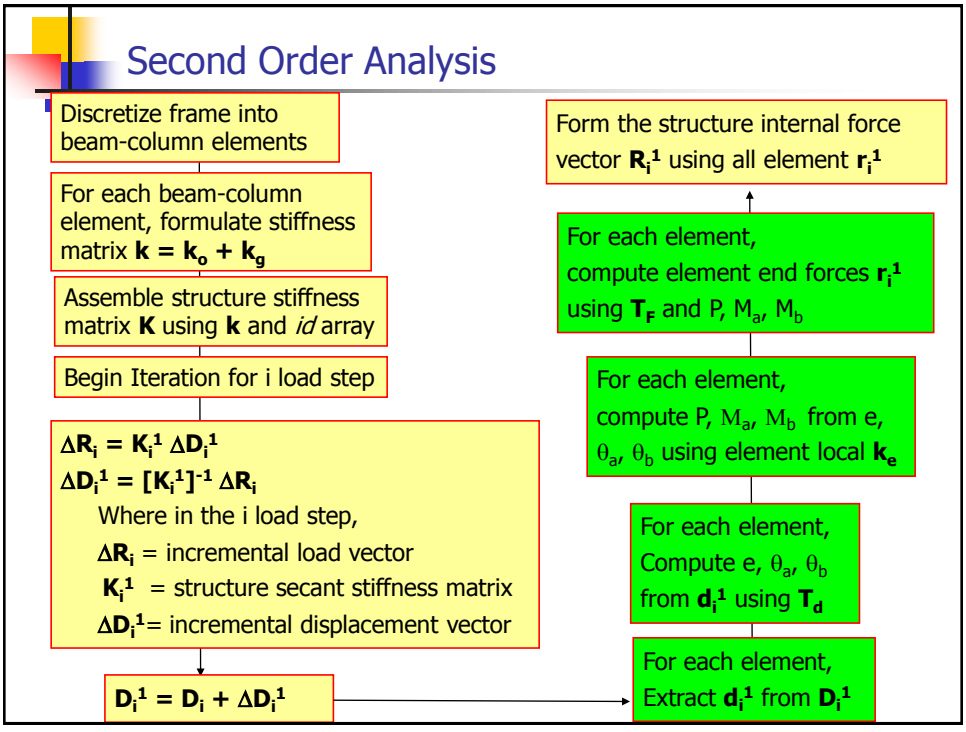
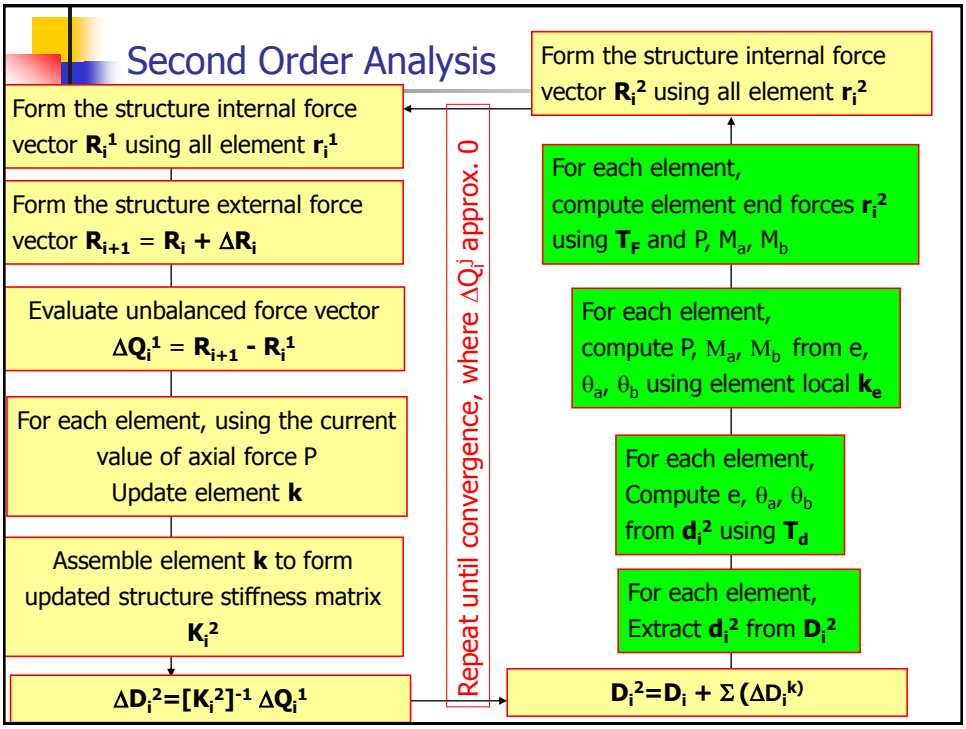
Second Order Analysis

- In the previous classes we looked at a method that determines the load corresponding to a state of bifurcation equilibrium of a perfect frame by eigenvalye analysis
- The system was assumed to be perfect, and there were no lateral deflection until the load reached P_{cr}.
- At P_{cr}, the original configuration of the frame becomes unstable, and with a slight perturbation the deflections start increasing without bound.
- However, if the system is not perfect, or it is subjected to lateral loads along with gravity loads, then the deflections will start increasing as soon as the loads are applied.
- However, for an elastic frame, the maximum load capacity will still be limited to that corresponding to P_{cr}

Second Order Analysis

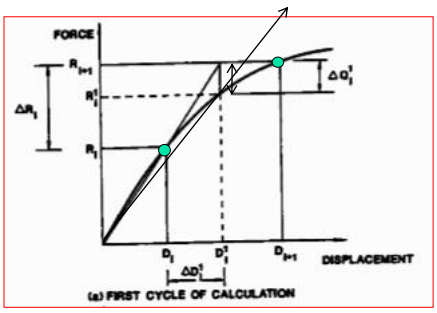
- To trace this curve, a complete load-deflection analysis of the frame is necessary. A second order analysis will generate this load-deflection curve
- In a second order analysis procedure, secondary effects as the P-δ and P-Δ effects, can be incorporated directly. As a result, the need for the B1 and B2 factors is eliminated.
- In a second-order analysis, the equilibrium equations are formulated with respect to the deformed geometry, which is not known in advance and constantly changing, we need to use an iterative technique to obtain solutions.
- In numerical implementation, the incremental load approach is popular. The load is divided into small increments and applied to the structure sequentially.

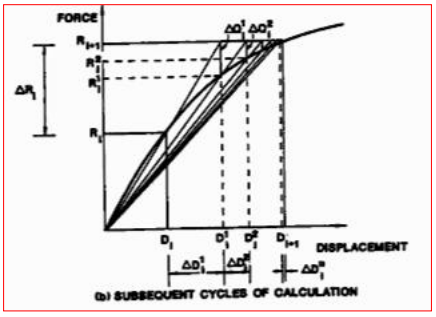

Second Order Analysis
Discretize frame into beam-column elements
For each beam-column element, formulate stiffness matrix $\mathbf{k} = \mathbf{k_o} + \mathbf{k_g}$
Assemble structure stiffness matrix **K** using **k** and *id* array
Begin Iteration for i load step
$\Delta \mathbf{R_i} = \mathbf{K_i^1}\ \Delta \mathbf{D_i^1}$
$\Delta \mathbf{D_i^1} = [\mathbf{K_i^1}]^{-1}\ \Delta \mathbf{R_i}$
Where in the i load step,
$\Delta \mathbf{R_i}$ = incremental load vector
$\mathbf{K_i^1}$ = structure secant stiffness matrix
$\Delta \mathbf{D_i^1}$= incremental displacement vector
$\mathbf{D_i^1} = \mathbf{D_i} + \Delta \mathbf{D_i^1}$
For each element, Extract $\mathbf{d_i^1}$ from $\mathbf{D_i^1}$
For each element, Compute e, θ_a, θ_b from $\mathbf{d_i^1}$ using $\mathbf{T_d}$
For each element, compute P, M_a, M_b from e, θ_a, θ_b using element local $\mathbf{k_e}$
For each element, compute element end forces $\mathbf{r_i^1}$ using $\mathbf{T_F}$ and P, M_a, M_b
Form the structure internal force vector $\mathbf{R_i^1}$ using all element $\mathbf{r_i^1}$


Second Order Analysis
Form the structure internal force vector $\mathbf{R_i^1}$ using all element $\mathbf{r_i^1}$
Form the structure external force vector $\mathbf{R_{i+1}} = \mathbf{R_i} + \Delta \mathbf{R_i}$
Evaluate unbalanced force vector $\Delta \mathbf{Q_i^1} = \mathbf{R_{i+1}} - \mathbf{R_i^1}$
For each element, using the current value of axial force P Update element **k**
Assemble element **k** to form updated structure stiffness matrix $\mathbf{K_i^2}$
$\Delta \mathbf{D_i^2} = [\mathbf{K_i^2}]^{-1}\ \Delta \mathbf{Q_i^1}$
Repeat until convergence, where ΔQ_i^j approx. 0
$\mathbf{D_i^2} = \mathbf{D_i} + \Sigma\ (\Delta \mathbf{D_i^k})$
For each element, Extract $\mathbf{d_i^2}$ from $\mathbf{D_i^2}$
For each element, Compute e, θ_a, θ_b from $\mathbf{d_i^2}$ using $\mathbf{T_d}$
For each element, compute P, M_a, M_b from e, θ_a, θ_b using element local $\mathbf{k_e}$
For each element, compute element end forces $\mathbf{r_i^2}$ using $\mathbf{T_F}$ and P, M_a, M_b
Form the structure internal force vector $\mathbf{R_i^2}$ using all element $\mathbf{r_i^2}$

Second Order Analysis

- After convergence, $\mathbf{D_{i+1} = D_i^n = D_i + \Sigma\, \Delta D_i^k}$
- Assumed another load increment, and go back to begining





AISC (2010) Specifications

Chapter C – Design for Stability

C1. General Stability Requirements

- 1. General Requirements
- Stability shall be provided for the structure as a whole and for each of its elements.
- The effects of all of the following on the stability of the structure and its elements shall be considered:
  - (1) Flexural, shear, and axial member deformations, and all other deformations that contribute to the displacements of the structure.
  - (2) second order effects (both P-⊗ and P-δ effects).
  - (3) geometric imperfections
  - (4) stiffness reductions due to inelasticity
  - (5) uncertainty in stiffness and strength
- All load-dependent effects shall be calculated at a level of loading corresponding to the LRFD load combinations

C1. General Stability Requirements

- Any rational methods of design for stability that consider all of the listed effects is permitted; this includes the methods identified in Sections C1.1 and C1.2
- C1.1 – Direct Analysis Method of Design
  - Consist of calculation of the *required strengths* in accordance with Section C2, and calculation of *available strengths* in accordance with C3 is permitted for all structures
- C1.2 Alternative Methods of Design
  - The *effective length* method and the *first-order analysis* in Appendix 7 are permitted as alternatives to the *direct analysis method* for structures that satisfy the constraints specified in that appendix.

C2. Calculation of Required Strengths

- For the *direct analysis method*, the *required strengths* of the components of the structure shall be determined from
  - Analysis conforming to Section C2.1.
  - Initial imperfections according to Section C2.2 and
  - stiffness according to Section C2.3.

C2.1 General Analysis Requirements

- (1) Account for member deformations (flexural, axial, shear) and all other component and connection deformations that contribute. Account for stiffness and stiffness reductions (according to C2.3) that contribute
- (2) Second-order analysis that considers both the P-Δ and P-δeffects.
  Use of approximate second-order analysis provided in App. 8 is permitted as an alternative to a rigorous second-order analysis
- (3) The analysis shall consider all gravity and other loads
- (4) For design by LRFD, the second order analysis shall be carried out under LRFD load combinations

C2.1 General Analysis Requirements

- (2) cont' … Second-order analysis requirements
  - A P-Δ only second-order analysis (one that neglects the effects of P-δresponse on the structure) is permitted when:
    - (a) structure supports gravity load using vertical members
    - (b) ratio of maximum second-order drift to maximum first-order drift is ≤ 1.7
    - (c) no more than 1/3 of the gravity load is supported by columns part of the MRFs in the direction of translation.
  - However, it is necessary in all cases to consider the P-δ effects in the evaluation of individual members subject to flexure and compression. This can be satisfied by using the B1 factor in App. 8.

C2 Consideration of Initial Imperfections

- The effects of initial imperfections on the stability of the structure shall be taken into account by :
  - (a) direct modeling of imperfections in the analysis as specified in C2.2a
  - (b) application of notional loads as specified in C2.2b
- These are imperfections in the location of points of intersection of members. Out-of-plumbness of columns.
  - Initial out-of-straightness of individual members is not addressed in this section. It is covered in the design of columns (compression members).

C2.2a Direct Modeling of Imperfections

- In all cases, it is permissible to account for the effect of initial imperfections by including them directly in the analysis.
- The structure shall be analyzed with points of intersection of members displaced from their nominal location.
- The magnitude of the initial displacement shall be the maximum amount considered in the design.
- The pattern of initial displacements shall be such that it provokes the destabilizing effect
- **User Note:** Initial displacements similar in configuration to both displacements due to loading and anticipated buckling modes should be considered in the modeling of imperfections. The magnitude of the initial displacements should be based on permissible construction tolerances (AISC CoSP) , or on actual imperfections if known

C2.2a Direct Modeling of Imperfections

- If the structure support gravity loads using vertical columns, and the ratio of maximum second-order drift to first-order drift is ≤ 1.7, then it is permissible to include imperfections only in analysis of gravity-only load combinations and not in analysis for combinations that include applied lateral loads.

C2.2b Use of Notional Loads

- For structures that support gravity loads using vertical columns, it is permissible to use notional loads to represent the effects of initial imperfections in accordance with this section. Nominal geometry used.
- (1) Notional loads shall be applied as lateral loads at all levels. These loads shall be additive to the other loads applied in all load combinations. The magnitude of the notional loads shall be:

$$N_i = 0.002\, a\, Y_i$$

  - N_i = notional load applied at level i (kips)
  - Y_i = gravity load applied at level i from load combos
- (2) The notional load Ni shall be distributed over that level in the same manner as the gravity load at that level, and applied in the direction providing greatest destabilization

C2.2b Use of Notional Loads

- (3) The notional load coefficient of 0.002 is based on the nominal initial out-of-story plumbness ratio of 1/500. If other value is justified, it is permitted to adjust the notional load coefficient proportionally.
- (4) If the ratio of the maximum second-order drift to maximum first-order drift is less than or equal to 1.7 in all stories, it is permitted to apply the notional lateral load in gravity only load combinations and not in combinations using other lateral loads

C2.3 Adjustments to Stiffness

- The analysis of structures to determine required strengths shall used reduced stiffness as follows:
  - A factor of 0.80 shall be applied to all stiffness that contributed to the structure stability. This factor can be applied to all stiffness in the structure.
  - An additional factor, τ_b shall be applied to the flexural stiffness of all members whose flexural stiffness are contributing to the stability

$$When\ \alpha P_r / P_y \le 0.5, \quad t_b = 1.0$$

$$When\ \alpha P_r / P_y > 0.5, \quad t_b = \frac{4(\alpha P_r / P_Y)}{1 - \alpha P_r / P_y}$$

- (3) Instead of using $\tau_b < 1.0$, it is permissible to use $\tau_b = 1.0$ and an additional notional load of $0.001\alpha Y_i$, which are to be added in the direction that causes more destabilization and also for gravity load combos.