Example: Inverse Power Method to Compute the Dominant Eigenvalue and Eigenvector

Define matrices A, B

$$A := \begin{pmatrix} 7 & -4 & 1 \\ -4 & 6 & -4 \\ 1 & -4 & 5 \end{pmatrix}$$

$$B := \begin{pmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix} \qquad E := (-1)\cdot A^{-1}\cdot B \qquad E = \begin{pmatrix} 0.2727 & 0.1818 & 0.0909 \\ -0.0455 & 0.6364 & 0.3182 \\ -0.0909 & 0.2727 & 0.6364 \end{pmatrix}$$

Initial Guess of Eigenvector

$$vo := \begin{pmatrix} 0.25 \\ 0.7 \\ 0.75 \end{pmatrix} \qquad v := vo$$

Iteration #1

$$VV := E\cdot v = \begin{pmatrix} 0.2636 \\ 0.6727 \\ 0.6455 \end{pmatrix} \qquad \max(VV) = 0.6727 \qquad VN := \frac{VV}{\max(VV)} = \begin{pmatrix} 0.3919 \\ 1 \\ 0.9595 \end{pmatrix}$$

$$L := \frac{1.0}{\max(VV)} = 1.486$$

Iteration #2

$v := VN$

$$VV := E\cdot v = \begin{pmatrix} 0.3759 \\ 0.9238 \\ 0.8477 \end{pmatrix} \qquad \max(VV) = 0.9238 \qquad VN := \frac{VV}{\max(VV)} = \begin{pmatrix} 0.4069 \\ 1 \\ 0.9176 \end{pmatrix}$$

$$LL := \frac{1.0}{\max(VV)} = 1.0824 \qquad LL - L = -0.404$$

Iteration #3

$L := LL$
$v := VN$

$$VV := E\cdot v = \begin{pmatrix} 0.3762 \\ 0.9098 \\ 0.8196 \end{pmatrix} \qquad \max(VV) = 0.9098 \qquad VN := \frac{VV}{\max(VV)} = \begin{pmatrix} 0.4135 \\ 1 \\ 0.9009 \end{pmatrix}$$

$$LL := \frac{1.0}{\max(VV)} = 1.0991 \qquad LL - L = 0.0167$$

Iteration #4

$L := LL$
$v := VN$

$$VV := E \cdot v = \begin{pmatrix} 0.3765 \\ 0.9042 \\ 0.8084 \end{pmatrix} \qquad \max(VV) = 0.9042 \qquad VN := \frac{VV}{\max(VV)} = \begin{pmatrix} 0.4164 \\ 1 \\ 0.8941 \end{pmatrix}$$

$$LL := \frac{1.0}{\max(VV)} = 1.1059 \qquad LL - L = 6.8137 \times 10^{-3}$$

Iteration #5

$L := LL$
$v := VN$

$$VV := E \cdot v = \begin{pmatrix} 0.3767 \\ 0.9019 \\ 0.8038 \end{pmatrix} \qquad \max(VV) = 0.9019 \qquad VN := \frac{VV}{\max(VV)} = \begin{pmatrix} 0.4176 \\ 1 \\ 0.8912 \end{pmatrix}$$

$$LL := \frac{1.0}{\max(VV)} = 1.1088 \qquad LL - L = 2.8185 \times 10^{-3}$$

Iteration #6

$L := LL$
$v := VN$

$$VV := E \cdot v = \begin{pmatrix} 0.3767 \\ 0.901 \\ 0.8019 \end{pmatrix} \qquad \max(VV) = 0.901 \qquad VN := \frac{VV}{\max(VV)} = \begin{pmatrix} 0.4181 \\ 1 \\ 0.8901 \end{pmatrix}$$

$$LL := \frac{1.0}{\max(VV)} = 1.1099 \qquad LL - L = 1.1732 \times 10^{-3}$$

Iteration #7

$L := LL$
$v := VN$

$$VV := E \cdot v = \begin{pmatrix} 0.3768 \\ 0.9006 \\ 0.8011 \end{pmatrix} \qquad \max(VV) = 0.9006 \qquad VN := \frac{VV}{\max(VV)} = \begin{pmatrix} 0.4184 \\ 1 \\ 0.8896 \end{pmatrix}$$

$$LL := \frac{1.0}{\max(VV)} = 1.1104 \qquad LL - L = 4.8997 \times 10^{-4}$$

Convergence

Eigenvalue, $LL = 1.1104$ Eigenvector, $VN = \begin{pmatrix} 0.4184 \\ 1 \\ 0.8896 \end{pmatrix}$

$$P := 1.1104 \cdot \left(\frac{16 \cdot E \cdot I}{L^2} \right) = 17.766 \cdot \frac{E \cdot I}{L^2}$$

Physical Meaning