.375 \end{pmatrix} \quad lb$$

Where the diagonal of element 4 is defined by the line y = -(1/3)x + 3 :

$$Q4bar(x,y) := t\cdot\int_{6}^{12}\int_{0}^{3-\frac{x}{3}} \left(Nbar_i(x,y)^T\right)\cdot Body\,dy\,dx$$

$$Q4(x,y) := \left(A4^{-1}\right)^T \cdot Q4bar(x,y)$$

$$Q4(x,y) \rightarrow \begin{pmatrix} 0 \\ 0.375 \\ 0 \\ -0.75 \\ 0 \\ 0.375 \end{pmatrix} \quad \text{lb}$$

Where the diagonal of element 5 is defined by the line y = -(1/3)x + 5 :

$$Q5bar(x,y) := t \cdot \int_{12}^{18} \int_{0}^{5-\frac{x}{3}} \left(Nbar_i(x,y)^T\right) \cdot Body \, dy \, dx$$

$$Q5(x,y) := \left(A5^{-1}\right)^T \cdot Q5bar(x,y)$$

$$Q5(x,y) \rightarrow \begin{pmatrix} 0 \\ -0.375 \\ 0 \\ 0.75 \\ 0 \\ -0.375 \end{pmatrix} \quad \text{lb}$$

Where the diagonal of element 6 is defined by the line y = -(1/3)x + 5 :

$$Q6bar(x,y) := t \cdot \int_{12}^{18} \int_{0}^{5-\frac{x}{3}} \left(Nbar_i(x,y)^T\right) \cdot Body \, dy \, dx$$

$$Q6(x,y) := \left(A6^{-1}\right)^T \cdot Q6bar(x,y)$$

$$Q6(x,y) \rightarrow \begin{pmatrix} 0 \\ 0.375 \\ 0 \\ -0.75 \\ 0 \\ 0.375 \end{pmatrix} \quad \text{lb}$$

$$Qe_{odd} := \begin{pmatrix} 0 \\ -0.375 \\ 0 \\ 0.75 \\ 0 \\ -0.375 \end{pmatrix} \text{ lb} \qquad Qe_{even} := \begin{pmatrix} 0 \\ 0.375 \\ 0 \\ -0.75 \\ 0 \\ 0.375 \end{pmatrix} \text{ lb}$$

Surface Traction:

Only on element 6.

$$Surface := \begin{pmatrix} 0 \\ -2400 \end{pmatrix} \text{ lb/in}^2$$

$$P6(x, y) := t \cdot \int_{-1}^{1} N6(18, y)^T \cdot Surface \, dy$$

$$P6(x, y) \rightarrow \begin{pmatrix} 0 \\ -600.0 \\ 0 \\ 0 \\ 0 \\ -600.0 \end{pmatrix} \text{ lb} \qquad Pe_6 := \begin{pmatrix} 0 \\ -600.0 \\ 0 \\ 0 \\ 0 \\ -600.0 \end{pmatrix} \text{ lb}$$

Combine force vectors for each element:

$Fe_1 := Qe_{odd}$

$Fe_2 := Qe_{even}$

$Fe_3 := Qe_{odd}$ $\qquad Fe_5 := Qe_{odd}$

$Fe_4 := Qe_{even}$ $\qquad Fe_6 := Qe_{even} + Pe_6$

Assembly of Global Stiffness Matrix:

$n\_elements := 6$ $n\_nodes := 8$ $n\_free := 2$ $n\_dof := n\_nodes \cdot n\_free$ $nod\_el := 3$

$$elem := \begin{pmatrix} 2 & 4 & 1 \\ 3 & 1 & 4 \\ 4 & 6 & 3 \\ 5 & 3 & 6 \\ 6 & 8 & 5 \\ 7 & 5 & 8 \end{pmatrix}$$

$ie := 1 .. n\_elements$ $j := 1 .. nod\_el$ $k := 0 .. n\_free - 1$

$m := 1 .. nod\_el \cdot n\_free$ $n := 1 .. nod\_el \cdot n\_free$

$$top_{ie, n\_free \cdot j - k} := n\_free \cdot elem_{ie, j} - k$$

$$top = \begin{pmatrix} 3 & 4 & 7 & 8 & 1 & 2 \\ 5 & 6 & 1 & 2 & 7 & 8 \\ 7 & 8 & 11 & 12 & 5 & 6 \\ 9 & 10 & 5 & 6 & 11 & 12 \\ 11 & 12 & 15 & 16 & 9 & 10 \\ 13 & 14 & 9 & 10 & 15 & 16 \end{pmatrix}$$

$$K_{n\_dof, n\_dof} := 0$$

$$K_{\left(top_{ie,m}, top_{ie,n}\right)} := K_{\left(top_{ie,m}, top_{ie,n}\right)} + Stiff_{m,n}$$

$K = $ (matrix below) $\cdot 10^6$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	5.25	0	-4.05	-1.35	-1.2	-0.9	0	2.25	0	0	0	0	0	0	0	0
2	0	11.25	-0.9	-10.8	-1.35	-0.45	2.25	0	0	0	0	0	0	0	0	0
3	-4.05	-0.9	5.25	2.25	0	0	-1.2	-1.35	0	0	0	0	0	0	0	0
4	-1.35	-10.8	2.25	11.25	0	0	-0.9	-0.45	0	0	0	0	0	0	0	0
5	-1.2	-1.35	0	0	10.5	2.25	-8.1	-2.25	-1.2	-0.9	0	2.25	0	0	0	0
6	-0.9	-0.45	0	0	2.25	22.5	-2.25	-21.6	-1.35	-0.45	2.25	0	0	0	0	0
7	0	2.25	-1.2	-0.9	-8.1	-2.25	10.5	2.25	0	0	-1.2	-1.35	0	0	0	0
8	2.25	0	-1.35	-0.45	-2.25	-21.6	2.25	22.5	0	0	-0.9	-0.45	0	0	0	0
9	0	0	0	0	-1.2	-1.35	0	0	10.5	2.25	-8.1	-2.25	-1.2	-0.9	0	2.25
10	0	0	0	0	-0.9	-0.45	0	0	2.25	22.5	-2.25	-21.6	-1.35	-0.45	2.25	0
11	0	0	0	0	0	2.25	-1.2	-0.9	-8.1	-2.25	10.5	2.25	0	0	-1.2	-1.35
12	0	0	0	0	2.25	0	-1.35	-0.45	-2.25	-21.6	2.25	22.5	0	0	-0.9	-0.45
13	0	0	0	0	0	0	0	0	-1.2	-1.35	0	0	5.25	2.25	-4.05	-0.9
14	0	0	0	0	0	0	0	0	-0.9	-0.45	0	0	2.25	11.25	-1.35	-10.8
15	0	0	0	0	0	0	0	0	0	2.25	-1.2	-0.9	-4.05	-1.35	5.25	0
16	0	0	0	0	0	0	0	0	2.25	0	-1.35	-0.45	-0.9	-10.8	0	11.25

Unit =lb/in

Assembly of Global Force Matrix:

$F_{n\_dof,1} := 0$

$F_{(top_{ie,m},1)} := F_{(top_{ie,m},1)} + (Fe_{ie})_{m,1}$

F =

	1
1	0
2	-1.13
3	0
4	-0.38
5	0
6	-0.75
7	0
8	0.75
9	0
10	-0.75
11	0
12	0.75
13	0
14	-599.63
15	0
16	-598.88

lb

Apply Boundary Conditions:

$F_{red} := \text{submatrix}(F, 5, 16, 1, 1)$

F_{red} =

	1
1	0
2	-0.75
3	0
4	0.75
5	0
6	-0.75
7	0
8	0.75
9	0
10	-599.63
11	0
12	-598.88

lb

$K_{red} := \text{submatrix}(K, 5, 16, 5, 16)$

$K_{red} =$ (×10^6 lb/in)

	1	2	3	4	5	6	7	8	9	10	11	12
1	10.5	2.25	-8.1	-2.25	-1.2	-0.9	0	2.25	0	0	0	0
2	2.25	22.5	-2.25	-21.6	-1.35	-0.45	2.25	0	0	0	0	0
3	-8.1	-2.25	10.5	2.25	0	0	-1.2	-1.35	0	0	0	0
4	-2.25	-21.6	2.25	22.5	0	0	-0.9	-0.45	0	0	0	0
5	-1.2	-1.35	0	0	10.5	2.25	-8.1	-2.25	-1.2	-0.9	0	2.25
6	-0.9	-0.45	0	0	2.25	22.5	-2.25	-21.6	-1.35	-0.45	2.25	0
7	0	2.25	-1.2	-0.9	-8.1	-2.25	10.5	2.25	0	0	-1.2	-1.35
8	2.25	0	-1.35	-0.45	-2.25	-21.6	2.25	22.5	0	0	-0.9	-0.45
9	0	0	0	0	-1.2	-1.35	0	0	5.25	2.25	-4.05	-0.9
10	0	0	0	0	-0.9	-0.45	0	0	2.25	11.25	-1.35	-10.8
11	0	0	0	0	0	2.25	-1.2	-0.9	-4.05	-1.35	5.25	0
12	0	0	0	0	2.25	0	-1.35	-0.45	-0.9	-10.8	0	11.25

$D := K_{red}^{-1} \cdot F_{red}$

$D =$ (in)

	1
1	$1.76 \cdot 10^{-3}$
2	$-6.55 \cdot 10^{-3}$
3	$-1.7 \cdot 10^{-3}$
4	$-6.48 \cdot 10^{-3}$
5	$2.84 \cdot 10^{-3}$
6	-0.02
7	$-2.67 \cdot 10^{-3}$
8	-0.02
9	$3.24 \cdot 10^{-3}$
10	-0.04
11	$-2.96 \cdot 10^{-3}$
12	-0.04

Displacement at free end = -0.040 inches.