

**TABLE C-C2.2**  
**Approximate Values of Effective Length Factor,  $K$**

| Buckled shape of column is shown by dashed line.                | (a)                                                                                                                                                      | (b)  | (c) | (d) | (e)  |
|-----------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|------|-----|-----|------|
|                                                                 |                                                                                                                                                          |      |     |     |      |
| Theoretical $K$ value                                           | 0.5                                                                                                                                                      | 0.7  | 1.0 | 1.0 | 2.0  |
| Recommended design value when ideal conditions are approximated | 0.65                                                                                                                                                     | 0.80 | 1.2 | 1.0 | 2.10 |
| End condition code                                              | Rotation fixed and translation fixed<br>Rotation free and translation fixed<br>Rotation fixed and translation free<br>Rotation free and translation free |      |     |     |      |

- The stiffness parameter  $L\sqrt{P/EI}$  of all columns is equal.
- Joint restraint is distributed to the column above and below the joint in proportion to  $EI/L$  for the two columns.
- All columns buckle simultaneously.
- No significant axial compression force exists in the girders.

The alignment chart for sidesway inhibited frames shown in Figure C-C2.3 is based on the following equation:

$$\frac{G_A G_B}{4} (\pi/K)^2 + \left( \frac{G_A + G_B}{2} \right) \left( 1 - \frac{\pi/K}{\tan(\pi/K)} \right) + \frac{2 \tan(\pi/2K)}{(\pi/K)} - 1 = 0$$

The alignment chart for sidesway uninhibited frames shown in Figure C-C2.4 is based on the following equation:

$$\frac{G_A G_B (\pi/K)^2 - 36}{6(G_A + G_B)} - \frac{(\pi/K)}{\tan(\pi/K)} = 0$$

where

$$G = \frac{\Sigma(E_c I_c / L_c)}{\Sigma(E_g I_g / L_g)} = \frac{\Sigma(EI/L)_c}{\Sigma(EI/L)_g}$$

The subscripts  $A$  and  $B$  refer to the joints at the ends of the column being considered. The symbol  $\Sigma$  indicates a summation of all members rigidly connected

to that joint and lying in the plane in which buckling of the column is being considered.  $E_c$  is the modulus of the column,  $I_c$  is the moment of inertia of the column, and  $L_c$  is the unsupported length of the column.  $E_g$  is the modulus of the girder,  $I_g$  is the moment of inertia of the girder, and  $L_g$  is the unsupported length of the girder or other restraining member.  $I_c$  and  $I_g$  are taken about axes perpendicular to the plane of buckling being considered. The alignment chart is valid for different materials if an appropriate effective rigidity,  $EI$ , is used in the calculation of  $G$ .

For column ends supported by, but not rigidly connected to, a footing or foundation,  $G$  is theoretically infinity but unless designed as a true friction-free pin, may be taken as 10 for practical designs. If the column end is rigidly attached to a properly designed footing,  $G$  may be taken as 1.0. Smaller values may be used if justified by analysis.

Theoretical  $K$  values obtained from the alignment charts for various idealized end conditions, rotation fixed or free and translation fixed or free, are shown in Table C-C2.2 along with practical recommendations for use in actual design.

It is important to remember that the alignment charts are based on the assumptions of idealized conditions previously discussed and that these conditions seldom exist

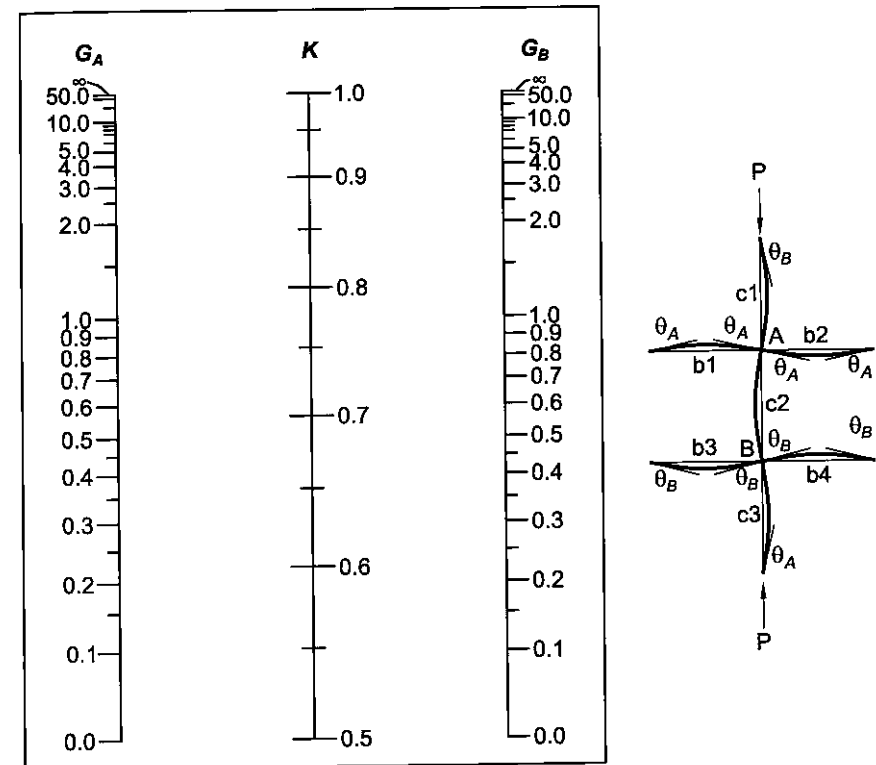


Fig. C-C2.3. Alignment chart—sidesway inhibited (braced frame).





W12

Table 4-1 (continued)  
Available Strength in  
Axial Compression, kips  
W Shapes

 $F_y = 50$  ksi

| Shape                                                                     |    | W12x           |              |                |              |                |              |                |              |                |              |
|---------------------------------------------------------------------------|----|----------------|--------------|----------------|--------------|----------------|--------------|----------------|--------------|----------------|--------------|
| Wt/ft                                                                     |    | 190            |              | 170            |              | 152            |              | 136            |              | 120            |              |
| Design                                                                    |    | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ |
|                                                                           |    | LRFD           | LRFD         | LRFD           | LRFD         | LRFD           | LRFD         | LRFD           | LRFD         | LRFD           | LRFD         |
| Effective length $KL$ (ft) with respect to least radius of gyration $r_y$ | 0  | 2510           | 2250         | 2010           | 1800         | 1590           | 1400         |                |              |                |              |
|                                                                           | 6  | 2420           | 2170         | 1940           | 1730         | 1530           | 1350         |                |              |                |              |
|                                                                           | 7  | 2390           | 2140         | 1910           | 1710         | 1510           | 1330         |                |              |                |              |
|                                                                           | 8  | 2360           | 2110         | 1880           | 1680         | 1480           | 1310         |                |              |                |              |
|                                                                           | 9  | 2320           | 2070         | 1850           | 1650         | 1460           | 1280         |                |              |                |              |
|                                                                           | 10 | 2270           | 2030         | 1820           | 1620         | 1430           | 1260         |                |              |                |              |
|                                                                           | 11 | 2230           | 1990         | 1780           | 1580         | 1390           | 1230         |                |              |                |              |
|                                                                           | 12 | 2180           | 1940         | 1730           | 1540         | 1360           | 1200         |                |              |                |              |
|                                                                           | 13 | 2120           | 1900         | 1690           | 1500         | 1320           | 1170         |                |              |                |              |
|                                                                           | 14 | 2070           | 1840         | 1640           | 1460         | 1290           | 1130         |                |              |                |              |
|                                                                           | 15 | 2010           | 1790         | 1600           | 1420         | 1250           | 1100         |                |              |                |              |
|                                                                           | 16 | 1950           | 1730         | 1540           | 1370         | 1210           | 1060         |                |              |                |              |
|                                                                           | 17 | 1880           | 1680         | 1490           | 1320         | 1160           | 1020         |                |              |                |              |
|                                                                           | 18 | 1820           | 1620         | 1440           | 1280         | 1120           | 985          |                |              |                |              |
|                                                                           | 19 | 1750           | 1560         | 1390           | 1230         | 1080           | 946          |                |              |                |              |
|                                                                           | 20 | 1690           | 1500         | 1330           | 1180         | 1030           | 907          |                |              |                |              |
|                                                                           | 22 | 1550           | 1380         | 1220           | 1080         | 944            | 828          |                |              |                |              |
|                                                                           | 24 | 1420           | 1250         | 1110           | 979          | 855            | 749          |                |              |                |              |
|                                                                           | 26 | 1280           | 1130         | 1000           | 881          | 768            | 672          |                |              |                |              |
|                                                                           | 28 | 1150           | 1010         | 895            | 786          | 684            | 597          |                |              |                |              |
|                                                                           | 30 | 1020           | 902          | 793            | 695          | 602            | 525          |                |              |                |              |
|                                                                           | 32 | 904            | 794          | 698            | 611          | 530            | 462          |                |              |                |              |
|                                                                           | 34 | 800            | 704          | 618            | 541          | 469            | 409          |                |              |                |              |
|                                                                           | 36 | 714            | 628          | 551            | 483          | 418            | 365          |                |              |                |              |
|                                                                           | 38 | 641            | 563          | 495            | 433          | 376            | 327          |                |              |                |              |
|                                                                           | 40 | 578            | 508          | 446            | 391          | 339            | 295          |                |              |                |              |

## Properties

|                                           |       |       |       |       |       |       |
|-------------------------------------------|-------|-------|-------|-------|-------|-------|
| $P_{wo}$ (kips)                           | 618   | 518   | 435   | 365   | 302   | 242   |
| $P_{wi}$ (kips/in.)                       | 53.0  | 48.0  | 43.5  | 39.5  | 35.5  | 30.5  |
| $P_{wb}$ (kips)                           | 3190  | 2370  | 1760  | 1320  | 958   | 608   |
| $P_{wb}$ (kips)                           | 847   | 684   | 551   | 439   | 343   | 276   |
| $L_p$ (ft)                                | 11.5  | 11.4  | 11.3  | 11.2  | 11.1  | 11.0  |
| $L_r$ (ft)                                | 87.3  | 78.5  | 70.6  | 63.3  | 56.5  | 50.7  |
| $A_g$ (in. <sup>2</sup> )                 | 55.8  | 50.0  | 44.7  | 39.9  | 35.3  | 31.2  |
| $I_x$ (in. <sup>4</sup> )                 | 1890  | 1650  | 1430  | 1240  | 1070  | 933   |
| $I_y$ (in. <sup>4</sup> )                 | 589   | 517   | 454   | 398   | 345   | 301   |
| $r_x$ (in.)                               | 3.25  | 3.22  | 3.19  | 3.16  | 3.13  | 3.11  |
| Ratio $r_x/r_y$                           | 1.79  | 1.78  | 1.77  | 1.77  | 1.76  | 1.76  |
| $P_{ex}(KL^2)/10^4$ (k-in. <sup>2</sup> ) | 54100 | 47200 | 40900 | 35500 | 30600 | 26700 |
| $P_{ey}(KL^2)/10^4$ (k-in. <sup>2</sup> ) | 16900 | 14800 | 13000 | 11400 | 9870  | 8620  |

LRFD

 $\Omega_c = 1.67$  $\phi_c = 0.90$ 

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Table 4-1 (continued)  
Available Strength in  
Axial Compression, kips  
W Shapes

 $F_y = 50$  ksi

W12

| Shape                                                                     |    | W12x           |              |                |              |                |              |                |              |                |              |
|---------------------------------------------------------------------------|----|----------------|--------------|----------------|--------------|----------------|--------------|----------------|--------------|----------------|--------------|
| Wt/ft                                                                     |    | 96             |              | 87             |              | 79             |              | 72             |              | 65             |              |
| Design                                                                    |    | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ |
|                                                                           |    | LRFD           | LRFD         | LRFD           | LRFD         | LRFD           | LRFD         | LRFD           | LRFD         | LRFD           | LRFD         |
| Effective length $KL$ (ft) with respect to least radius of gyration $r_y$ | 0  | 1270           | 1150         | 1040           | 951          | 859            |              |                |              |                |              |
|                                                                           | 6  | 1220           | 1110         | 1000           | 913          | 824            |              |                |              |                |              |
|                                                                           | 7  | 1200           | 1090         | 987            | 899          | 811            |              |                |              |                |              |
|                                                                           | 8  | 1180           | 1070         | 971            | 884          | 798            |              |                |              |                |              |
|                                                                           | 9  | 1160           | 1050         | 952            | 867          | 782            |              |                |              |                |              |
|                                                                           | 10 | 1140           | 1030         | 932            | 849          | 765            |              |                |              |                |              |
|                                                                           | 11 | 1110           | 1010         | 910            | 828          | 747            |              |                |              |                |              |
|                                                                           | 12 | 1080           | 980          | 887            | 807          | 727            |              |                |              |                |              |
|                                                                           | 13 | 1050           | 953          | 862            | 784          | 706            |              |                |              |                |              |
|                                                                           | 14 | 1020           | 924          | 836            | 761          | 685            |              |                |              |                |              |
|                                                                           | 15 | 990            | 895          | 809            | 736          | 662            |              |                |              |                |              |
|                                                                           | 16 | 957            | 864          | 781            | 710          | 639            |              |                |              |                |              |
|                                                                           | 17 | 923            | 833          | 752            | 684          | 615            |              |                |              |                |              |
|                                                                           | 18 | 888            | 801          | 723            | 657          | 591            |              |                |              |                |              |
|                                                                           | 19 | 852            | 769          | 694            | 630          | 566            |              |                |              |                |              |
|                                                                           | 20 | 816            | 736          | 664            | 603          | 541            |              |                |              |                |              |
|                                                                           | 22 | 744            | 670          | 603            | 548          | 491            |              |                |              |                |              |
|                                                                           | 24 | 672            | 605          | 544            | 493          | 442            |              |                |              |                |              |
|                                                                           | 26 | 602            | 541          | 486            | 440          | 393            |              |                |              |                |              |
|                                                                           | 28 | 534            | 479          | 430            | 389          | 347            |              |                |              |                |              |
|                                                                           | 30 | 469            | 420          | 376            | 340          | 303            |              |                |              |                |              |
|                                                                           | 32 | 412            | 369          | 331            | 299          | 267            |              |                |              |                |              |
|                                                                           | 34 | 365            | 327          | 293            | 265          | 236            |              |                |              |                |              |
|                                                                           | 36 | 326            | 292          | 261            | 236          | 211            |              |                |              |                |              |
|                                                                           | 38 | 292            | 262          | 234            | 212          | 189            |              |                |              |                |              |
|                                                                           | 40 | 264            | 236          | 212            | 191          | 171            |              |                |              |                |              |

## Properties

|                                           |       |       |       |       |       |
|-------------------------------------------|-------|-------|-------|-------|-------|
| $P_{wo}$ (kips)                           | 206   | 181   | 157   | 136   | 117   |
| $P_{wi}$ (kips/in.)                       | 27.5  | 25.8  | 23.5  | 21.5  | 19.5  |
| $P_{wb}$ (kips)                           | 445   | 366   | 278   | 213   | 159   |
| $P_{wb}$ (kips)                           | 228   | 185   | 152   | 126   | 103   |
| $L_p$ (ft)                                | 10.9  | 10.8  | 10.8  | 10.7  | 11.9  |
| $L_r$ (ft)                                | 46.6  | 43.0  | 39.9  | 37.4  | 35.1  |
| $A_g$ (in. <sup>2</sup> )                 | 28.2  | 25.6  | 23.2  | 21.1  | 19.1  |
| $I_x$ (in. <sup>4</sup> )                 | 833   | 740   | 662   | 597   | 533   |
| $I_y$ (in. <sup>4</sup> )                 | 270   | 241   | 216   | 195   | 174   |
| $r_x$ (in.)                               | 3.09  | 3.07  | 3.05  | 3.04  | 3.02  |
| Ratio $r_x/r_y$                           | 1.76  | 1.75  | 1.75  | 1.75  | 1.75  |
| $P_{ex}(KL^2)/10^4$ (k-in. <sup>2</sup> ) | 23800 | 21200 | 18900 | 17100 | 15300 |
| $P_{ey}(KL^2)/10^4$ (k-in. <sup>2</sup> ) | 7730  | 6900  | 6180  | 5580  | 4980  |

LRFD

 $\Omega_c = 1.67$  $\phi_c = 0.90$ 

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**5**  
COMPOSITE  
PIPE 3 1/2—PIPE 3

**Table 4-20 (continued)**  
**Available Strength in**  
**Axial Compression, kips**  
**Concrete Filled Pipe**

$F_y = 35 \text{ ksi}$   
 $f'_c = 5 \text{ ksi}$

| Shape                      |    | Pipe 3 1/2     |              | Pipe 3         |              |                |              |
|----------------------------|----|----------------|--------------|----------------|--------------|----------------|--------------|
|                            |    | XS             | Std          | XXS            | XS           | Std            |              |
| Wall Thickness, in.        |    | 0.296          | 0.211        | 0.559          | 0.280        | 0.201          |              |
| Steel Wt/ft                |    | 12.5           | 9.12         | 18.6           | 10.3         | 7.58           |              |
| Design                     |    | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ |
|                            |    | LRFD           | LRFD         | LRFD           | LRFD         | LRFD           | LRFD         |
| Effective length (KL) (ft) | 0  | 123            | 102          | 151            | 98.5         | 81.5           |              |
|                            | 6  | 102            | 84.4         | 117            | 77.5         | 63.5           |              |
|                            | 7  | 95.6           | 78.8         | 107            | 71.0         | 58.5           |              |
|                            | 8  | 88.5           | 72.9         | 96.1           | 64.3         | 52.5           |              |
|                            | 9  | 81.2           | 66.8         | 85.1           | 57.4         | 47.5           |              |
|                            | 10 | 73.6           | 60.5         | 74.4           | 50.5         | 41.5           |              |
|                            | 11 | 66.1           | 54.2         | 64.1           | 43.9         | 36.0           |              |
|                            | 12 | 58.8           | 48.1         | 54.3           | 37.6         | 30.5           |              |
|                            | 13 | 51.7           | 42.3         | 46.3           | 32.1         | 26.2           |              |
|                            | 14 | 45.0           | 36.7         | 39.9           | 27.6         | 22.5           |              |
|                            | 15 | 39.2           | 31.9         | 34.8           | 24.1         | 19.7           |              |
|                            | 16 | 34.4           | 28.1         | 30.5           | 21.2         | 17.3           |              |
|                            | 17 | 30.5           | 24.9         | 27.1           | 18.7         | 15.3           |              |
|                            | 18 | 27.2           | 22.2         |                | 16.7         | 13.7           |              |
|                            | 19 | 24.4           | 19.9         |                | 15.0         | 12.3           |              |
|                            | 20 | 22.0           | 18.0         |                |              |                |              |
|                            | 21 | 20.0           | 16.3         |                |              |                |              |
|                            | 22 |                | 14.8         |                |              |                |              |

**Properties**

|                                       |                                                               |      |      |      |      |
|---------------------------------------|---------------------------------------------------------------|------|------|------|------|
| $\phi_b M_n$ kip-ft                   | 10.7                                                          | 7.96 | 12.8 | 7.64 | 5.75 |
| $P_o(KL)^2/10^4$ kip-in. <sup>2</sup> | 193                                                           | 157  | 171  | 119  | 97.1 |
| LRFD                                  | Note: Heavy line indicates KL/r equal to or greater than 200. |      |      |      |      |
| $\Omega_c = 2.00$                     | $\phi_c = 0.75$                                               |      |      |      |      |

**Table 4-21**  
**Stiffness Reduction Factor**

$\tau_a$

| LRFD | $F_y$ , ksi |      |        |      |        |      |        |      |      |        |
|------|-------------|------|--------|------|--------|------|--------|------|------|--------|
|      | $P_u/A_g$   | 35   | 38     | 42   | 46     | 50   |        |      |      |        |
|      |             | LRFD | LRFD   | LRFD | LRFD   | LRFD | LRFD   | LRFD | LRFD | LRFD   |
| 45   |             | —    | —      | —    | —      | —    | —      | —    | —    | —      |
| 44   |             | —    | —      | —    | —      | —    | —      | —    | —    | 0.0599 |
| 43   |             | —    | —      | —    | —      | —    | —      | —    | —    | 0.118  |
| 42   |             | —    | —      | —    | —      | —    | —      | —    | —    | 0.175  |
| 41   |             | —    | —      | —    | —      | —    | 0.0262 | —    | —    | 0.231  |
| 40   |             | —    | —      | —    | —      | —    | 0.0905 | —    | —    | 0.285  |
| 39   |             | —    | —      | —    | —      | —    | 0.153  | —    | —    | 0.338  |
| 38   |             | —    | —      | —    | —      | —    | 0.214  | —    | —    | 0.389  |
| 37   |             | —    | —      | —    | 0.0570 | —    | 0.274  | —    | —    | 0.438  |
| 36   |             | —    | —      | —    | 0.127  | —    | 0.331  | —    | —    | 0.486  |
| 35   |             | —    | —      | —    | 0.194  | —    | 0.387  | —    | —    | 0.532  |
| 34   |             | —    | —      | —    | 0.260  | —    | 0.441  | —    | —    | 0.577  |
| 33   |             | —    | —      | —    | 0.323  | —    | 0.492  | —    | —    | 0.620  |
| 32   |             | —    | 0.0334 | —    | 0.384  | —    | 0.542  | —    | —    | 0.660  |
| 31   | 0.0429      | —    | 0.115  | —    | 0.443  | —    | 0.590  | —    | —    | 0.699  |
| 30   | 0.127       | —    | 0.194  | —    | 0.500  | —    | 0.636  | —    | —    | 0.736  |
| 29   | 0.207       | —    | 0.270  | —    | 0.554  | —    | 0.679  | —    | —    | 0.771  |
| 28   | 0.285       | —    | 0.344  | —    | 0.606  | —    | 0.720  | —    | —    | 0.804  |
| 27   | 0.360       | —    | 0.414  | —    | 0.655  | —    | 0.759  | —    | —    | 0.835  |
| 26   | 0.431       | —    | 0.481  | —    | 0.701  | —    | 0.796  | —    | —    | 0.863  |
| 25   | 0.500       | —    | 0.545  | —    | 0.745  | —    | 0.830  | —    | —    | 0.890  |
| 24   | 0.564       | —    | 0.606  | —    | 0.786  | —    | 0.861  | —    | —    | 0.913  |
| 23   | 0.626       | —    | 0.663  | —    | 0.823  | —    | 0.890  | —    | —    | 0.934  |
| 22   | 0.683       | —    | 0.716  | —    | 0.858  | —    | 0.915  | —    | —    | 0.953  |
| 21   | 0.736       | —    | 0.766  | —    | 0.890  | —    | 0.938  | —    | —    | 0.969  |
| 20   | 0.786       | —    | 0.811  | —    | 0.917  | —    | 0.957  | —    | —    | 0.982  |
| 19   | 0.831       | —    | 0.853  | —    | 0.942  | —    | 0.974  | —    | —    | 0.992  |
| 18   | 0.871       | —    | 0.890  | —    | 0.962  | —    | 0.986  | —    | —    | 0.998  |
| 17   | 0.907       | —    | 0.922  | —    | 0.979  | —    | 0.996  | —    | —    | 1.00   |
| 16   | 0.937       | —    | 0.949  | —    | 0.991  | —    | 1.00   | —    | —    |        |
| 15   | 0.962       | —    | 0.971  | —    | 0.999  | —    |        | —    | —    |        |
| 14   | 0.982       | —    | 0.988  | —    | 1.00   | —    |        | —    | —    |        |
| 13   | 0.995       | —    | 0.998  | —    |        | —    |        | —    | —    |        |
| 12   | 1.00        | —    | 1.00   | —    |        | —    |        | —    | —    |        |
| 11   |             | —    |        | —    |        | —    |        | —    | —    |        |
| 10   |             | —    |        | —    |        | —    |        | —    | —    |        |
| 9    |             | —    |        | —    |        | —    |        | —    | —    |        |
| 8    |             | —    |        | —    |        | —    |        | —    | —    |        |
| 7    |             | —    |        | —    |        | —    |        | —    | —    |        |
| 6    |             | —    |        | —    |        | —    |        | —    | —    |        |
| 5    |             | —    |        | —    |        | —    |        | —    | —    |        |

Indicates stiffness reduction factor is not applicable because the required strength exceeds the available strength for  $KL/r = 0$ .

in real structures. Therefore, adjustments are required when these assumptions are violated and the alignment charts are still to be used. Adjustments for common design conditions that apply to both sidesway conditions are:

1. To account for inelasticity in columns, replace  $(E_c I_c)$  with  $\tau_a(E_c I_c)$  for columns in the expression for  $G_A$  and  $G_B$ . The stiffness reduction factor,  $\tau_a$ , is discussed later in this section.
2. For girders containing significant axial load, multiply the  $(EI/L)_g$  by factor  $(1-Q/Q_{cr})$  where  $Q$  is the axial load in the girder and  $Q_{cr}$  is in-plane buckling load of the girder based on  $K = 1.0$ .

For sidesway uninhibited frames, these adjustments for different beam end conditions may be made:

1. If the far end of a girder is fixed, multiply the  $(EI/L)_g$  of the member by 2.
2. If the far end of the girder is pinned, multiply the  $(EI/L)_g$  of the member by 1.5.

For sidesway uninhibited frames and girders with different boundary conditions, the modified girder length,  $L'_g$ , should be used in place of the actual girder length.

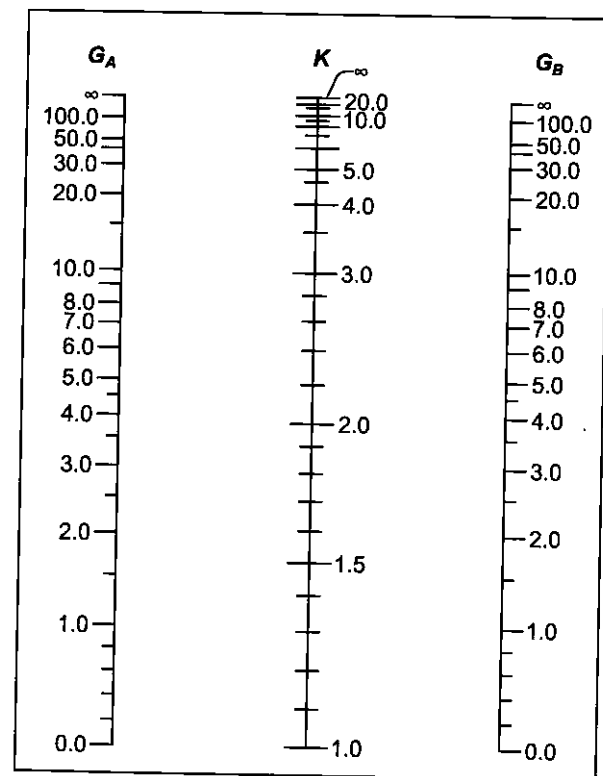


Fig. C-C2.4. Alignment chart—sidesway uninhibited (moment frame).

where

$$L'_g = L_g (2 - M_F/M_N)$$

$M_F$  is the far end girder moment and  $M_N$  is the near end girder moment from a first-order lateral analysis of the frame. The ratio of the two moments is positive if the girder is in reverse curvature. If  $M_F/M_N$  is more than 2.0, then  $L'_g$  becomes negative, in which case  $G$  is negative and the alignment chart equation must be used.

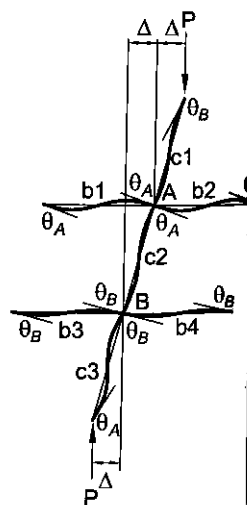
1. If the far end of a girder is fixed, multiply the  $(EI/L)_g$  of the member by  $2/3$ .
2. If the far end of the girder is pinned, multiply the  $(EI/L)_g$  of the member by 0.5.

One important assumption in the development of the alignment charts is that all beam-column connections are fully restrained (FR connections). As seen above, when the far end of a beam does not have an FR connection that behaves as assumed, an adjustment must be made. When a beam connection at the column under consideration is a shear only connection—that is, there is no moment—then that beam can not participate in the restraint of the column and it cannot be considered in the  $\Sigma(EI/L)_g$  term of the equation for  $G$ . Only FR connections can be used directly in the determination of  $G$ . PR connections with a documented moment-rotation response can be utilized, but the  $(EI/L)_g$  of each beam must be adjusted to account for the connection flexibility. The ASCE Task Committee on Effective Length (ASCE, 1997) provides a detailed discussion of frame stability with PR connections.

**Amplified First-Order Elastic Analysis (Section C2.1b).** In this application of the effective length factor,  $K$  is used in the determination of the elastic critical buckling load,  $P_{e1}$ , for a member, or  $\Sigma P_{e2}$ , for a building story. These elastic critical buckling loads are then used for calculation of the corresponding amplification factors  $B_1$  and  $B_2$ .

$B_1$  is used to estimate the  $P-\delta$  effects on the nonsway moments,  $M_{nt}$ , in axially loaded members.  $K_1$  is calculated in the plane of bending on the basis of no translation of the ends of the member and is normally set to 1.0, unless a smaller value is justified on the basis of analysis. There are also  $P-\delta$  effects on the sway moments,  $M_{lt}$ , as explained previously in the discussion of Equation C2-6b.

$B_2$  is used to determine the  $P-\Delta$  effect on the various components of moment, braced and/or combined framing systems.  $K_2$  is calculated in the plane of bending through a sidesway buckling analysis.  $K_2$  may be determined from the sidesway uninhibited alignment chart, Figure C-C2.4, without any correction for story buckling discussed later.  $\Sigma P_{e2}$  from the lateral load resisting columns with  $K_2$  calculated in this way is an accurate estimate of the story elastic sidesway buckling strength. The contribution to the story sidesway buckling strength from leaning columns is zero, and therefore, these columns are not included in the summation in Equation C2-6a. However, the total story vertical load, including all columns in the story, is used for  $\alpha \Sigma P_{nt}$  in Equation C2-3.





#### E4. COMPRESSIVE STRENGTH FOR TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

This section applies to singly symmetric and unsymmetric members, and certain doubly symmetric members, such as cruciform or built-up columns with compact and noncompact sections, as defined in Section B4 for uniformly compressed elements. These provisions are not required for single angles, which are covered in Section E5.

The nominal compressive strength,  $P_n$ , shall be determined based on the limit states of flexural-torsional and torsional buckling, as follows:

$$P_n = F_{cr} A_g \quad (E4-1)$$

(a) For double-angle and tee-shaped compression members:

$$F_{cr} = \left( \frac{F_{cry} + F_{crz}}{2H} \right) \left[ 1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}} \right] \quad (E4-2)$$

where  $F_{cry}$  is taken as  $F_{cr}$  from Equation E3-2 or E3-3, for flexural buckling about the y-axis of symmetry and  $\frac{KL}{r} = \frac{KL}{r_y}$ , and

$$F_{crz} = \frac{GJ}{A_g \bar{r}_o^2} \quad (E4-3)$$

(b) For all other cases,  $F_{cr}$  shall be determined according to Equation E3-2 or E3-3, using the torsional or flexural-torsional elastic buckling stress,  $F_e$ , determined as follows:

(i) For doubly symmetric members:

$$F_e = \left[ \frac{\pi^2 EC_w}{(K_z L)^2} + GJ \right] \frac{1}{I_x + I_y} \quad (E4-4)$$

(ii) For singly symmetric members where y is the axis of symmetry:

$$F_e = \left( \frac{F_{ey} + F_{ez}}{2H} \right) \left[ 1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}} \right] \quad (E4-5)$$

(iii) For unsymmetric members,  $F_e$  is the lowest root of the cubic equation:

$$(F_e - F_{ex})(F_e - F_{ey})(F_e - F_{ez}) - F_e^2(F_e - F_{ey}) \left( \frac{x_o}{\bar{r}_o} \right)^2 - F_e^2(F_e - F_{ex}) \left( \frac{y_o}{\bar{r}_o} \right)^2 = 0 \quad (E4-6)$$

where

$A_g$  = gross area of member, in.<sup>2</sup> (mm<sup>2</sup>)

$C_w$  = warping constant, in.<sup>6</sup> (mm<sup>6</sup>)

$$\bar{r}_o^2 = x_o^2 + y_o^2 + \frac{I_x + I_y}{A_g} \quad (E4-7)$$

$$H = 1 - \frac{x_o^2 + y_o^2}{\bar{r}_o^2} \quad (E4-8)$$

$$F_{ex} = \frac{\pi^2 E}{\left( \frac{K_x L}{r_x} \right)^2} \quad (E4-9)$$

$$F_{ey} = \frac{\pi^2 E}{\left( \frac{K_y L}{r_y} \right)^2} \quad (E4-10)$$

$$F_{ez} = \left( \frac{\pi^2 EC_w}{(K_z L)^2} + GJ \right) \frac{1}{A_g \bar{r}_o^2} \quad (E4-11)$$

$G$  = shear modulus of elasticity of steel = 11,200 ksi (77 200 MPa)

$I_x, I_y$  = moment of inertia about the principal axes, in.<sup>4</sup> (mm<sup>4</sup>)

$J$  = torsional constant, in.<sup>4</sup> (mm<sup>4</sup>)

$K_z$  = effective length factor for torsional buckling

$x_o, y_o$  = coordinates of shear center with respect to the centroid, in. (mm)

$\bar{r}_o$  = polar radius of gyration about the shear center, in. (mm)

$r_y$  = radius of gyration about y-axis, in. (mm)

**User Note:** For doubly symmetric I-shaped sections,  $C_w$  may be taken as  $I_y h_o^2/4$ , where  $h_o$  is the distance between flange centroids, in lieu of a more precise analysis. For tees and double angles, omit term with  $C_w$  when computing  $F_{ez}$  and take  $x_o$  as 0.

#### SINGLE ANGLE COMPRESSION MEMBERS

The nominal compressive strength,  $P_n$ , of single angle members shall be determined in accordance with Section E3 or Section E7, as appropriate, for axially loaded members, as well as those subject to the slenderness modification of Section E5(a) or E5(b), provided the members meet the criteria imposed.

The effects of eccentricity on single angle members are permitted to be neglected when the members are evaluated as axially loaded compression members using one of the effective slenderness ratios specified below, provided that: (1) members are loaded at the ends in compression through the same one leg; (2) members are attached by welding or by minimum two-bolt connections; and (3) there are no intermediate transverse loads.