### ­Chapter 5. Design of Beams – Flexure and Shear

**5.1 Section force-deformation response & Plastic Moment (Mp)**

* A beam is a structural member that is subjected primarily to transverse loads and negligible axial loads.
* The transverse loads cause internal shear forces and bending moments in the beams as shown in Figure 1 below.

**Figure 1.** Internal shear force and bending moment diagrams for transversely loaded beams.

* These internal shear forces and bending moments cause longitudinal axial stresses and shear stresses in the cross-section as shown in the Figure 2 below.

Curvature =  = /(d/2)   (Planes remain plane)

**Figure 2.** Longitudinal axial stresses caused by internal bending moment.

* Steel material follows a typical stress-strain behavior as shown in Figure 3 below.

**Figure 3.** Typical steel stress-strain behavior.

* If the steel stress-strain curve is approximated as a bilinear elasto-plastic curve with yield stress equal to y, then the section Moment - Curvature (M-) response for monotonically increasing moment is given by Figure 4.

**Figure 4.** Section Moment - Curvature (M-) behavior.

* In Figure 4, My is the moment corresponding to first yield and Mp is the plastic moment capacity of the cross-section.
* The ratio of Mp to My is called as the shape factor *f* for the section.
* For a rectangular section, *f* is equal to 1.5. For a wide-flange section, *f* is equal to 1.1.
* Calculation of Mp: Cross-section subjected to either +y or -y at the plastic limit. See Figure 5 below.

(a) General cross-section (b) Stress distribution (c) Force distribution



(d) Equations

**Figure 5.** Plastic centroid and Mp for general cross-section.

* The plastic centroid for a general cross-section corresponds to the axis about which the total area is equally divided, i.e., A1 = A2 = A/2
* The plastic centroid is not the same as the elastic centroid or center of gravity (c.g.) of the cross-section.
* As shown below, the c.g. is defined as the axis about which A1y1 = A2y2.

* For a cross-section with at-least one axis of symmetry, the neutral axis corresponds to the centroidal axis in the elastic range. However, at Mp, the neutral axis will correspond to the plastic centroidal axis.
* **For a doubly symmetric cross-section, the elastic and the plastic centroid lie at the same point.**
* Mp = y x A/2 x (y1+y2)
* As shown in Figure 5, y1 and y­2 are the distance from the plastic centroid to the centroid of area A1 and A2, respectively.
* A/2 x (y1+y2) is called **Z,** the plastic section modulus of the cross-section. Values for Z are tabulated for various cross-sections in the properties section of the LRFD manual.
* **Mp = 0.90 Z Fy - See Spec. F2.1 (Page 16.1-47)

My = Fy S for homogenous cross-sections

 = Fyf S for hybrid sections.

where,

Mp = plastic moment

My = moment corresponding to onset of yielding at the extreme fiber from an elastic stress distribution

Z = plastic section modulus from the Properties section of the AISC manual.

S = elastic section modulus, also from the Properties section of the AISC manual.

**Example 2.1** Determine the elastic section modulus, S, plastic section modulus, Z, yield moment, My, and the plastic moment Mp, of the cross-section shown below. What is the design moment for the beam cross-section? Assume 50 ksi steel.

* Ag = 12 x 0.75 + (16 - 0.75 - 1.0) x 0.5 + 15 x 1.0 = 31.125 in2

Af1 = 12 x 0.75 = 9 in2

Af2 = 15 x 1.0 = 15.0 in2

Aw = 0.5 x (16 - 0.75 - 1.0) = 7.125 in2

* distance of elastic centroid from bottom = 



Ix = 120.753/12 + 9.09.0062 + 0.514.253/12 + 7.1251.5062 + 15.013/12 +

 156.1192 = 1430 in4

 Sx = Ix / (16-6.619) = 152.44 in3

 My-x = Fy Sx = 7622 kip-in. = 635.17 kip-ft.

* distance of plastic centroid from bottom = 



y1=centroid of top half-area about plastic centroid = in.

y2=centroid of bottom half-area about plastic centroid =

 in.

Zx = A/2 x (y1 + y2) = 15.5625 x (10.5746 + 1.5866) = 189.26 in3

Mp-x = Zx Fy = 189.26 x 50 = 9462.93 kip-in. = 788.58 kip-ft.

* Design strength according to AISC Chapter F= bMp= 0.9 x 788.58 = 709.72 kip-ft.
* Reading Assignment – AISC Specification Chapter F.

#### **5.2 Local buckling of beam section – Compact and Non-compact**

* Mp, the plastic moment capacity for the steel shape, is calculated by assuming a plastic stress distribution (+ or - y) over the cross-section.
* The development of a plastic stress distribution over the cross-section can be hindered by two different length effects:

(1) *Local buckling* of the individual plates (flanges and webs) of the cross-section before

they develop the compressive yield stress y.

(2) *Lateral-torsional buckling* of the unsupported length of the beam / member before

the cross-section develops the plastic moment Mp.

**Figure 7.** Local buckling of flange due to compressive stress ()

* The analytical equations for local buckling of steel plates with various edge conditions and the results from experimental investigations have been used to develop limiting slenderness ratios for the individual plate elements of the cross-sections.
* See Spec. B4 (Pages 16.1-14 and 16.1-15), Table B4.1b (Page 16.1-17)
* Steel sections are classified as compact, noncompact, or slender depending upon the slenderness () ratio of the individual plates of the cross-section.
	+ *Compact section* if all elements of cross-section have  ≤ p
	+ *Non-compact sections* if any one element of the cross-section has p ≤  ≤ r
	+ *Slender section* if any element of the cross-section has r ≤ 
* It is important to note that:
	+ If  ≤ p, then the individual plate element can develop and sustain y for large values of  before local buckling occurs.
	+ If p ≤  ≤ r, then the individual plate element can develop y but cannot sustain it before local buckling occurs.
	+ If r ≤ , then elastic local buckling of the individual plate element occurs.

**Figure 8.** Stress-strain response of plates subjected to axial compression and local buckling.

* Thus, slender sections cannot develop Mp due to elastic local buckling. Non-compact sections can develop My but not Mp before local buckling occurs. Only compact sections can develop the plastic moment Mp.
* All rolled wide-flange shapes are **compact** with the following exceptions, which are non-compact.
	+ W40x174, W21x48, W14x99, W14x90, W12x65, W10x12, W8x31, W8x10, W6x15, W6x9, W6x8.5, M4x6 (made from A992)
* The definition of  and the values for p and r for the individual elements of various cross-sections are given in Table B4.1b on page 16.1-17. For example,

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Section | **Plate element** | **** | **p** | **r** |
| Wide-flange | Flange | bf/2tf | 0.38  | 1.0 |
| Web | h/tw | 3.76  | 5.70  |
| Channel | Flange | bf/tf | 0.38  | 1.0 |
| Web | h/tw | 3.76  | 5.70  |
| Square or Rect. Box | Flange | (b-3t)/t | 1.12  | 1.40  |
| Web | (b-3t)/t | 2.42  | 5.70  |

* Note that the slenderness limits (p and r) and the definition of plate slenderness (b/t) ratio depend upon the boundary conditions for the plate.
* If the plate is supported along *two edges* parallel to the direction of compression force, then it is a *stiffened* element. For example, the webs of W shapes
* If the plate is supported along only *one edge* parallel to the direction of the compression force, then it is an *unstiffened* element. Ex., the flanges of W shapes.

In CE470 we will try to design all beam sections to be compact from a local buckling standpoint

# 5.3 Lateral-Torsional Buckling

* The laterally unsupported length of a beam-member can undergo lateral-torsional buckling due to the applied flexural loading (bending moment).

**Figure 9.** Lateral-torsional buckling of a wide-flange beam subjected to constant moment.

* Lateral-torsional buckling is fundamentally similar to the flexural buckling or flexural-torsional buckling of a column subjected to axial loading.
	+ The similarity is that it is also a bifurcation-buckling type phenomenon.
	+ The differences are that lateral-torsional buckling is caused by flexural loading (M), and the buckling deformations are coupled in the lateral and torsional directions.
* There is one very important difference. For a column, the axial load causing buckling remains constant along the length. But, for a beam, usually the lateral-torsional buckling causing bending moment M(x) varies along the unbraced length.
	+ The worst situation is for beams subjected to uniform bending moment along the unbraced length. Why?

**5.4.1 Lateral-torsional buckling – Uniform bending moment**

* Consider a beam that is simply-supported at the ends and subjected to four-point loading as shown below. The beam center-span is subjected to uniform bending moment M. Assume that lateral supports are provided at the load points.

* Laterally unsupported length = Lb.
* If the laterally unbraced length Lb is less than or equal to a plastic length Lp then lateral torsional buckling is not a problem and the beam will develop its plastic strength Mp.
* Lp = 1.76 ry x  - for I members & channels (See Pg. **16.1**-48)
* If Lb is greater than Lp then lateral torsional buckling will occur and the moment capacity of the beam will be reduced below the plastic strength Mp as shown in Figure 10 below.

**Figure 10.** Moment capacity (Mn) versus unsupported length (Lb).

* As shown in Figure 10 above, the lateral-torsional buckling moment (M­n = M­cr) is a function of the laterally unbraced length Lb and can be calculated using the equation:

Mn = Mcr = 

where, Mn = moment capacity

Lb = laterally unsupported length.

 Mcr = critical lateral-torsional buckling moment.

 E = 29000 ksi; G = 11,200 ksi

 Iy = moment of inertia about minor or y-axis (in4)

 J = torsional constant (in4) from the AISC manual.

 Cw = warping constant (in6) from the AISC manual.

* This equation is valid for **ELASTIC** lateral torsional buckling only (like the Euler equation). That is it will work only as long as the cross-section is elastic and no portion of the cross-section has yielded.
* As soon as any portion of the cross-section reaches the yield stress Fy, the elastic lateral torsional buckling equation cannot be used.
* Lr is the unbraced length that corresponds to a lateral-torsional buckling moment
Mr = 0.70 Sx Fy.
* Mr will cause yielding of the cross-section due to residual stresses.
* When the unbraced length is less than Lr, then the elastic lateral torsional buckling equation cannot be used.
* When the unbraced length (Lb) is less than Lr but more than the plastic length Lp, then the lateral-torsional buckling Mn is given by the equation below:
* If Lp ≤ Lb ≤ Lr, then 
* This is linear interpolation between (Lp, Mp) and (Lr, Mr)
* See Figure 10 again.

### 5.4.2 Moment Capacity of beams subjected to non-uniform bending moments

* As mentioned previously, the case with uniform bending moment is worst for lateral torsional buckling.
* For cases with non-uniform bending moment, the lateral torsional buckling moment **is greater** than that for the case with uniform moment.
* The AISC specification (Spec. F1 at Page 16.1-46) says that:
* The lateral torsional buckling moment for non-uniform bending moment case

= **Cb x** lateral torsional buckling moment for uniform moment case.

* **Cb** is always greater than 1.0 for non-uniform bending moment.
* **Cb** is equal to 1.0 for uniform bending moment.
* Sometimes, if you cannot calculate or figure out **Cb**, then it can be conservatively assumed as 1.0.
* 

where, **Mmax** = magnitude of maximum bending moment in **Lb**

 **MA** = magnitude of bending moment at quarter point of **Lb**

 **MB** = magnitude of bending moment at half point of **Lb**

 **MC** = magnitude of bending moment at three-quarter point of **Lb**

* The moment capacity Mn for the case of non-uniform bending moment
* Mn = **Cb** x {Mn for the case of uniform bending moment} **≤ Mp**
* Important to note that the increased moment capacity for the non-uniform moment case cannot possibly be more than **Mp.**
* Therefore, if the calculated values is greater than **Mp**, then you have to reduce it to **Mp**

**Figure 11.** Moment capacity versus Lb for non-uniform moment case.

# 5.4 Flexural Deflection of Beams – Serviceability

* Steel beams are designed for the factored design loads. The moment capacity, i.e., the factored moment strength (bMn) should be greater than the moment (Mu) caused by the factored loads.
* A *serviceable* structure is one that performs satisfactorily, not causing discomfort or perceptions of unsafety for the occupants or users of the structure.
* For a beam, being serviceable usually means that the deformations, primarily the vertical slag, or deflection, must be limited.
* The maximum deflection of the designed beam is checked at the service-level loads. The deflection due to service-level loads must be less than the specified values.
* The AISC Specification gives little guidance other than a statement in Chapter L, “*Serviceability Design Considerations*,” that deflections should be checked. Appropriate limits for deflection can be found from the governing building code for the region.
* The following values of deflection are typical maximum allowable total (service live load) deflections.
* Plastered floor construction – L/360
* Unplastered floor construction – L/240
* Unplastered roof construction – L/180
* In the following examples, we will assume that local buckling and lateral-torsional buckling are not controlling limit states, i.e, the beam section is compact and laterally supported along the length.

# Example 5.2 Design a simply supported beam subjected to uniformly distributed dead load of 450 lbs./ft. and a uniformly distributed live load of 550 lbs./ft. The dead load does not include the self-weight of the beam. The length of the beam is 30 ft.

* **Step I.** Calculate the factored design loads (without self-weight).

wU = 1.2 wD + 1.6 wL = 1.42 kips / ft.

MU = wu L2 / 8 = 1.42 x 302 / 8 = 159.75 kip-ft.

* **Step II.** Select the lightest section from the AISC Manual design tables.

From page 3-26 of the AISC manual, select **W14 x 30** made from 50 ksi steel with bMp = 177.0 kip-ft.

* **Step III.** Check deflection at service live loads.

 = 5 w L4 / (384 E Ix) = 5 x (0.55/12) x (30 x 12)4 / (384 x 29000 x 291)

 = 1.18 in. > L/360 (1 in.) - for plastered floor construction

* **Step V.** Redesign with service-load deflection as design criteria

L /360 = 1.0 in. > 5 w L4/(384 E Ix)

Therefore, Ix > 345.65 in4

Select the section from the *moment of inertia* selection tables in the AISC manual. See page 3-26 of the AISC manual – select **W16 x 31.**

**W16 x 31** with **Ix** = 375 in4 and **bMp** = 203 kip-ft. (50 ksi steel).

Deflection at service load =  = 0.922 in. < L/360 - **OK!**

##### ***Note that the serviceability design criteria controlled the design and the section***

**Example 5.3** Design the beam shown below. The unfactored dead and live loads are shown in the Figure.

* **Step I.** Calculate the factored design loads (without self-weight).

wu = 1.2 wD + 1.6 wL = 1.2 x 0.67 + 1.6 x 0.75 = 2.004 kips / ft.

Pu = 1.2 PD + 1.6 PL = 1.2 x 0 + 1.6 x 10 = 16.0 kips

Mu = wU L2 / 8 + PU L / 4 = 225.45 + 120 = 345.45 kip-ft.

* **Step II.** Select the lightest section from the AISC Manual design tables.

From page ­3-25 of the AISC manual, select **W21 x 44** made from 50 ksi steel with bMp = 358.0 kip-ft.

Self-weight = wsw = 44 lb./ft.

* **Step IV.** Check deflection at service live loads.

Service loads

* Distributed load = w = 0.75 kips/ft.
* Concentrated load = P = L = 10 kips = 10 kips

Deflection due to uniform distributed load = d = 5 w L4 / (384 EI)

Deflection due to concentrated load = c = P L3 / (48 EI)

## Therefore, service-load deflection =  = d + c

 = 5 x 1.464 x 3604 / (384 x 29000 x 12 x 843) + 10 x 3603 / (48 x 29000 x 843)

 = 0.56 + 0.3976 = 0.957 in.

Assuming plastered floor construction, max = L/360 = 360/360 = 1.0 in.

Therefore,  < max **- OK!**

**5.5 Beam Design**

**Example 5.4**

Design the beam shown below. The unfactored uniformly distributed live load is equal to 3 kips/ft. There is no dead load. Lateral support is provided at the end reactions.

**Step I.** Calculate the factored loads assuming a reasonable self-weight.

Assume self-weight = wsw = 100 lbs./ft.

Dead load = wD = 0 + 0.1 = 0.1 kips/ft.

Live load = wL = 3.0 kips/ft.

Ultimate load = wu = 1.2 wD + 1.6 wL = 4.92 kips/ft.

Factored ultimate moment = Mu = wu L2/8 = 354.24 kip-ft.

**Step II.** Determine unsupported length Lb and Cb

There is only one unsupported span with Lb = 24 ft.

Cb = 1.14 for the parabolic bending moment diagram, See values of Cb shown in **Table 3-1** on **page 3-18** of the AISC manual. .

**Step III.** Select a wide-flange shape

The moment capacity of the selected section **bMn > Mu**  (Note **b = 0.9**)

**bMn** = moment capacity = **Cb** x (**bMn** for the case with **uniform moment**) ≤ **bMp**

* Pages 3-99 to 3-142 in the AISC-LRFD manual, show the plots of **bMn-Lb** for the case of uniform bending moment (Cb=1.0)
* Therefore, in order to select a section, calculate **Mu/Cb** and use it with Lb to find the first section with a **solid line** as shown in class.
* Mu/Cb = 354.24/1.14 = 310.74 kip-ft.
* Select W16 x 67 (50 ksi steel) with bMn =328 kip-ft. for Lb = 24 ft. and Cb =1.0
* For the case with Cb = 1.14,

bMn = 1.14 x 328 = 373.92 kip-ft., **which must be ≤ bMp = 488 kip-ft**. **OK!**

* *Thus, W16 x 67 made from 50 ksi steel with moment capacity equal to 373 kip-ft. for an unsupported length of 24 ft. is the designed section.*

**Step IV.** Check for local buckling.

 = bf / 2tf = 7.67; Corresponding p = 0.38 (E/Fy)0.5 = 9.152

Therefore,  < p - compact flange

 = h/tw = 33.5; Corresponding p = 3.76 (E/Fy)0.5 = 90.5

Therefore,  < p - compact web

Compact section. - OK!

* *This example demonstrates the method for designing beams and accounting for Cb > 1.0*

Example 5.5

Design the beam shown below. The concentrated live loads acting on the beam are shown in the Figure. The beam is laterally supported at the load and reaction points.

**Step I.** Assume a self-weight and determine the factored design loads

Let, w­sw = 100 lbs./ft. = 0.1 kips/ft.

 PL = 30 kips

 Pu = 1.6 PL = 48 kips

 wu = 1.2 x wsw = 0.12 kips/ft.

The reactions and bending moment diagram for the beam are shown below.

**Step II.** Determine Lb, Cb, Mu, and Mu/Cb for all spans.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Span | **Lb****(ft.)** | **Cb** | **Mu****(kip-ft.)** | **Mu/Cb****(kip-ft.)** |
| AB | 12 | 1.67 | 550.6 | 329.7 |
| BC | 8 | 1.0(assume) | 550.6 | 550.6 |
| CD | 10 | 1.67 | 524.0 | 313.8 |

*It is important to note that it is possible to have different Lb and Cb values for different laterally unsupported spans of the same beam.*

**Step III.** Design the beam and check all laterally unsupported spans

Assume that **span BC** is the controlling span because it has the largest **Mu/Cb** although the corresponding **Lb** is the smallest.

**From the AISC-LRFD manual select W24 x 68 made from 50 ksi steel (page 3-122)**

Check the selected section for spans **AB, BC, and CD**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Span** | **Lb****(ft.)** | **bMn** **for C­b = 1.0***from \_\_\_\_\_\_* | **Cb** | **bMn** **for Cb value***col. 3 x col. 4* | **bMp***limit* |
| AB | 12 | 549 | 1.67 | 916.8 | 664 kip-ft |
| BC | 8 | 633 | 1.0 | 633 |  |
| CD | 10 | 591 | 1.67 | 986.97 | 664 kip-ft. |

Thus, for span AB, bMn = 664 kip-ft. > Mu - OK!

 for span BC, bMn = 633.0 kip-ft. > Mu -OK!

 For span CD, bMn = 664 kip-ft. > Mu -OK!

**Step IV.** Check for local buckling

 = bf / 2tf = 7.67; Corresponding p = 0.38 (E/Fy)0.5 = 9.152

Therefore,  < p - compact flange

 = h/tw = 50; Corresponding p = 3.76 (E/Fy)0.5 = 90.55

Therefore,  < p - compact web

Compact section. - OK!

*This example demonstrates the method for designing beams with several laterally unsupported spans with different Lb and Cb values.*

**Example 5.6**

Design the simply-supported beam shown below. The uniformly distributed dead load is equal to 1 kips/ft. and the uniformly distributed live load is equal to 2 kips/ft. A concentrated live load equal to 10 kips acts at the mid-span. Lateral supports are provided at the end reactions and at the mid-span.

**Step I.** Assume the self-weight and calculate the factored design loads.

Let, wsw = 100 lbs./ft. = 0.1 kips/ft.

wD = 1+ 0.1 = 1.1 kips/ft.

wL = 2.0 kips/ft.

wu = 1.2 wD + 1.6 wL = 4.52 kips/ft.

Pu = 1.6 x 10 = 16.0 kips

The reactions and the bending moment diagram for the factored loads are shown below.

**Step II.** Calculate Lb and Cb for the laterally unsupported spans.

Since this is a symmetric problem, need to consider only span AB

Lb = 12 ft.; 

M(x) = 62.24 x – 4.52 x2/2

Therefore,

MA = M(x = 3 ft.) = 166.38 kip-ft. - quarter-point along Lb = 12 ft.

MB = M(x = 6 ft.) = 292.08 kip-ft. - half-point along Lb = 12 ft.

MC = M(x = 9ft.) = 377.1 kip-ft -three-quarter point along Lb= 12 ft.

Mmax = M(x = 12 ft.) = 421.44 kip-ft. - maximum moment along Lb =12ft.

 Therefore, Cb = 1.37

**Step III.** Design the beam section

 Mu = Mmax = 421.44 kip-ft.

 Lb = 12.0 ft.; Cb = 1.37

 Mu/Cb = 421.44/1.37 = 307.62 kip-ft.

* Select W21 x 48 made from 50 ksi with bMn = ­­­312 kip-ft. for Lb = 12.0 ft. and Cb =1.0
* For Cb = 1.37, bMn = \_427.44 k-ft., but must be ≤ bMp = 398 k-ft.
* Therefore, for Cb =1.37, bMn = 398 k-ft. < Mu

**Step IV.** Redesign the section

* Select the next section with greater capacity than W21 x 48
* Select W18 x 55 with bMn = 335 k-ft. for Lb = 12 ft. and Cb = 1.0

For Cb = 1.37, bMn = 335 x 1.37 = 458.95 k-ft. but must be ≤ bMp = 420 k-ft.

Therefore, for Cb = 1.37, bMn = 420 k-ft., which is < Mu (421.44 k-ft), (**NOT OK!**)

* Select W 21 x 55 with bMn = 376 k-ft. for Lb = 12 ft. and Cb = 1.0

For Cb 1.37, bMn = 376 x 1.37 = 515.12 k-ft., but must be ≤ bMp = 473 k-ft.

Therefore, for Cb = 1.37, bMn = 473 k-ft, which is > Mu (421.44 k-ft). (**OK!**)

**Step V.** Check for local buckling.

 = bf / 2tf = 7.87; Corresponding p = 0.38 (E/Fy)0.5 = 9.152

Therefore,  < p - compact flange

 = h/tw = 49.0; Corresponding p = 3.76 (E/Fy)0.5 = 90.55

Therefore,  < p - compact web

Compact section. **- OK!**

*This example demonstrates the calculation of Cb and the iterative design method.*