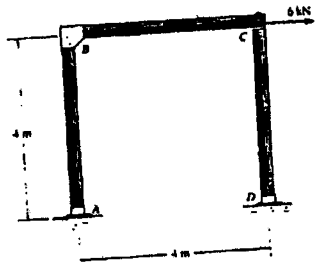
*11-20. The wood frame is subjected to the load of 6 kN. Determine the moments at the fixed joints A, B, and D. The joint at C is pinned. EI is constant.



$\theta_A = \theta_D = \psi_{BC} = 0$

$\psi_{AB} = \psi_{DC} = \psi$

Applying Eqs. 11-8 and 11-10,

$$M_{AB} = \frac{2EI}{4}(\theta_B - 3\psi) + 0$$

$$M_{BA} = \frac{2EI}{4}(2\theta_B - 3\psi) + 0$$

$$M_{BC} = \frac{3EI}{4}(\theta_B) + 0$$

$$M_{DC} = \frac{3EI}{4}(-\psi) + 0$$

Moment equilibrium at B: (1)

$$M_{BA} + M_{BC} = 0$$

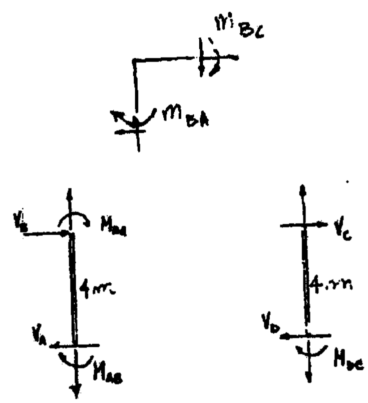
Force equilibrium for entire frame:

$\xrightarrow{+}\Sigma F_x = 0;$ $\quad 6 - V_A - V_D = 0$

From FBDs of members AB and CD:

$\Sigma M_B = 0;$ $\quad V_A = -\left(\frac{M_{AB} + M_{BA}}{4}\right)$

$\Sigma M_D = 0;$ $\quad V_D = \frac{-M_{DC}}{4}$



Thus,

$$6 + \frac{M_{AB} + M_{BA}}{4} + \frac{M_{DC}}{4} = 0 \qquad (2)$$

From Eq.(1):

$$\frac{2EI}{4}(2\theta_B - 3\psi) + \frac{3EI}{4}\theta_B = 0$$

$$\theta_B = \frac{6}{7}\psi \qquad (3)$$

From Eq.(2):

$$6 + \frac{1}{4}\left(\frac{2EI}{4}(\theta_B - 3\psi) + \frac{2EI}{4}(2\theta_B - 3\psi) + \frac{3EI}{4}(-\psi)\right) = 0$$

$$\frac{96}{EI} + 6\theta_B - 15\psi = 0 \qquad (4)$$

Solving Eqs. (3) and (4),

$$\theta_B = \frac{8.348}{EI}$$

$$\psi = \frac{9.739}{EI}$$

Thus:

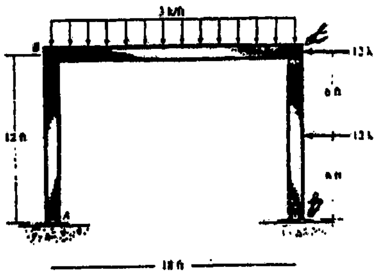
$M_{AB} = -10.4$ kN·m **Ans**

$M_{BA} = -6.26$ kN·m **Ans**

$M_{BC} = 6.26$ kN·m **Ans**

$M_{DC} = -7.30$ kN·m **Ans**

11-21. Determine the moments at the ends of each member. Assume *A* and *D* are pins and *B* and *C* are fixed-connected joints *EI* is the same for all members.



$FEM_{BC} = -\dfrac{wL^2}{12} = -81, \qquad FEM_{CB} = \dfrac{wL^2}{8} = 81$

$FEM_{CD} = -\dfrac{3PL}{16} = -27$

$\psi_{BC} = 0$

$\psi_{AB} = \psi_{CD} = \psi$

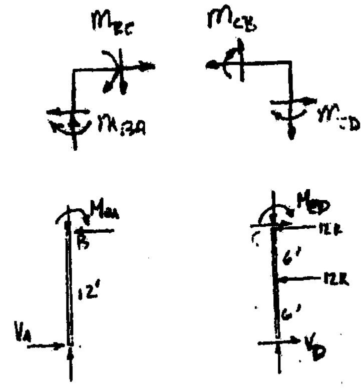
Applying Eqs. 11−8 and 11−10,

$M_{BA} = \dfrac{3EI}{12}(\theta_B + \psi)$

$M_{BC} = \dfrac{2EI}{18}(2\theta_B + \theta_C) - 81$

$M_{CB} = \dfrac{2EI}{18}(2\theta_C + \theta_B) + 81$

$M_{CD} = \dfrac{3EI}{12}(\theta_C + \psi) - 27$



Moment equilibrium at *B* and *C*:

$M_{BA} + M_{BC} = 0; \quad 0.472EI\theta_B + 0.11EI\theta_C + 0.25EI\psi = 81 \qquad (1)$

$M_{CB} + M_{CD} = 0; \quad 0.11EI\theta_B + 0.472EI\theta_C + 0.25EI\psi = -54 \qquad (2)$

From FBDS of *AB* and *DC*:

$+\Sigma M_B = 0; \qquad M_{BA} - 12V_A = 0$

$V_A = \dfrac{M_{BA}}{12}$

$+\Sigma M_C = 0; \qquad M_{CD} + 12(6) - 12V_D = 0$

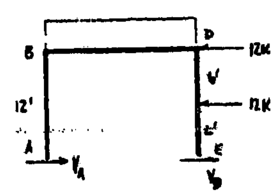
$V_D = \dfrac{M_{CD}}{12} + 6$

From FBD of frame:

$\xrightarrow{+} \Sigma F_x = 0; \qquad V_A + V_D = 24$

$M_{BA} + M_{CD} = 216$

$0.25EI\theta_B + 0.25EI\theta_C + 0.5EI\psi = 243 \qquad (3)$



Solving Eqs. (1),(2) and (3):

$\theta_B = -\dfrac{137.077}{EI}$

$\theta_C = -\dfrac{510.923}{EI}$

$\psi = \dfrac{810}{EI}$

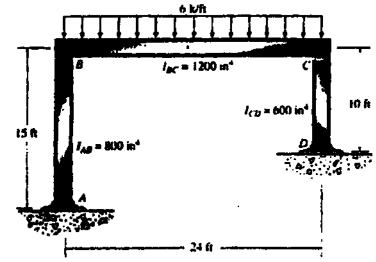
$M_{BA} = 168\ \text{k}\cdot\text{ft}$ **Ans**

$M_{BC} = -168\ \text{k}\cdot\text{ft}$ **Ans**

$M_{CB} = -47.8\ \text{k}\cdot\text{ft}$ **Ans**

$M_{CD} = 47.8\ \text{k}\cdot\text{ft}$ **Ans**

11-22. Determine the moments acting at the ends of each member. Assume the supports at A and D are fixed. The moment of inertia of each member is indicated in the figure. $E = 29(10^3)$ ksi.

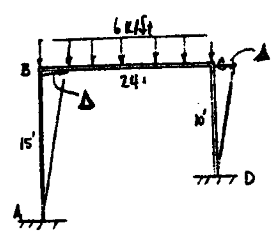


$$FEM_{BC} = -\frac{wL^2}{12} = -288, \qquad FEM_{CB} = \frac{wL^2}{12} = 288$$

$$\theta_A = \theta_D = \psi_{BC} = 0$$

$$\psi_{AB} = \psi = \frac{\Delta}{15}$$

$$\psi_{DC} = \frac{\Delta}{10} = 1.5\psi$$



$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + FEM_N$$

$$M_{AB} = \frac{2E(800)}{15}(\theta_B - 3\psi)$$

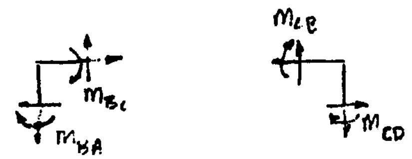
$$M_{BA} = \frac{2E(800)}{15}(2\theta_B - 3\psi)$$

$$M_{BC} = \frac{2E(1200)}{24}(2\theta_B + \theta_C) - 288$$

$$M_{CB} = \frac{2E(1200)}{24}(2\theta_C + \theta_B) + 288$$

$$M_{CD} = \frac{2E(600)}{10}(2\theta_C - 3(1.5)\psi)$$

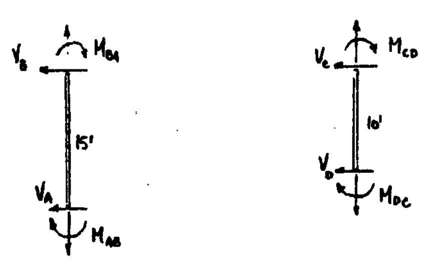
$$M_{DC} = \frac{2E(600)}{10}(\theta_C - 3(1.5)\psi)$$



Moment equilibrium at B and C:

$M_{BA} + M_{BC} = 0; \quad 413.3E\theta_B + 100E\theta_C - 320E\psi = 288$ (1)

$M_{CB} + M_{CD} = 0: \quad 100E\theta_B + 440E\theta_C - 540E\psi = -288$ (2)



From FBDs of members AB and CD,

$\curvearrowleft + \Sigma M_B = 0; \quad M_{BA} + M_{AB} + V_A(15) = 0$

$\curvearrowleft + \Sigma M_C = 0; \quad M_{CD} + M_{DC} + V_D(10) = 0$

For the entire frame,

$\Sigma F_x = 0; \quad V_A + V_D = 0$

Thus,

$$M_{BA} + M_{AB} + 1.5(M_{CD} + M_{DC}) = 0$$

$320E\theta_B + 540E\theta_C - 2260E\psi = 0$ (3)

Solving Eqs. (1),(2) and (3),

$$\theta_B = \frac{0.84589}{E}$$

$$\theta_C = -\frac{0.99016}{E}$$

$$\psi = -\frac{0.11681}{E}$$

Thus,

$M_{AB} = 128$ k·ft **Ans**

$M_{BA} = 218$ k·ft **Ans**

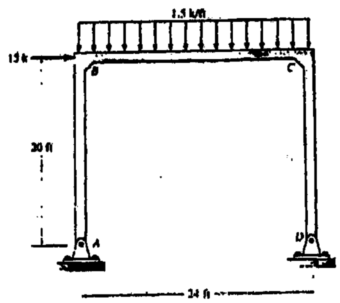
$M_{BC} = -218$ k·ft **Ans**

$M_{CB} = 175$ k·ft **Ans**

$M_{CD} = -175$ k·ft **Ans**

$M_{DC} = -55.7$ k·ft **Ans**

11-23. Determine the moments acting at the ends of each member of the frame. EI is the same for all members.



$$FEM_{BC} = -\frac{wL^2}{12} = -72, \qquad FEM_{CB} = \frac{wL^2}{12} = 72$$

$\psi_{BC} = 0$

$\psi_{AB} = \psi_{DC} = \psi$

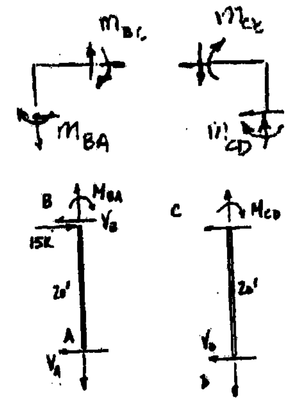
Applying Eqs. 11 − 8 and 11 − 10,

$$M_{BA} = \frac{3EI}{20}(\theta_B - \psi)$$

$$M_{BC} = \frac{2EI}{24}(2\theta_B + \theta_C) - 72$$

$$M_{CB} = \frac{2EI}{24}(2\theta_C + \theta_B) + 72$$

$$M_{CD} = \frac{3EI}{20}(\theta_C - \psi)$$



Moment equilibrium at B and C:

$M_{BA} + M_{BC} = 0;\quad 0.317EI\theta_B + 0.083EI\theta_C - 0.15EI\psi = 72$ (1)

$M_{CB} + M_{CD} = 0;\quad 0.083EI\theta_B + 0.317EI\theta_C - 0.15EI\psi = -72$ (2)

From the FBDs of members AB and CD:

$\curvearrowleft + \Sigma M_B = 0;\quad M_{BA} + 20V_A = 0$

$\curvearrowleft + \Sigma M_C = 0;\quad M_{CD} + 20V_D = 0$

For the frame

$\Sigma F_x = 0;\quad V_A + V_D = 15$

Thus

$$M_{BA} + M_{CD} = -300$$

$0.075EI\theta_B + 0.075EI\theta_C - 0.15EI\psi = -150$ (3)

Solving Eqs. (1),(2) and (3),

$$\theta_B = \frac{908.5714}{EI}$$

$$\theta_C = \frac{291.4285}{EI}$$

$$\psi = \frac{1600}{EI}$$

Hence,

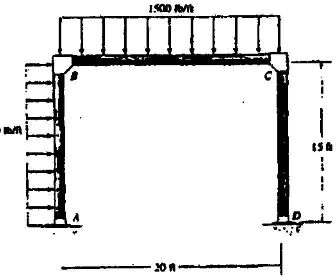
$M_{BA} = -104 \text{ k}\cdot\text{ft}$ **Ans**

$M_{BC} = 104 \text{ k}\cdot\text{ft}$ **Ans**

$M_{CB} = 196 \text{ k}\cdot\text{ft}$ **Ans**

$M_{CD} = -196 \text{ k}\cdot\text{ft}$ **Ans**

*11-24. Determine the moments acting at the ends of each member. EI is the same for all members. Assume all joints are fixed.



$FEM_{AB} = -\frac{wL^2}{12} = -15, \quad FEM_{BA} = \frac{wL^2}{12} = 15$

$FEM_{BC} = -\frac{wL^2}{12} = -50, \quad FEM_{CB} = \frac{wL^2}{12} = 50$

$\theta_A = \theta_D = \psi_{BC} = 0$

$\psi_{AB} = \psi_{CD} = \psi$

$M_N = 2E(\frac{I}{L})(2\theta_N + \theta_F - 3\psi) + FEM_N$

$M_{AB} = \frac{2EI}{15}(\theta_B - 3\psi) - 15$

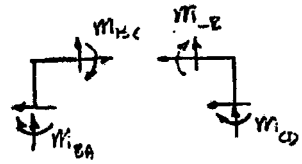
$M_{BA} = \frac{2EI}{15}(2\theta_B - 3\psi) + 15$

$M_{BC} = \frac{2EI}{20}(2\theta_B + \theta_C) - 50$

$M_{CB} = \frac{2EI}{20}(2\theta_C + \theta_B) + 50$

$M_{CD} = \frac{2EI}{15}(2\theta_C - 3\psi)$

$M_{DC} = \frac{2EI}{15}(\theta_C - 3\psi)$



Moment equilibrium at B and C:

$M_{BA} + M_{BC} = 0; \quad 0.4667EI\theta_B + 0.1EI\theta_C - 0.4EI\psi = 35 \quad (1)$

$M_{CB} + M_{CD} = 0; \quad 0.1EI\theta_B + 0.4667EI\theta_C - 0.4EI\psi = -50 \quad (2)$

From FBDs of AB and DC:

$\curvearrowleft + \Sigma M_B = 0; \quad M_{BA} + M_{AB} + V_A(15) - 90 = 0$

$\curvearrowleft + \Sigma M_C = 0; \quad M_{CD} + M_{DC} + V_D(15) = 0$

From FBD of frame,

$V_A + V_D = 12$

Thus,

$M_{BA} + M_{AB} + M_{CD} + M_{DC} = -90$

Substituting,

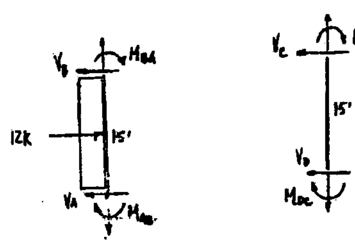
$0.4EI\theta_B + 0.4EI\theta_C - 1.6EI\psi = -90 \quad (3)$

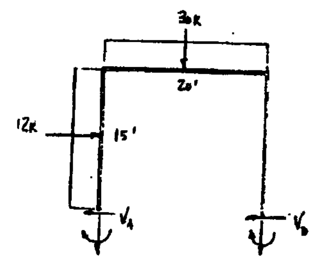
Solving Eqs. (1), (2) and (3),

$\theta_B = \frac{156.818}{EI}$

$\theta_C = -\frac{75}{EI}$

$\psi = \frac{76.704}{EI}$





Thus,

$M_{AB} = -24.8$ k·ft **Ans**

$M_{BA} = 26.1$ k·ft **Ans**

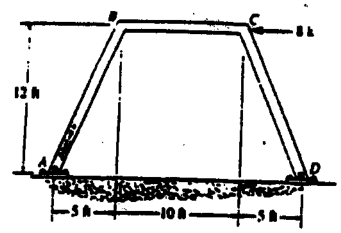
$M_{BC} = -26.1$ k·ft **Ans**

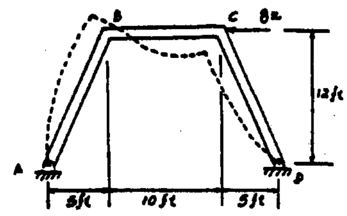
$M_{CB} = 50.7$ k·ft **Ans**

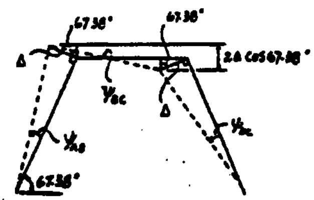
$M_{CD} = -50.7$ k·ft **Ans**

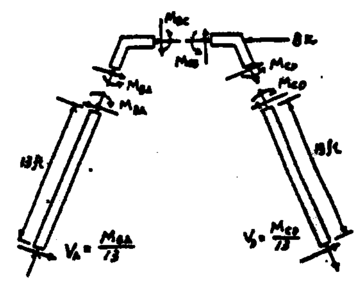
$M_{DC} = -40.7$ k·ft **Ans**

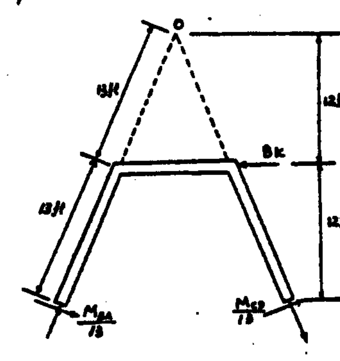
11-25. Determine the moment at each joint of the battered-column frame. The supports at A and D are pins. EI is constant.











$$(FEM)_{BA} = (FEM)_{BC} = (FEM)_{CB} = (FEM)_{CD} = 0$$

$$\psi_{AB} = \psi_{DC} = \frac{\Delta}{13} \qquad \psi_{BC} = \frac{2\Delta \cos 67.38°}{10}$$

$$\psi_{AB} = \psi_{DC} = \psi_{BC}$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (FEM)_N$$

$$M_{BA} = 3E\left(\frac{I}{13}\right)(\theta_B + \psi_{AB}) + 0$$

$$M_{BA} = 0.2308EI(\theta_B + \psi_{AB}) \qquad (1)$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{BC} = 2E\left(\frac{I}{10}\right)(2\theta_B + \theta_C - 3\psi_{AB}) + 0$$

$$M_{BC} = 0.2EI(2\theta_B + \theta_C - 3\psi_{AB}) + 0 \qquad (2)$$

$$M_{CB} = 2E\left(\frac{I}{10}\right)(2\theta_C + \theta_B - 3\psi_{AB}) + 0$$

$$M_{CB} = 0.2EI(2\theta_C + \theta_B - 3\psi_{AB}) + 0 \qquad (3)$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (FEM)_N$$

$$M_{CD} = 3E\left(\frac{I}{13}\right)(\theta_C + \psi_{AB}) + 0$$

$$M_{CD} = 0.2308EI(\theta_C + \psi_{AB}) \qquad (4)$$

Equilibrium

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

$$M_{CD} + M_{CB} = 0 \qquad (6)$$

$$\circlearrowleft + \Sigma M_O = 0: \quad \frac{M_{BA}}{13}(26) + \frac{M_{CD}}{13}(26) - 8(12) = 0$$

$$2M_{BA} + 2M_{CD} - 96 = 0 \qquad (7)$$

Solving these equations :

$$\theta_B = \theta_C = \frac{32}{EI}$$

$$\psi_{AB} = \frac{72}{EI}$$

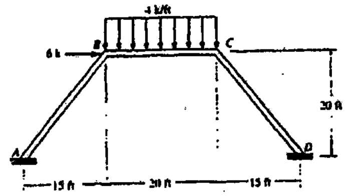
$M_{BA} = 24$ k·ft **Ans**

$M_{BC} = -24$ k·ft **Ans**

$M_{CB} = -24$ k·ft **Ans**

$M_{CD} = 24$ k·ft **Ans**

11–26 Determine the moments acting at the supports A and D of the battered-column frame. Take $E = 29(10^3)$ ksi, $I = 600\ \text{in}^4$.



$FEM_{BC} = -\frac{wL^2}{12} = -1600$ k·in., $\quad FEM_{CB} = \frac{wL^2}{12} = 1600$ k·in.

$\theta_A = \theta_D = 0$

$\psi_{AB} = \psi_{CD} = \frac{\Delta}{25}$

$\psi_{BC} = -\frac{1.2\Delta}{20}$

$\psi_{BC} = -1.5\psi_{CD} = -1.5\psi_{AB}$

$\psi' = -1.5\psi \quad$ (where $\psi' = \psi_{BC}, \psi = \psi_{AB} = \psi_{CD}$)

$M_N = 2E(\frac{I}{L})(2\theta_N + \theta_F - 3\psi) + FEM_N$

$M_{AB} = 2E(\frac{600}{25(12)})(0 + \theta_B - 3\psi) + 0 = 116{,}000\theta_B - 348{,}000\psi$

$M_{BA} = 2E(\frac{600}{25(12)})(2\theta_B + 0 - 3\psi) + 0 = 232{,}000\theta_B - 348{,}000\psi$

$M_{BC} = 2E(\frac{600}{20(12)})(2\theta_B + \theta_C - 3(-1.5\psi)) - 1600$

$= 290{,}000\theta_B + 145{,}000\theta_C + 652{,}500\psi - 1600$

$M_{CB} = 2E(\frac{600}{20(12)})(2\theta_C + \theta_B - 3(-1.5\psi)) + 1600$

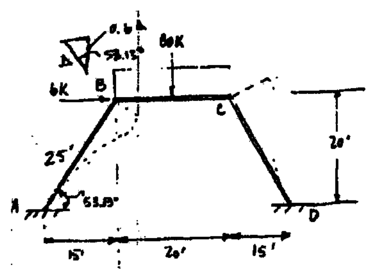
$= 290{,}000\theta_C + 145{,}000\theta_B + 652{,}500\psi + 1600$

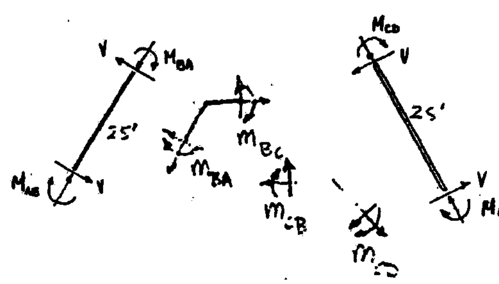
$M_{CD} = 2E(\frac{600}{25(12)})(2\theta_C + 0 - 3\psi) + 0$

$= 232{,}000\theta_C - 348{,}000\psi$

$M_{DC} = 2E(\frac{600}{25(12)})(0 + \theta_C - 3\psi) + 0$

$= 116{,}000\theta_C - 348{,}000\psi$





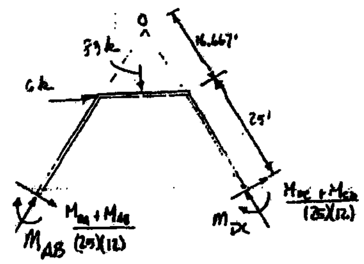
Moment equilibrium at B and C:

$M_{BA} + M_{BC} = 0$

$522{,}000\theta_B + 145{,}000\theta_C + 304{,}500\psi = 1600 \quad (1)$

$M_{CB} + M_{CD} = 0$

$145{,}000\theta_B + 522{,}000\theta_C + 304{,}500\psi = -1600 \quad (2)$



using the FBD of the frame.

$\curvearrowleft + \Sigma M_O = 0;$

$M_{AB} + M_{DC} - (\frac{M_{BA} + M_{AB}}{25(12)})(41.667)(12) - (\frac{M_{DC} + M_{CD}}{25(12)})(41.667)(12) - 6(13.333)(12) = 0$

$-0.667M_{AB} - 0.667M_{DC} - 1.667M_{BA} - 1.667M_{CD} - 960 = 0$

$464{,}000\theta_B + 464{,}000\theta_C - 1{,}624{,}000\psi = -960 \quad (3)$

Solving Eqs.(1),(2) and (3),

$\theta_B = 0.004030$ rad

$\theta_C = -0.004458$ rad

$\psi = 0.0004687$ in.

$M_{AB} = 25.4$ k·ft **Ans**

$M_{BA} = 64.3$ k·ft

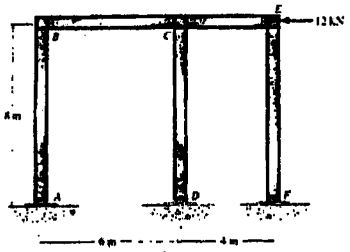
$M_{BC} = -64.3$ k·ft

$M_{CB} = 99.8$ k·ft

$M_{CD} = -99.8$ k·ft

$M_{DC} = -56.7$ k·ft **Ans**

11-29 Wind loads are transmitted to the frame at joint E. If A, B, E, D, and F are all pin-connected and C is fixed-connected, determine the moments at joint C and draw the bending-moment diagrams for the girders BCE. EI is constant.



Prob. 10-24

$\psi_{BC} = \psi_{CE} = 0$

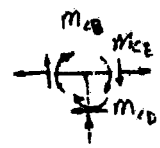
$\psi_{AB} = \psi_{CD} = \psi_{EF} = \psi$

Applying Eq. 11 – 10,

$M_{CB} = \dfrac{3EI}{6}(\theta_C - 0) + 0$

$M_{CE} = \dfrac{3EI}{4}(\theta_C - 0) + 0$

$M_{CD} = \dfrac{3EI}{8}(\theta_C - \psi) + 0 \qquad (1)$



Moment equilibrium at C:

$M_{CB} + M_{CE} + M_{CD} = 0$

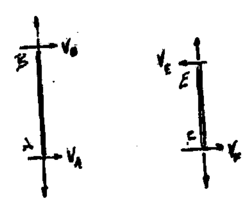
$\dfrac{3EI}{6}(\theta_C) + \dfrac{3EI}{4}\theta_C + \dfrac{3EI}{8}(\theta_C - \psi) = 0$

$\psi = 4.333\theta_C \qquad (2)$

From FBDs of members AB and EF:

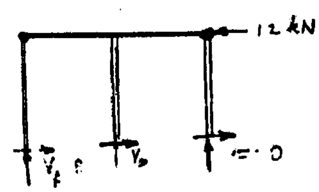
$\curvearrowleft + \Sigma M_B = 0; \quad V_A = 0$

$\curvearrowleft + \Sigma M_E = 0; \quad V_F = 0$



Since AB and FE are two-force members, then for the entire frame:

$\xrightarrow{+} \Sigma F_x = 0; \quad V_D - 12 = 0; \quad V_D = 12 \text{ kN}$



From FBD of member CD:

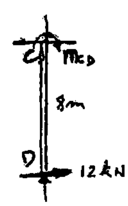
$\curvearrowleft + \Sigma M_C = 0; \quad M_{CD} - 12(8) = 0$

$M_{CD} = 96 \text{ kN·m} \qquad$ **Ans**

From Eq.(1),

$96 = \dfrac{3}{8}EI(\theta_C - 4.333\theta_C)$

$\theta_C = \dfrac{-76.8}{EI}$



From Eq.(2),

$\psi = \dfrac{-332.8}{EI}$

Thus,

$M_{CB} = -38.4 \text{ kN·m} \qquad$ **Ans**

$M_{CE} = -57.6 \text{ kN·m} \qquad$ **Ans**