

CHAPTER 4. COMPRESSION MEMBER DESIGN

4.1 INTRODUCTORY CONCEPTS

- Compression Members: Structural elements that are subjected to axial compressive forces only are called *columns*. Columns are subjected to axial loads through the centroid.
- Stress: The stress in the column cross-section can be calculated as

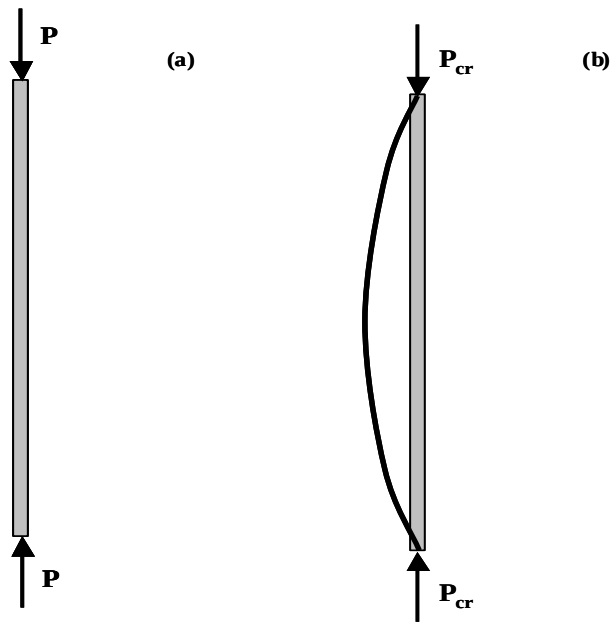
$$f = \frac{P}{A} \quad (2.1)$$

where f is assumed to be uniform over the entire cross-section.

- This ideal state is never reached. The stress-state will be non-uniform due to:
 - Accidental eccentricity of loading with respect to the centroid
 - Member out-of-straightness (crookedness), or
 - Residual stresses in the member cross-section due to fabrication processes.
- Accidental eccentricity and member out-of-straightness can cause bending moments in the member. However, these are secondary and are usually ignored.
- Bending moments cannot be neglected if they are acting on the member. Members with axial compression and bending moment are called *beam-columns*.

4.2 COLUMN BUCKLING

- Consider a long slender compression member. If an axial load P is applied and increased slowly, it will ultimately reach a value P_{cr} that will cause buckling of the column (Figure 1). P_{cr} is called the critical buckling load of the column.



What is buckling?

Buckling occurs when a straight column subjected to axial compression suddenly undergoes bending as shown in the Figure 1(b). Buckling is identified as a failure limit-state for columns.

Figure 1. Buckling of axially loaded

compression members

- The critical buckling load P_{cr} for columns is theoretically given by Equation (4.1)

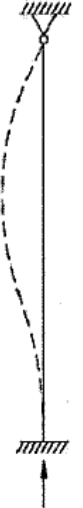
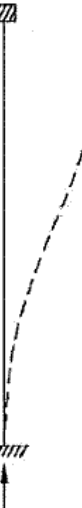
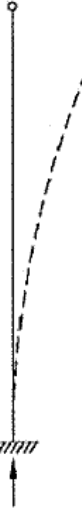
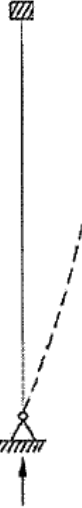
$$P_{cr} = \frac{\pi^2 E I}{(K L)^2} \quad (4.1)$$

where, I = moment of inertia about axis of buckling

K = effective length factor based on end boundary conditions

- Effective length factors are given on page 16.1-511 (Table C-A-7.1) of the AISC manual.

TABLE C-A-7.1
Approximate Values of Effective
Length Factor, K

Buckled shape of column is shown by dashed line	(a) 	(b) 	(c) 	(d) 	(e) 	(f) 
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0
End condition code	    Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free					

- In examples, homeworks, and exams please state clearly whether you are using the theoretical value of K or the recommended design values.

EXAMPLE 4.1 Determine the buckling strength of a W 12 x 50 column. Its length is 20 ft. For major axis buckling, it is pinned at both ends. For minor buckling, is it pinned at one end and fixed at the other end.

Solution

Step I. Visualize the problem

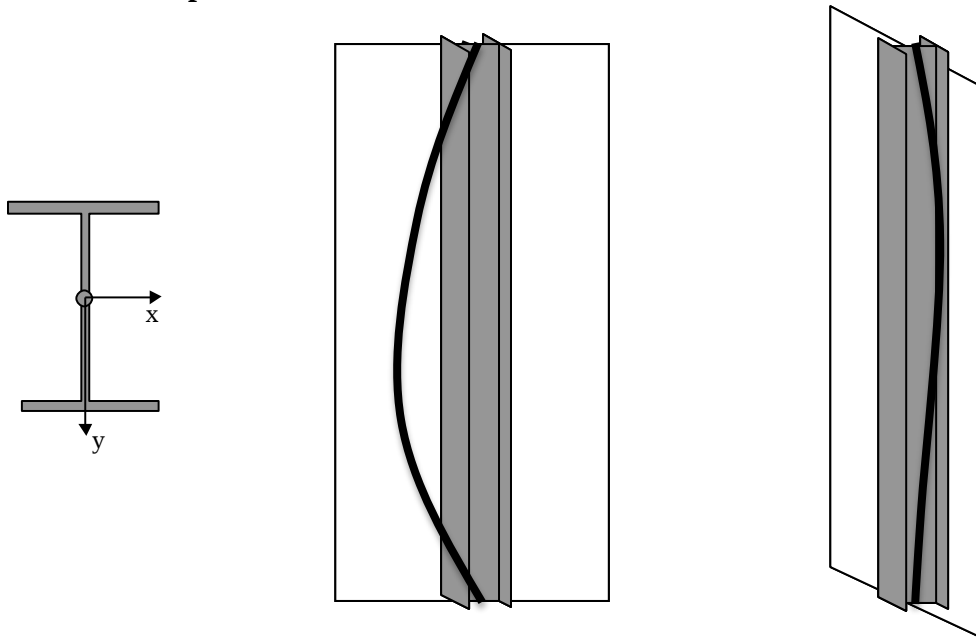


Figure 2. (a) Cross-section; (b) major-axis buckling; (c) minor-axis buckling

- For the W12 x 50 (or any wide flange section), x is the major axis and y is the minor axis.

Major axis means axis about which it has greater moment of inertia ($I_x > I_y$)

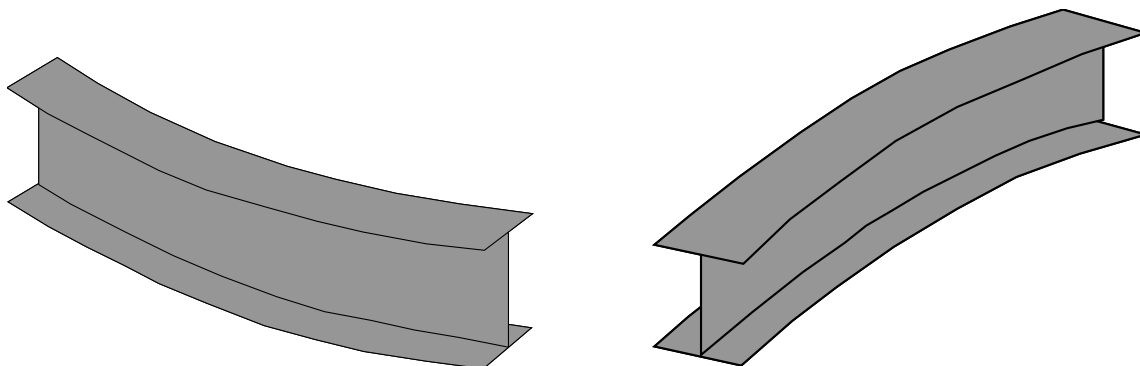


Figure 3. (a) Major axis buckling; (b) minor axis buckling

Step II. Determine the effective lengths

- According to Table C-A-7.1 of the AISC Manual (see page 16.1 - 511):
 - For pin-pin end conditions about the major axis
 $K_x = 1.0$ (theoretical value); and $K_x = 1.0$ (recommended design value)
 - For pin-fix end conditions about the minor axis
 $K_y = 0.7$ (theoretical value); and $K_y = 0.8$ (recommended design value)
- According to the problem statement, the unsupported length for buckling about the major (x) axis = $L_x = 20$ ft.
- The unsupported length for buckling about the minor (y) axis = $L_y = 20$ ft.
- Effective length for major (x) axis buckling = $K_x L_x = 1.0 \times 20 = 20$ ft. = 240 in.
- Effective length for minor (y) axis buckling = $K_y L_y = 0.8 \times 20 = 16$ ft. = 192 in.

Step III. Determine the relevant section properties

- For W12 x 50: elastic modulus = $E = 29000$ ksi (constant for all steels)
- For W12 x 50: $I_x = 391$ in⁴. $I_y = 56.3$ in⁴ (see pages 1-26 and 1-27 of the AISC manual)

Step IV. Calculate the buckling strength

- Critical load for buckling about x - axis = $P_{cr-x} = \frac{\pi^2 E I_x}{(K_x L_x)^2} = \frac{\pi^2 \times 29000 \times 391}{(240)^2}$

$$P_{cr-x} = 1942.9 \text{ kips}$$

- Critical load for buckling about y-axis = $P_{cr-y} = \frac{\pi^2 E I_y}{(K_y L_y)^2} = \frac{\pi^2 \times 29000 \times 56.3}{(192)^2}$

$$P_{cr-y} = 437.12 \text{ kips}$$

- Buckling strength of the column = smaller (P_{cr-x}, P_{cr-y}) = $P_{cr} = 437.12$ kips

Minor (y) axis buckling governs.

- **Notes:**

- *Minor axis buckling usually governs for all doubly symmetric cross-sections. However, for some cases, major (x) axis buckling can govern.*
- *Note that the steel yield stress was irrelevant for calculating this buckling strength.*

4.3 INELASTIC COLUMN BUCKLING

- Let us consider the previous example. According to our calculations $P_{cr} = 437$ kips. This P_{cr} will cause a uniform stress $f = P_{cr}/A$ in the cross-section
- For W12 x 50, $A = 14.6 \text{ in}^2$. Therefore, for $P_{cr} = 437$ kips; $f = 30$ ksi
The calculated value of f is within the elastic range for a 50 ksi yield stress material.
- However, if the unsupported length was only 10 ft., $P_{cr} = \frac{\pi^2 E I_y}{(K_y L_y)^2}$ would be calculated as 1748 kips, and $f = 119.73$ ksi.
- This value of f is ridiculous because the material will yield at 50 ksi and never develop $f = 119.73$ ksi. The member would yield before buckling.
- **Equation (4.1) is valid only when the material everywhere in the cross-section is in the elastic region. If the material goes inelastic then Equation (4.1) becomes useless and cannot be used.**
- What happens in the inelastic range?
Several other problems appear in the inelastic range.
 - The member out-of-straightness has a significant influence on the buckling strength in the inelastic region. It must be accounted for.

- The residual stresses in the member due to the fabrication process causes yielding in the cross-section much before the uniform stress f reaches the yield stress F_y .
- The shape of the cross-section (W, C, etc.) also influences the buckling strength.
- In the inelastic range, the steel material can undergo strain hardening.

All of these are very advanced concepts and beyond the scope of CE470. You are welcome to CE579 to develop a better understanding of these issues.

- So, what should we do? We will directly look at the AISC Specifications for the strength of compression members, i.e., Chapter E (page 16.1-31 of the AISC manual).

4.4 AISC SPECIFICATIONS FOR COLUMN STRENGTH

- The AISC specifications for column design are based on several years of research.
- These specifications account for the elastic and inelastic buckling of columns including all issues (member crookedness, residual stresses, accidental eccentricity etc.) mentioned above.
- The specification presented here (AISC Spec E3) will work for all doubly symmetric cross-sections and channel sections.
- The design strength of columns for the flexural buckling limit state is equal to $\phi_c P_n$

Where, $\phi_c = 0.9$ (Resistance factor for compression members)

$$P_n = A_g F_{cr} \quad (4.2)$$

- When $\frac{KL}{r} \leq 4.71 \sqrt{\frac{E}{F_y}}$ (or $\frac{F_y}{F_e} \leq 2.25$)

$$\blacksquare F_{cr} = \left(0.658^{\frac{F_y}{F_e}} \right) F_y \quad (4.3)$$

- When $\frac{KL}{r} > 4.71 \sqrt{\frac{E}{F_y}}$ (or $\frac{F_y}{F_e} > 2.25$)

$$F_{cr} = [0.877 F_e] \quad (4.4)$$

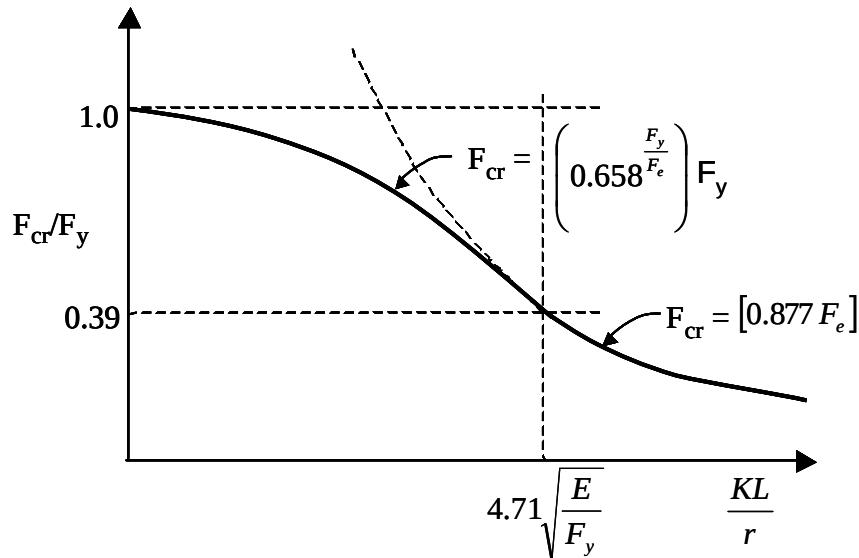
Where, $F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$ (4.5)

A_g = gross member area;

K = effective length factor

L = unbraced length of the member;

r = corresponding radius of gyration



- Note that the original Euler buckling equation is $P_{cr} = \frac{\pi^2 EI}{(KL)^2}$

$$\therefore F_e = \frac{P_{cr}}{A_g} = \frac{\pi^2 E}{(KL)^2} \times \frac{I}{A_g} = \frac{\pi^2 E}{(KL)^2} \times r^2 = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

- Note that the AISC equation for $\frac{KL}{r} > 4.71 \sqrt{\frac{E}{F_y}}$ is $F_{cr} = 0.877 F_e$
 - The 0.877 factor tries to account for initial crookedness.
- For a given column section:
 - Calculate I , A_g , r

- Determine effective length $K L$ based on end boundary conditions.
- Calculate F_e , F_y/F_e or $4.71 \sqrt{\frac{E}{F_y}}$
- If (KL/r) greater than $4.71 \sqrt{\frac{E}{F_y}}$, *elastic buckling* occurs and use Equation (4.4)
- If (KL/r) is less than or equal to $4.71 \sqrt{\frac{E}{F_y}}$, *inelastic buckling* occurs and use Equation (4.3)
- Note that the column can develop its yield strength F_y as (KL/r) approaches zero.

4.5 COLUMN STRENGTH

- In order to simplify calculations, the AISC specification includes Tables.
 - Table 4-22 on pages **4-322 to 4-326** shows KL/r vs. $\phi_c F_{cr}$ for various steels.
 - You can calculate KL/r for the column, then read the value of $\phi_c F_{cr}$ from this table
 - The column strength will be equal to $\phi_c F_{cr} \times A_g$

EXAMPLE 4.2 Calculate the design strength of W14 x 74 with length of 20 ft. and pinned ends.

A36 steel is used.

Solution

- Step I. Calculate the effective length and slenderness ratio for the problem

$$K_x = K_y = 1.0$$

$$L_x = L_y = 240 \text{ in.}$$

$$\text{Major axis slenderness ratio} = K_x L_x / r_x = 240 / 6.04 = 39.735$$

$$\text{Minor axis slenderness ratio} = K_y L_y / r_y = 240 / 2.48 = 96.77$$

- Step II. Calculate the elastic critical buckling stress

The governing slenderness ratio is the larger of ($K_x L_x / r_x$, $K_y L_y / r_y$)

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 * 29000}{(96.77)^2} = 30.56 \text{ ksi}$$

Check the limits

$$\left(\frac{KL}{r} > 4.71 \sqrt{\frac{E}{F_y}}\right) \text{ or } \left(\frac{F_y}{F_e} > 2.25\right)$$

$$4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29000}{36}} = 133.68$$

$$\text{Since } \frac{KL}{r} \leq 4.71 \sqrt{\frac{E}{F_y}}; \quad \text{Therefore, } F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right) F_y$$

Therefore, $F_{cr} = 21.99 \text{ ksi}$

$$\text{Design column strength} = \phi_c P_n = 0.9 (A_g F_{cr}) = 0.9 (21.8 \text{ in}^2 \times 21.99 \text{ ksi}) = 431.4 \text{ kips}$$

Design strength of column = 431 kips

- Check calculated values with Table 4-22. For $KL/r = 97$, $\phi_c F_{cr} = 19.7 \text{ ksi}$

4.6 LOCAL BUCKLING LIMIT STATE

- The AISC specifications for column strength assume that column buckling is the governing limit state. However, if the column section is made of thin (slender) plate elements, then failure can occur due to *local buckling* of the flanges or the web.

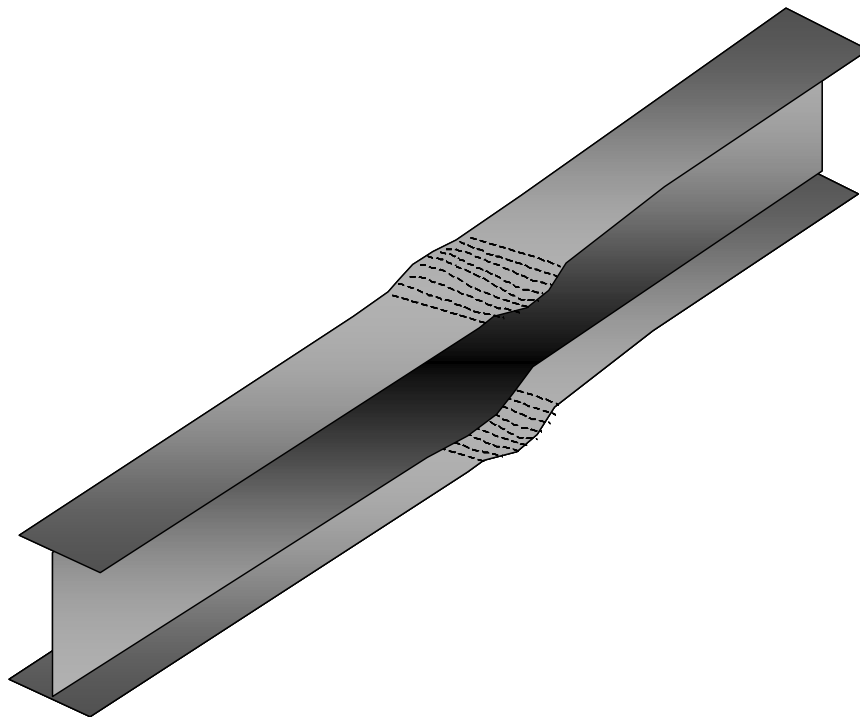


Figure 4. Local buckling of columns

- If *local buckling* of the individual plate elements occurs, then the column may not be able to develop its buckling strength.
- Therefore, the local buckling limit state must be prevented from controlling the column strength.
- Local buckling depends on the slenderness (width-to-thickness b/t ratio) of the plate element and the yield stress (F_y) of the material.
- Each plate element must be stocky enough, i.e., have a b/t ratio that prevents local buckling from governing the column strength.

- The AISC specification B4.1 (Page 16.1-14) provides the slenderness (b/t) limit that the individual plate elements must satisfy so that *local buckling* does not control.
- For compression, the AISC specification provides slenderness limit (λ_r) for the local buckling of plate elements.

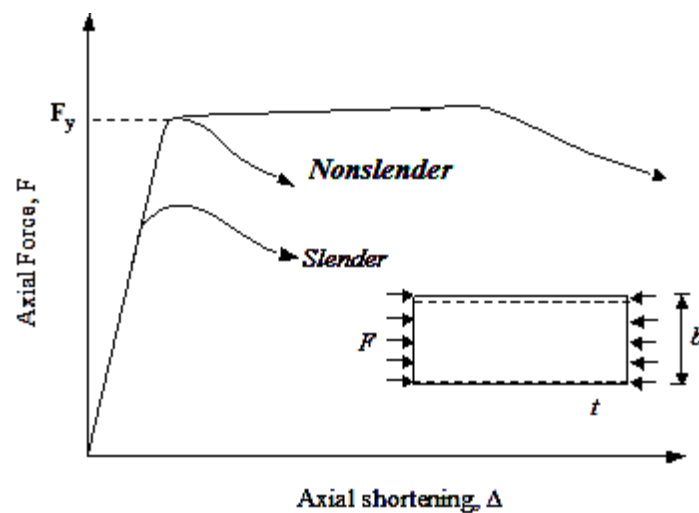


Figure 5. Local buckling behavior and classification of plate elements

- If the slenderness ratio (b/t) of the plate element is greater than λ_r then it is *slender*. It will locally buckle in the elastic range *before* reaching F_y
- If the slenderness ratio (b/t) of the plate element is less than λ_r , then it is *non-slender*. It will not locally buckle *in elastic range before* reaching F_y
- If any one plate element is slender, then the cross-section is slender.
- The slenderness limit λ_r for various plate elements with different boundary conditions are given in **Table B4.1a** on page **16.1-16** of the AISC Spec.
- Note that the slenderness limit (λ_r) and the definition of plate slenderness (b/t) ratio depend upon the boundary conditions for the plate.
 - If the plate is supported along *two edges* parallel to the direction of compression force, then it is a *stiffened* element. For example, the webs of W shapes

- If the plate is supported along only *one edge* parallel to the direction of the compression force, then it is an *unstiffened* element, e.g., the flanges of W shapes.
- The local buckling limit state can **be prevented from controlling** the column strength by using sections that **are nonslender**.
 - If all the elements of the cross-section have calculated slenderness (b/t) ratio less than λ_r , then the local buckling limit state will not control.
 - For the **definitions of b/t and λ_r for various situations see Table B4.1a and Spec B4.1.**

EXAMPLE 4.3 Determine the local buckling slenderness limit and evaluate the W14 x 74 section used in Example 4.2. Does local buckling limit the column strength?

Solution

- Step I. Calculate the slenderness limits

See Table B4.1a on page 16.1-16.

- For the flanges of I-shape sections

$$\lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 0.56 \times \sqrt{\frac{29000}{36}} = 15.9$$

- For the webs of I-shapes section

$$\lambda_r = 1.49 \times \sqrt{\frac{E}{F_y}} = 1.49 \times \sqrt{\frac{29000}{36}} = 42.3$$

- Step II. Calculate the slenderness ratios for the flanges and webs of W14 x 74

- For the flanges of I-shape member, $b = b_f/2 = \text{flange width} / 2$

Therefore, $b/t = b_f/2t_f$.

For W 14 x 74, $b_f/2t_f = 6.43$

(See Page 1-24 in AISC)

- For the webs of I shaped member, $b = h$

h is the clear distance between flanges less the fillet / corner radius of each flange

For W14 x 74, $h/t_w = 24.17$

(See Page 1-24 in AISC)

- Step III. Make the comparisons and comment

For the flanges, $b/t < \lambda_r$. Therefore, the flange is nonslender

For the webs, $h/t_w < \lambda_r$. Therefore the web is nonslender

Therefore, **the section is nonslender.**

Therefore, local buckling will not limit the column strength.

4.7 COLUMN DESIGN

- The AISC manual has tables for column strength. See page 4-12 onwards.
- For wide flange sections, *the column buckling strength ($\phi_c P_n$) is tabulated with respect to the effective length about the minor axis $K_y L_y$ in Table 4-1.*
 - The table takes the $K_y L_y$ value for a section, internally calculates the $K_y L_y / r_y$, and then calculates the *tabulated* column strength using either Equation E3-2 or E3-3 of the specification.
- If you want to use the Table 4-1 for calculating the column **strength for buckling about the major axis**, then do the following:
 - Take the major axis $K_x L_x$ value. Calculate an equivalent **$(KL)_{eq} = \frac{K_x L_x}{r_x / r_y}$**
 - Use the calculated $(KL)_{eq}$ value to find $(\phi_c P_n)$ the column strength for buckling about the *major axis* from Table (4-1)
- For example, consider a W14 x 74 column with $K_y L_y = 20$ ft. and $K_x L_x = 25$ ft.
 - Material has yield stress = 50 ksi (always in Table 4-1).

- See Table 4-1, for $K_y L_y = 20$ ft., $\phi P_n = 495$ kips (minor axis buckling strength)
- r_x/r_y for W14x74 = 2.44 from Table 4-1 (see page 4-16 of AISC).
- For $K_x L_x = 25$ ft., $(KL)_{eq} = 25/2.44 = 10.25$ ft.
- For $(KL)_{eq} = 10.25$ ft., $\phi P_n = 819.5$ kips (major axis buckling strength)
- If calculated value of $(KL)_{eq} < K_y L_y$ then minor axis buckling will govern.

EXAMPLE 4.4 Determine the design strength of an ASTM A992 W14 x 132 that is part of a braced frame. Assume that the physical length $L = 30$ ft., the ends are pinned and the column is braced at the ends only for the X-X axis and braced at the ends and mid-height for the Y-Y axis.

Solution

- Step I. Calculate the *effective lengths*.

For W14 x 132: $r_x = 6.28$ in; $r_y = 3.76$ in; $A_g = 38.8$ in²

$K_x = 1.0$ and $K_y = 1.0$

$L_x = 30$ ft. and $L_y = 15$ ft.

$K_x L_x = 30$ ft. and $K_y L_y = 15$ ft.

- Step II. Determine the governing slenderness ratio

$$K_x L_x / r_x = 30 \times 12 \text{ in.} / 6.28 \text{ in.} = 57.32$$

$$K_y L_y / r_y = 15 \times 12 \text{ in.} / 3.76 \text{ in.} = 47.87$$

The larger slenderness ratio, therefore, **buckling about the major axis will govern** the column strength.

- Step III. Calculate the column strength

$$K_x L_x = 30 \text{ ft.} \quad \text{Therefore, } (KL)_{eq} = \frac{K_x L_x}{r_x / r_y} = \frac{30}{6.28 / 3.76} = 17.96 \text{ ft.}$$

From Table 4-1, for $(KL)_{eq} = 18.0$ ft. $\phi_c P_n = 1370$ kips (design column strength)

- Step IV. Check the local buckling limits

$$\text{For the flanges, } b_f/2t_f = 7.14 < \lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 13.5$$

$$\text{For the web, } h/t_w = 15.5 < \lambda_r = 1.49 \times \sqrt{\frac{E}{F_y}} = 35.9$$

Therefore, the section is nonslender. OK.

EXAMPLE 4.5 A compression member is subjected to service loads of 165 kips dead load and 535 kips of live load. The member is 26 ft. long and pinned at each end. Use A992 (50 ksi) steel and select a W shape

Solution

- Calculate the factored design load P_u

$$P_u = 1.2 P_D + 1.6 P_L = 1.2 \times 165 + 1.6 \times 535 = 1054 \text{ kips}$$

- Select a W shape from the AISC manual Tables

For $K_y L_y = 26$ ft. and required strength = 1054 kips

- Select W14 x 145 from page 4-15. It has $\phi_c P_n = 1230$ kips
- Select W12 x 170 from page 4-18. It has $\phi_c P_n = 1130$ kips
- No W10 will work. See Page 4-21
- W14 x 145 is the lightest.
- Note that column sections are usually W12 or W14. Usually sections bigger than W14 are usually not used as columns.

4.8 DESIGN OF SINGLY SYMMETRIC CROSS-SECTIONS

- So far, we have been talking about doubly symmetric wide-flange (I-shaped) sections and channel sections. These rolled shapes always fail by *flexural* buckling.
- Singly symmetric (Tees and double angle) sections fail either by *flexural* buckling about the axis of non-symmetry or by *flexural-torsional* buckling about the axis of symmetry and the longitudinal axis.

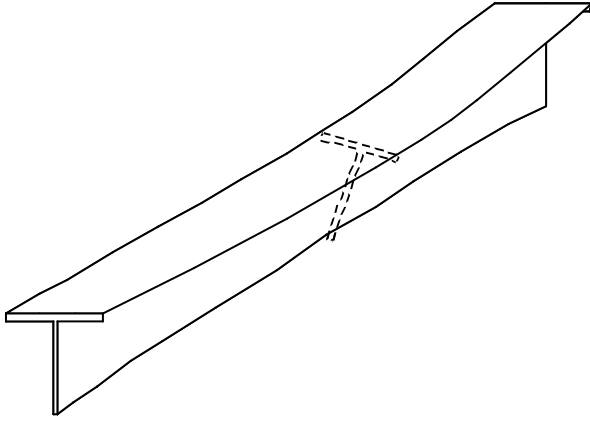


Figure 6(a). Flexural torsional buckling

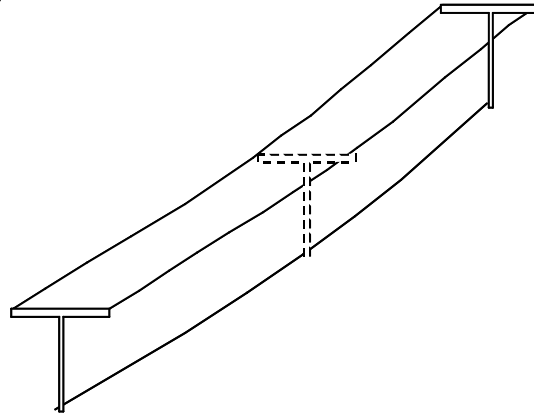
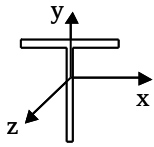


Figure 6(b). Flexural buckling



Flexural buckling will occur about the x-axis

Flexural-torsional buckling will occur about the y and z-axis

Smaller of the two will govern the design strength

Figure 6(c). Singly symmetric cross-section

- The AISC specification for flexural-torsional buckling is given by Spec. E4. (Page 16.1-34)

$$\text{Design strength} = \phi_c P_n = 0.90 A_g F_{cr} \quad (1)$$

$$\text{Where, } F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2} \right) \left[1 - \sqrt{1 - \frac{4 F_{cry} F_{crz} H}{(F_{cry} + F_{crz})^2}} \right] \quad (2)$$

$$F_{cry} = \text{critical stress for buckling about the y-axis, see Spec. E3.} \quad (3)$$

$$F_{crz} = \frac{GJ}{A \bar{r}_o^2} \quad (4)$$

$$\bar{r}_o^2 = \text{polar radius of gyration about shear center (in.)} = y_o^2 + \frac{I_x + I_y}{A} \quad (5)$$

$$H = 1 - \frac{y_o^2}{\bar{r}_o^2} \quad (6)$$

$$y_o = \text{distance between shear center and centroid (in.)} \quad (7)$$

- The section properties for calculating the flexural-torsional buckling strength F_{crft} are given as follows:

$$- \quad G = \frac{E}{2(1+\nu)}$$

Shape	Where are the constants?
W, M, S, HP, WT, MT, ST	J, C_w are given in the Tables in the manual. The manual companion CD includes $\bar{r}_o$ and H for WT, MT, and ST shapes
C	J, C_w , $\bar{r}_o$, H
MC, Angles	J, C_w , $\bar{r}_o$. In addition the manual companion CD gives values of H for MC and angle shapes
Double Angles	$\bar{r}_o$, H (J and C_w values are twice that of the values for single angles).

The manual does not give the values for $\bar{r}_o$, H for tees. However, they are easy to calculate if x_o and y_o are known. x_o and y_o are the shear center coordinates with respect to the centroid. The shear center for Tees is located at the web-flange junction.

- The design tables for WT shapes given in Table 4-7 on page 4-89 to 4-119. These design tables include the axial compressive strength for flexural buckling about the x axis and flexural-torsional buckling about the y and z axis.

EXAMPLE 4.6 Calculate the design compressive strength of a WT12 x 81. The effective length with respect to x-axis is 25ft. 6in. The effective length with respect to the y-axis is 20 ft. and the effective length with respect to z-axis is 20ft. A992 steel is used.

Solution

- **Step I.** Buckling strength about x-axis

$$\frac{K_x L}{r_x} = \frac{25.5 \times 12}{3.50} = 87.43$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 29000}{(87.43)^2} = 37.44 \text{ ksi}$$

$$4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29000}{50}} = 113.4$$

- Since $KL/r < 4.71 \sqrt{E/F_y}$

Equation E3 – 2 applies

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/37.44)} 50 = 28.59 \text{ ksi}$$

- The design strength for x-axis buckling is $\phi P_n = 0.9 A_g F_{cr} = 0.9 \times 28.59 \times 23.9 = 615 \text{ kips}$

Compare with tabulated design strength for buckling about x-axis in Table 4-7

- **Step II.** Flexural-torsional buckling about the y and z axes

- Calculate F_{cry} and F_{crz} then calculate F_{cr} and ϕP_n

Compute F_{cry} using AISCE3. Use AISCE3 – 4

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 29000}{(78.69)^2} = 46.22 \text{ ksi}$$

- Since $K_y L_y / r_y < 4.71 \sqrt{E/F_y} = 113.4$

$$F_{cry} = 0.658^{(F_y/F_e)} F_y = 31.79 \text{ ksi}$$

$$F_{crz} = \frac{GJ}{A_g r_o^2} = \frac{11200 \times 9.22}{23.9 \times 25.87} = 167 \text{ ksi}$$

$$x_0 = 0$$

$$y_0 = \bar{y} - \frac{t_f}{2} = 2.70 - 1.22/2 = 2.090 \text{ in.}$$

$$r_o^2 = x_0^2 + y_0^2 + \frac{I_x + I_y}{A_g} = 0 + 2.09^2 + \frac{293 + 221}{23.9} = 25.87 \text{ in}^2$$

$$- \quad H = 1 - \frac{x_0^2 + y_0^2}{r_o^2} = 0.8312$$

$$\therefore F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2H} \right) \left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}} \right] = 30.63 \text{ ksi}$$

$$\therefore \phi F_{cr} A_g = 0.9 \times 30.63 \times 23.9 = 658.85 \text{ ksi}$$

Compare with values in Table 4.7

- **Step III.** Design strength and check local buckling

$$\text{Flanges: } b_f/2t_f = 13/(2 \times 1.22) = 5.33, \text{ which is } < \lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 13.5$$

$$\text{Stem of Tee: } d/t_w = 10.9/0.65 = 17.73, \text{ which is } < \lambda_r = 0.75 \times \sqrt{\frac{E}{F_y}} = 18.06$$

Local buckling is not a problem. Design strength = 615 kips. X-axis flexural buckling governs.

4.9 DESIGN OF DOUBLE ANGLE SECTIONS

- Double-angle sections are very popular as compression members in trusses and bracing members in frames.
 - These sections consist of two angles placed back-to-back and connected together using bolts or welds.
 - You have to make sure that the two single angle sections are connected such that they do not buckle (individually) between the connections along the length.
 - The AISC specification E6.2 requires that $\mathbf{Ka/r_i}$ of the individual single angles $< \frac{3}{4}$ of the governing $\mathbf{KL/r}$ of the double angle.
 - where, a is the distance between connections and r_i is the smallest radius of gyration of the single angle (see dimensions in Table 1-7 of the AISC Specs.)
- Double-angle sections can fail by flexural buckling about the x-axis or flexural torsional buckling about the y and z axes.
 - For flexural buckling about the x-axis, the moment of inertia I_{x-2L} of the double angle will be equal to two times the moment of inertia I_{x-L} of each single angle.
 - For flexural torsional buckling, there is a slight problem. The double angle section will have some *additional flexibility* due to the intermittent connectors. This added flexibility will depend on the connection parameters.
- According to AISC Specification E6.1, a modified $(KL/r)_m$ must be calculated for the double angle section for buckling about the y-axis to account for this added flexibility
 - Intermediate connectors that are snug-tight bolted
$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{a}{r_i}\right)^2}$$
 - Intermediate connectors that are welded or fully tensioned bolted:

a) when $\alpha/r_i \leq 40$

$$\left(\frac{KL}{r}\right)_m = \left(\frac{KL}{r}\right)_o$$

b) when $\alpha/r_i > 40$

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2}$$

where,

r_i = minimum radius of gyration of individual angle, in.

α = distance between connectors, in.

K_i = 0.5 for angles back-to-back

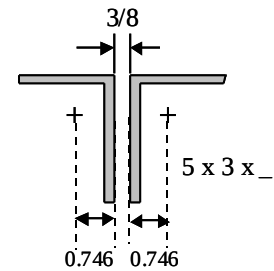
$\left(\frac{KL}{r}\right)_o$ = slenderness ratio of built-up member acting as a unit

EXAMPLE 4.7 Calculate the design strength of the compression member

shown in the figure. Two angles, 5 x 3 x ½ are oriented with the long legs

back-to-back and separated by 3/8 in. The effective length KL is 16 ft.

A36 steel is used. Assume three welded intermediate connectors

**Solution**

Step I. Determine the relevant properties from the AISC manual

Property	Single angle	Double angle
A_g	3.75 in ²	7.5 in ²
r_x	1.58 in.	1.58 in.
r_y	0.824 in.	1.24 in.
r_z	0.642 in.	-----
J	0.322 in ⁴	0.644 in ⁴
$\bar{r}_o$		2.51 in.
H		0.646
AISC Manual	---	Table 1-15 on pages 1-104 and 1-105

Step II. Calculate the x-axis buckling strength

- $KL/r_x = 16 \times 12 / 1.58 = 121.5$

$$\frac{K_x L}{r_x} = \frac{16 \times 12}{1.58} = 121.5$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 29000}{(121.5)^2} = 19.39 \text{ ksi}$$

$$4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29000}{36}} = 133.7$$

- Since $KL/r < 4.71 \sqrt{E/F_y}$

Equation E3-2 applies

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(36/19.39)} 36 = 16.55 \text{ ksi}$$

- $\phi P_n = 0.90 \times 16.55 \times (2 \times 3.75) = 111.71 \text{ kips}$

Step III. Calculate $(KL/r)_m$ for y-axis buckling

- $(KL/r)_0 = KL/r_y = 16 \times 12 / 1.24 = 154.8$
- Connector spacing $= \alpha = 16 \times 12 / 4 \text{ spaces} = 48 \text{ in.}$
- $\alpha/r_i = 48 / 0.642 = 74.77 > 40$, hence

$$\begin{aligned} \left(\frac{KL}{r} \right)_m &= \sqrt{\left(\frac{KL}{r} \right)_o^2 + \left(\frac{K_i \alpha}{r_i} \right)^2} \\ &= \sqrt{(154.8)^2 + \left(\frac{24}{0.642} \right)^2} = 159.25 \end{aligned}$$

Step IV. Calculate flexural torsional buckling strength.

Compute F_{cry} using AISCE3. Use AISCE3-4

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 29000}{(159.25)^2} = 11.29 \text{ ksi}$$

$$\text{Since } K_y L_y / r_y > 4.71 \sqrt{E/F_y} = 134$$

$$F_{cry} = 0.877 F_e = 9.9 \text{ ksi}$$

$$F_{crz} = \frac{GJ}{A_g r_o^2} = \frac{11200 \times 2 \times 0.322}{7.5 \times 2.51^2} = 152.6 \text{ ksi}$$

$$\bullet \quad F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2 H} \right) \left[1 - \sqrt{1 - \frac{4 F_{cry} F_{crz} H}{(F_{cry} + F_{crz})^2}} \right] = \left(\frac{9.9 + 152.6}{2 \times 0.646} \right) \left[1 - \sqrt{1 - \frac{4 \times 9.9 \times 152.6 \times 0.646}{(9.9 + 152.6)^2}} \right]$$

$$F_{cr} = 9.67 \text{ ksi}$$

$$\bullet \quad \phi P_n = 0.90 \times F_{cr} \times A_g = 0.90 \times 9.67 \times 7.50 = 65.27 \text{ kips}$$

Flexural torsional buckling strength controls. The design strength of the double angle member is 65.27 kips.

Step V. Compare with design strengths in Table 4-9 (page 4-138) of the AISC manual

- ϕP_n for x-axis buckling with unsupported length = 16 ft. = 112 kips
- ϕP_n for y-z axis buckling with unsupported length = 16 ft. = 62.6 kips

These results indicate excellent correlation between the calculations in steps II to IV and the tabulated values.

Design tables for double angle compression members are given in the AISC manual. See Tables 4-8, 4-9 and 4-10 on pages 4-122 to 4-160

- In these Tables $F_y = 36 \text{ ksi}$
- Back to back distance = 3/8 in.
- Design strength for buckling about x axis
- Design strength for flexural torsional buckling accounting for the *modified* slenderness ratio depending on the number of intermediate connectors.
- These design Tables can be used to design compression members as double angle sections.

NOTE: For Torsional and Flexural-Torsional Buckling of members other than double-angle or tee-shaped member, refer section E-4 (Page 16.1-34).