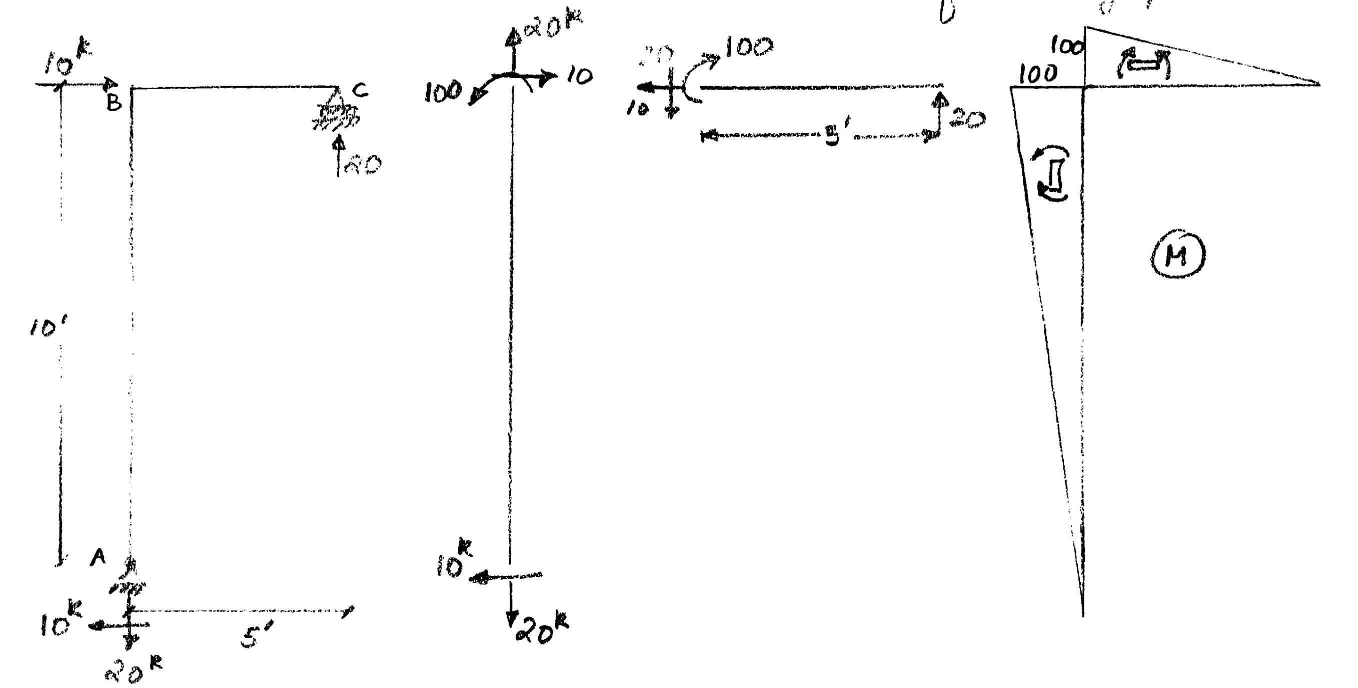
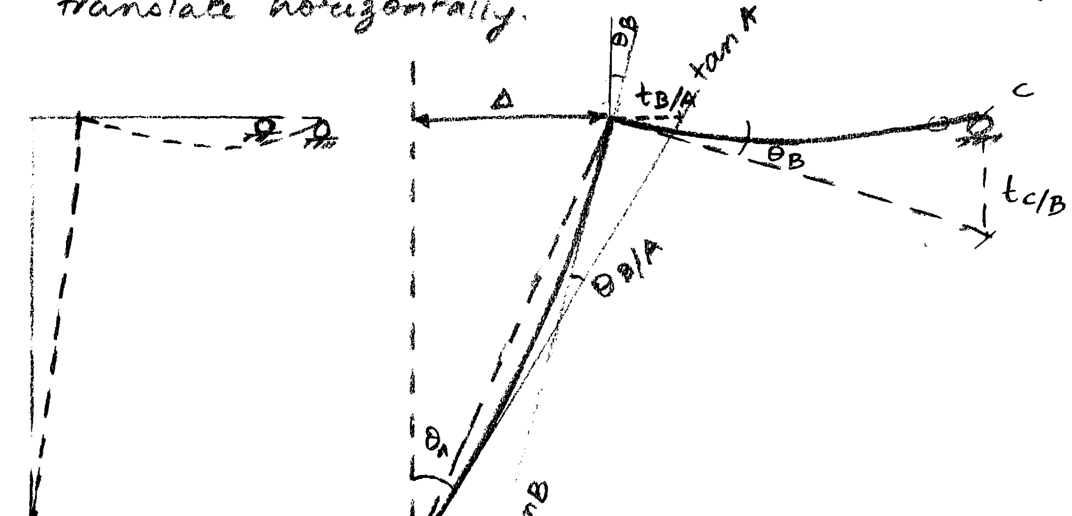
Use moment-area method to calculate the following parameters



Step 1. Sketch the deflected shape of the frame.
Please note that support C is a roller, i.e., free to translate horizontally.



Step 2.

Calculate $t_{C/B}$ and θ_B. What is the sign of θ_B.

$$t_{C/B} = \left(\frac{100}{EI} \times 5' \times \frac{1}{\cancel{2}}\right) \times \frac{\cancel{2}}{3} \times 5' = \frac{2500}{3EI} \text{ k-ft}^3$$

$$\theta_B = t_{C/B} \times \frac{1}{5'} = \frac{\overset{500}{\cancel{2500}}}{3EI \times \cancel{5}} \text{ k-ft}^2 = \frac{500}{3EI} \text{ k-ft}^2$$

$$\theta_B = -\frac{500}{3EI} \text{ k-ft}^2$$

<u>Step 3.</u> Calculate $\theta_{B/A}$ and θ_A

$$\theta_{B/A} = \frac{1}{2} \times 10' \times \frac{100}{EI} \text{ k-ft} = \frac{500}{EI} \text{ k-ft}^2$$

$$\theta_A = \theta_B - \theta_{B/A} = -\frac{500}{3EI} - \frac{500}{EI} = \frac{-2000}{3EI} \text{ k-ft}^2$$

<u>Step 4.</u> Calculate $t_{B/A}$

$$t_{B/A} = \frac{1}{2} \times 10' \times \frac{100}{EI} \text{ k-ft} \times \frac{1}{3} \times 10' = \frac{5000}{3EI}$$

<u>Step 5</u> Calculate frame sway Δ

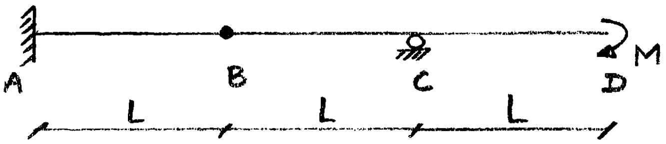
$$\Delta + t_{B/A} = \theta_A \times 10'$$

sign dropped because geometry being used

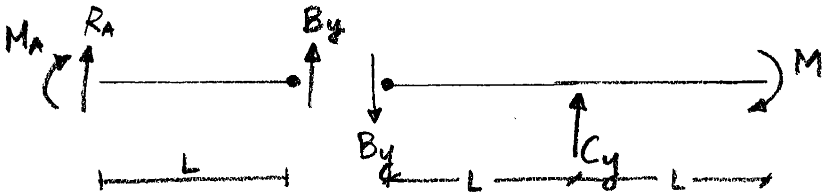
$$= \frac{\cancel{-}2000}{3EI} \times 10' = \frac{20000}{3EI}$$

$$\therefore \Delta = \frac{20000}{3EI} - \frac{5000}{3EI} = \frac{15000}{3EI} = \frac{5000}{EI}$$

PROBLEM 2.



Step 1. Calculate reactions



$+\uparrow \sum F_y = 0$

$\therefore R_A + B_y = 0$

$\therefore R_A = -B_y$

$R_A = -\frac{M}{L}$

$\circlearrowleft + \sum M_A = 0$

$\therefore -M_A + B_y \cdot L = 0$

$\therefore M_A = B_y \cdot L$

$M_A = M$

$\circlearrowleft + \sum M_B = 0$

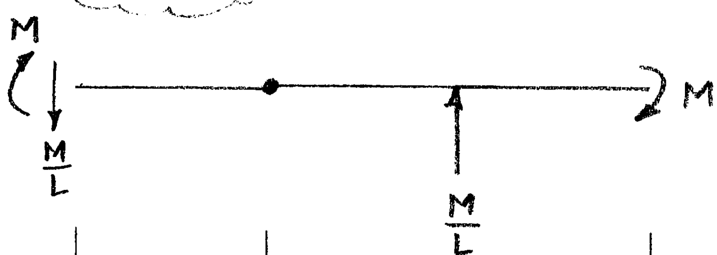
$C_y \cdot L - M = 0$

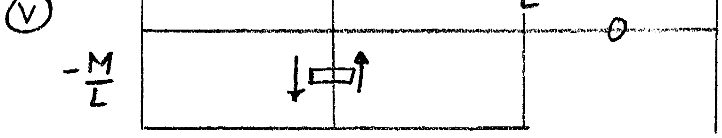
$\therefore C_y = \frac{M}{L}$

$+\uparrow \sum F_y = 0$

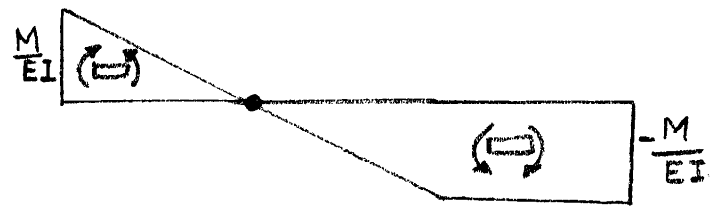
$\therefore -B_y + C_y = 0$

$\therefore B_y = C_y = \frac{M}{L}$

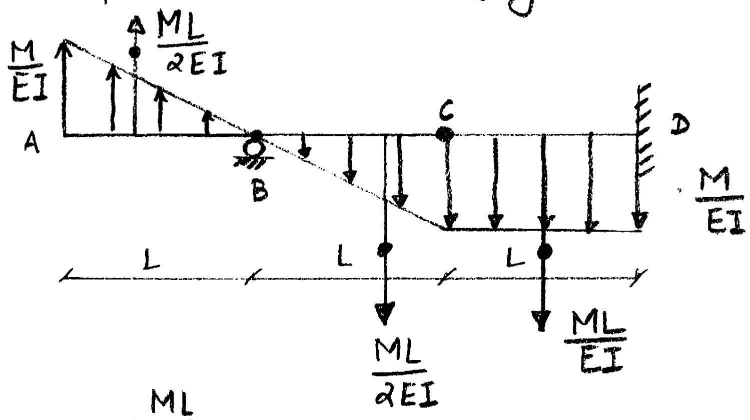


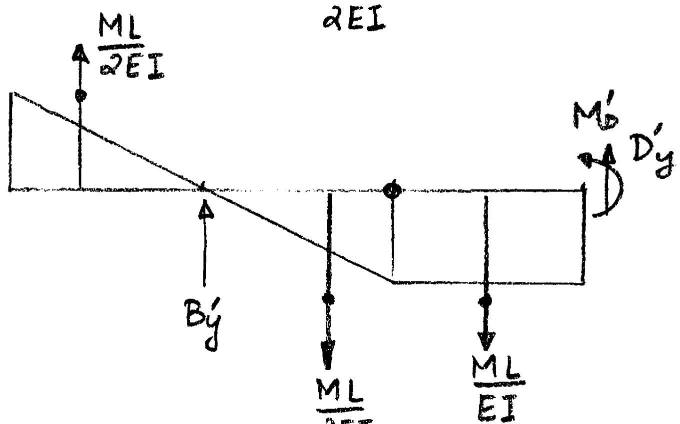


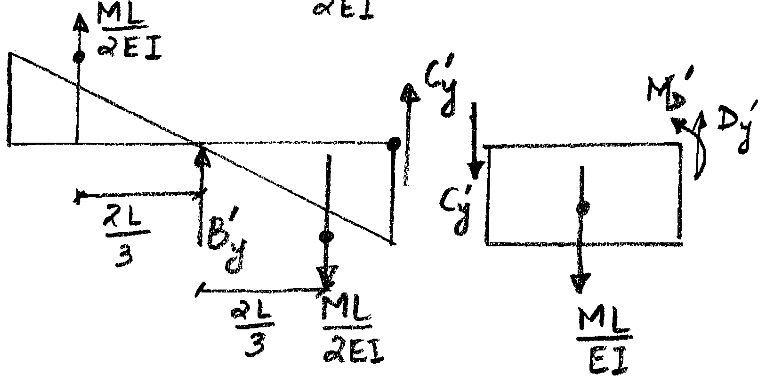
Step 2: Develop the $\frac{M}{EI}$ diagram for the beam



Step 3 Develop conjugate beam model







$\circlearrowleft + \sum M_c' = 0$

$$\therefore \frac{ML}{2EI} \times \frac{L}{3} - B_y' \times L - \frac{ML}{2EI} \times \left(L + \frac{2L}{3}\right) = 0$$

$$\therefore B_y' L = \frac{ML^2}{6EI} - \frac{5ML^2}{6EI}$$

$$\therefore B_y' = -\frac{4ML}{6EI} = -\frac{2}{3}\frac{ML}{EI}$$

$\uparrow + \sum F_y' = 0$

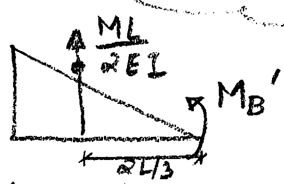
$$\therefore \cancel{\frac{ML}{2EI}} + B_y' - \cancel{\frac{ML}{2EI}} + C_y' = 0$$

$$\therefore C_y' = -B_y'$$

$$\therefore C_y' = \frac{2}{3}\frac{ML}{EI} \uparrow$$

$$\therefore V_c' = -\frac{2}{3}\frac{ML}{EI}$$

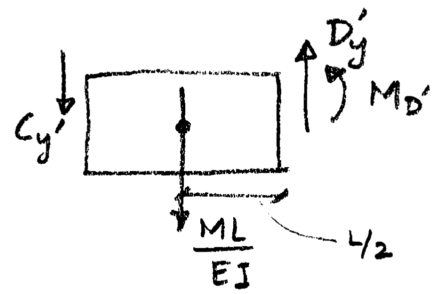
$$\therefore \theta_c = -\frac{2}{3}\frac{ML}{EI}$$



$\circlearrowleft + \sum M_B' = 0 \quad \therefore M_B' - \frac{ML}{2EI} \times \frac{2L}{3} = 0$

$$\therefore M_B' = \frac{ML^2}{3EI}$$

$$\therefore \Delta_B = \frac{ML^2}{3EI} \uparrow$$



$\circlearrowleft + \sum M_D' = 0$

$\therefore M_D' - \frac{ML^2}{2EI} + C_y' \times L = 0$

$\therefore M_D' - \frac{ML^2}{2EI} + \frac{2}{3}\frac{ML^2}{EI} = 0$

$\therefore M_D' = \frac{3-4}{6EI} ML^2 = -\frac{ML^2}{6EI}$

$\therefore \Delta_D = -\frac{ML^2}{6EI} \downarrow$

$+\uparrow \sum F_y = 0 \quad \therefore -C_y' + D_y' - \frac{ML}{EI} = 0$

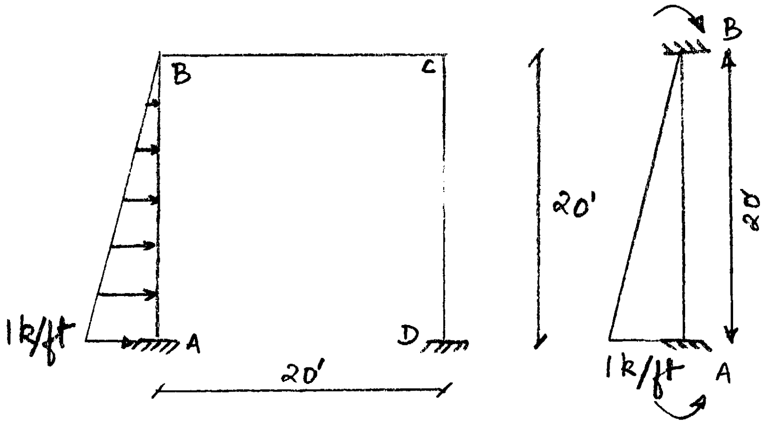
$\therefore D_y' = C_y' + \frac{ML}{EI} = \frac{2}{3}\frac{ML}{EI} + \frac{ML}{EI} = \frac{5ML}{3EI}$

$\therefore D_y' = \frac{5ML}{3EI} \uparrow$

$\therefore V_D' = -\frac{5}{3}\frac{ML}{EI}$

$\therefore \theta_D = -\frac{5}{3}\frac{ML}{EI}.$

PROBLEM NO. 3



$$FEM\Big|_{BA} = +\frac{wL^2}{30} = \frac{1\times 20^2}{30} = \frac{40\cancel{0}}{3\cancel{0}}$$
$$= +13.33$$

$$FEM\Big|_{AB} = -\frac{wL^2}{20}$$
$$= -\frac{1\times 20^2}{20} = -20\ k\text{-}ft$$

$$M_{AB} = -20 + 2E\times\frac{I}{20}\left(2\cancel{\theta_A}^{\,0} + \theta_B - 3\psi_{AB}\right) \quad \text{———} \quad (1)$$

$$M_{BA} = +13.33 + 2E\frac{I}{20}\left(\cancel{\theta_A}^{\,0} + 2\theta_B - 3\psi_{AB}\right) \quad \text{———} \quad (2)$$

$$M_{BC} = 0 + 2E\frac{I}{20}\left(2\theta_B + \theta_C\right) \quad \text{———} \quad (3)$$

$$\psi_{AB} = \psi_{CD} = \frac{\Delta}{20'} \quad \text{———} \quad (6)$$

$$M_{CB} = 0 + 2E\frac{I}{20}\left(\theta_B + 2\theta_C\right) \quad \text{———} \quad (4)$$

$$M_{CD} = 0 + 3E\frac{I}{20}\left(\theta_C - \psi_{CD}\right) \quad \text{———} \quad (5)$$

$$M_{BA} + M_{BC} = 0 \quad \text{———} \quad (7)$$

$$\therefore +13.33 + \frac{2EI}{20}\left(2\theta_B - 3\psi\right) + \frac{2EI}{20}\left(2\theta_B + \theta_C\right) = 0$$

$$\therefore +13.33 + \frac{2EI}{20}\left(4\theta_B + \theta_C - 3\psi\right) = 0 \quad \text{———} \quad (8)$$

$$M_{CB} + M_{CD} = 0 \quad \text{———} \quad (9)$$

$$\frac{2EI}{20}\left(\theta_B + 2\theta_C\right) + \frac{3EI}{20}\left(\theta_C - \psi\right) = 0$$

$$\therefore \frac{EI}{20}\left(2\theta_B + 4\theta_C + 3\theta_C - 3\psi\right) = 0$$

$$\therefore \quad 2\theta_B + 7\theta_c - 3\psi = 0$$

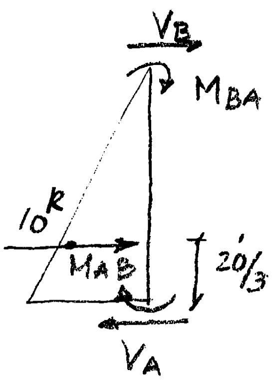
$$\therefore \quad \psi = \frac{2}{3}\theta_B + \frac{7}{3}\theta_c \quad \text{——} \quad (10)$$

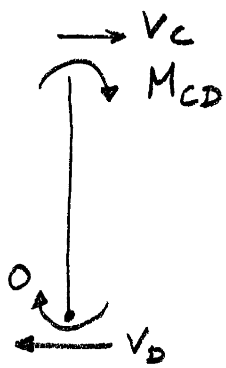
Subs (10) in (8)

$$\therefore \quad +13.33 + \frac{2EI}{20}\left(4\theta_B + \theta_c - 2\theta_B - 7\theta_c\right) = 0$$

$$\therefore \quad +13.33 + \frac{2EI}{20}\left(2\theta_B - 6\theta_c\right) = 0$$

$$\therefore \quad +13.33 + \frac{4EI}{20}\left(\theta_B - 3\theta_c\right) = 0 \quad \text{——} \quad (11)$$





$$\circlearrowleft + \ \sum M_A = 0$$

$$-M_{AB} - M_{BA} - 10 \times \frac{20}{3} - V_B \times 20' = 0$$

$$\therefore \quad V_B = -\frac{M_{AB} + M_{BA}}{20'} - \frac{\overset{10}{\cancel{200}}}{3 \cdot \cancel{20}}$$

$$\overset{+}{\rightarrow} \sum F_x = 0$$

$$\therefore \quad 10 + V_B - V_A = 0$$

$$\therefore \quad V_A = 10 + V_B$$

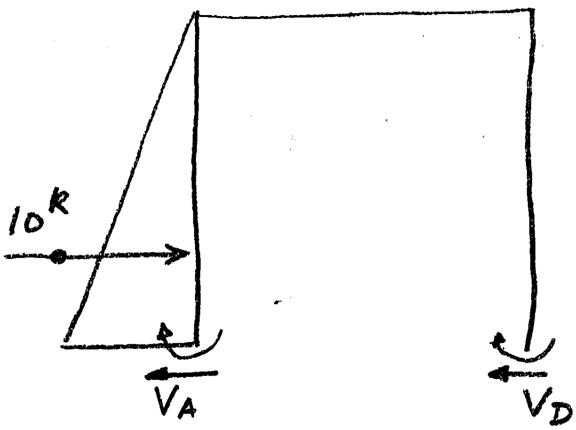
$$\boxed{V_A = \frac{20}{3} - \frac{M_{AB} + M_{BA}}{20'}} \quad \text{——} \quad (12)$$

$$\circlearrowleft + \ \sum M_D = 0$$

$$\therefore \quad 0 - M_{CD} - V_c \times 20' = 0$$

$$\therefore \quad V_c = -M_{CD}/20'$$

$$\boxed{V_D = V_c = -\frac{M_{CD}}{20'}} \longrightarrow (13)$$



$$\therefore \quad \sum F_x = 0$$

$$\therefore \quad 10 - V_A - V_D = 0 \qquad \text{——} \quad (14)$$

$$\therefore \quad 10 + \frac{M_{AB} + M_{BA}}{20} - \frac{20}{3} + \frac{M_{CD}}{20} = 0$$

$$\therefore \quad 200 + M_{AB} + M_{BA} + M_{CD} - \frac{400}{3} = 0$$

$$\therefore \quad \frac{200}{3} + M_{AB} + M_{BA} + M_{CD} = 0$$

$$\therefore \quad \frac{200}{3} - 20 + \frac{2EI}{20}\left(\theta_B - 3\psi\right) + 13.33 + \frac{2EI}{20}\left(2\theta_B - 3\psi\right) + \frac{3EI}{20}\left(\theta_c - \psi\right) = 0$$

$$\therefore \quad 60 + \frac{EI}{20}\left(6\theta_B + 3\theta_c - 15\psi\right) = 0$$

$$\therefore \quad 60 + \frac{EI}{20}\left(6\theta_B + 3\theta_c - 5\left(2\theta_B + 7\theta_c\right)\right) = 0$$

$$\therefore \quad 60 + \frac{EI}{20}\left(-4\theta_B - 32\theta_c\right) = 0$$

$$\therefore \quad 60 + \frac{4EI}{20}\left(-\theta_B - 8\theta_c\right) = 0 \qquad \text{——} \quad (15)$$

Solve (11) & (15) simultaneously;

(11) + (15)

$$\therefore \quad 13.33 + \frac{4EI}{20}\left\{\cancel{\theta_B} - 3\theta_c\right\} + 60 + \frac{4EI}{20}\left\{-\cancel{\theta_B} - 8\theta_c\right\} = 0$$

$$\therefore \quad 73.333 + \frac{4EI}{20}\left(-11\,\theta_c\right) = 0$$

$$\therefore \quad \theta_c = \frac{33.333}{EI}$$

Substitute in (11)

$$+13.333 + \frac{\cancel{4}EI}{\cancel{20}_5}\cdot\theta_B - \frac{\overset{3}{\cancel{12}}\cancel{EI}}{\cancel{20}_5} \times \frac{33.333}{\cancel{EI}} = 0$$

$$\theta_B = \frac{33.334}{EI}$$

$$\psi = \frac{2}{3}\theta_B + \frac{7}{3}\theta_c = \;=\; 3\times\theta_c \;=\; \frac{100}{EI}$$

$$M_{AB} = -20 + \frac{\cancel{2EI}}{\cancel{20}_{10}}\left\{\frac{33.333}{\cancel{EI}} - 3\times\frac{100}{\cancel{EI}}\right\}$$

$$M_{AB} = -46.67 \text{ k-ft}$$

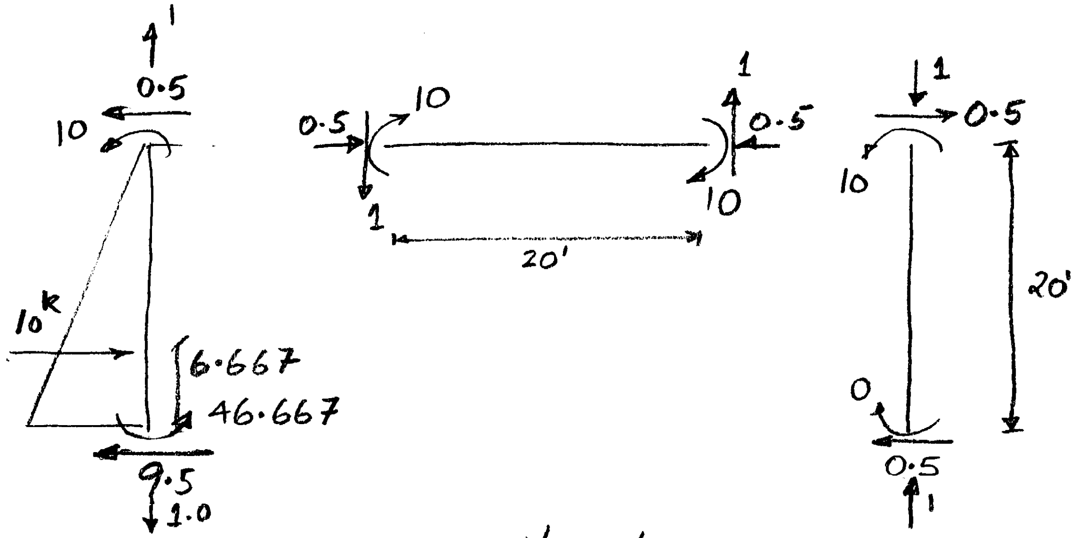
$$M_{CB} = \frac{\cancel{2}EI}{\cancel{20}_{10}}\left(\theta_B + 2\theta_c\right) = \frac{EI}{10}\left(3\times\frac{33.333}{\cancel{EI}}\right) = +10 \text{ k-ft}$$

$$M_{BA} = +13.33 + \frac{\cancel{EI}}{10}\left\{2\times\frac{33.333}{\cancel{EI}} - 3\times\frac{100}{\cancel{EI}}\right\}$$

$$M_{BA} = -10 \text{ k-ft}$$

$$M_{CD} = \frac{3\cancel{EI}}{20}\left(\frac{33.333 - 100}{\cancel{EI}}\right) = -10 \text{ k-ft}$$

$$M_{BC} = \frac{2EI}{20}\left(2\theta_B + \theta_c\right) = \frac{\cancel{EI}}{10}\left(3\times\frac{33.333}{\cancel{EI}}\right) = +10 \text{ k-ft}$$



$46.667 + 10 - 10 \times 6.667 + V_B \times 20' = 0$