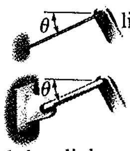
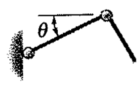
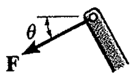
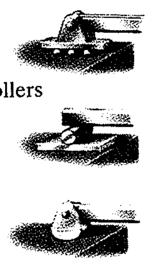
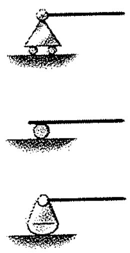
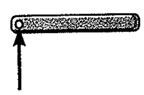
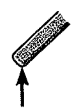
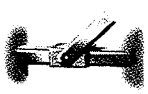
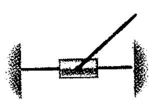
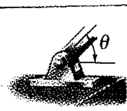
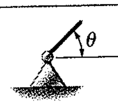
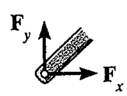
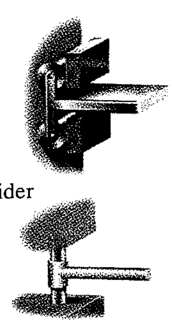
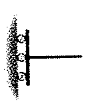
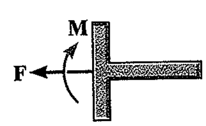
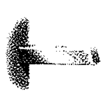
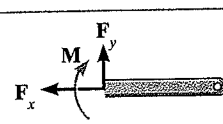
TABLE 2–1 • Supports for Coplanar Structures

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1)  light cable weightless link			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2)  rollers rocker			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3)  smooth contacting surface			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(4)  smooth pin-connected collar			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(5)  smooth pin or hinge			Two unknowns. The reactions are two force components.
(6)  slider fixed-connected collar			Two unknowns. The reactions are a force and a moment.
(7)  fixed support			Three unknowns. The reactions are the moment and the two force components.

The ability to reduce an actual structure to an idealized form, as shown by these examples, can only be gained by experience. To provide practice at doing this, the example problems and the problems for solution throughout this book are presented in somewhat realistic form, and the associated problem statements aid in explaining how the connections and supports can be modeled by those listed in Table 2–1. In engineering practice, if it becomes doubtful as to how to model a structure or transfer the loads to the members, it is best to consider *several* idealized structures and loadings and then design the actual structure so that it can resist the loadings in all the idealized models.

EXAMPLE 2-1

The floor of a classroom is supported by the bar joists shown in Fig 2–15*a*. Each joist is 15 ft long and they are spaced 2.5 ft on centers. The floor itself is made from lightweight concrete that is 4 in. thick. Neglect the weight of the joists and the corrugated metal deck, and determine the load that acts along each joist.

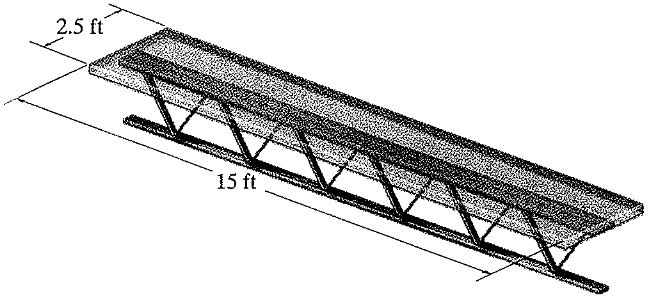
(a)

Solution

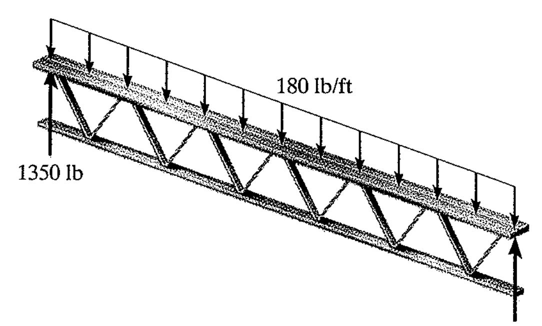
The dead load on the floor is due to the weight of the concrete slab. From Table 1–3 for 4 in. of lightweight concrete it is $(4)(8\ \text{lb/ft}^2) = 32\ \text{lb/ft}^2$. From Table 1–4, the live load for a classroom is $40\ \text{lb/ft}^2$. Thus the total floor load is $32\ \text{lb/ft}^2 + 40\ \text{lb/ft}^2 = 72\ \text{lb/ft}^2$. For the floor system, $L_1 = 2.5$ ft and $L_2 = 15$ ft. Since $L_2/L > 2$ the concrete slab is treated as a one-way slab. The tributary area for each joist is shown in Fig. 2–15*b*. Therefore the uniform load along its length is

$$w = 72\ \text{lb/ft}^2(2.5\ \text{ft}) = 180\ \text{lb/ft}$$

This loading and the end reactions on each joist are shown in Fig. 2–15*c*.



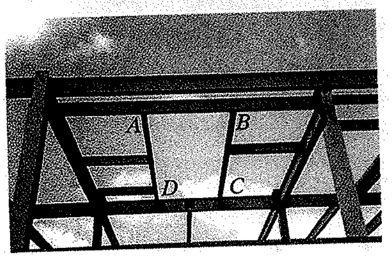
(b)



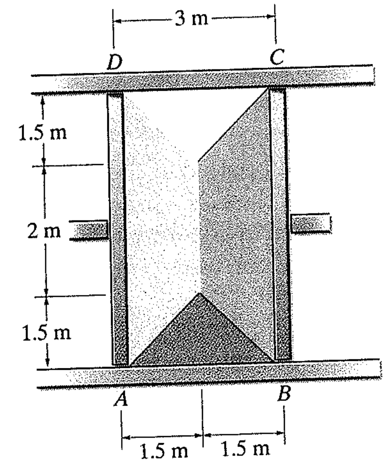
(c)

Fig. 2–15

EXAMPLE 2-2



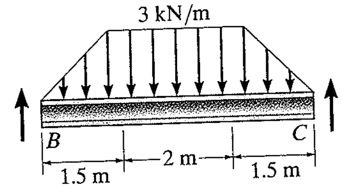
The flat roof of the steel-frame building in Fig. 2–16*a* is intended to support a total load of 2 kN/m^2 over its surface. If the span of beams *AD* and *BC* is 5 m and the space between them (*AB* and *DC*) is 3 m, determine the roof load within region *ABCD* that is transmitted to beam *BC*.



(a)

Solution

In this case $L_1 = 5$ m and $L_2 = 3$ m. Since $L_2/L_1 < 2$, we have two-way slab action. The tributary loading is shown in Fig. 2–16*b*, where the shaded trapezoidal area of loading is transmitted to member *BC*. The peak intensity of this loading is $(2\text{ kN/m}^2)(1.5\text{ m}) = 3\text{ kN/m}$. As a result, the distribution of load along *BC* is shown in Fig. 2–16*b*. This process of tributary load transmission should *also* be calculated for the two square regions to the right of *BC* in Fig. 2–16*a*, and this additional load should then be placed on *BC*.



(b)

Fig. 2–16

2–2 Principle of Superposition

The principle of superposition forms the basis for much of the theory of structural analysis. It may be stated as follows: *The total displacement or internal loadings (stress) at a point in a structure subjected to several external loadings can be determined by adding together the displacements or internal loadings (stress) caused by each of the external loads acting separately*. For this statement to be valid it is necessary that a *linear relationship* exist among the loads, stresses, and displacements.

Two requirements must be imposed for the principle of superposition to apply:

1. The material must behave in a linear-elastic manner, so that Hooke's law is valid, and therefore the load will be proportional to displacement.
2. The geometry of the structure must not undergo significant change when the loads are applied, i.e., small displacement theory applies. Large displacements will significantly change the position and orientation of the loads. An example would be a cantilevered thin rod subjected to a force at its end.

Throughout this text, these two requirements will be satisfied. Here only linear-elastic material behavior occurs; and the displacements produced by the loads will not significantly change the directions of applied loadings nor the dimensions used to compute the moments of forces.

The "shear walls" on the sides of this building are used to strengthen the structure when it is subjected to large hurricane wind loadings applied to the front or back of the building.

2–3 Equations of Equilibrium

It may be recalled from statics that a structure or one of its members is in equilibrium when it maintains a balance of force and moment. In general this requires that the force and moment equations of equilibrium be satisfied along three independent axes, namely,

$$\begin{array}{lll} \Sigma F_x = 0 & \Sigma F_y = 0 & \Sigma F_z = 0 \\ \Sigma M_x = 0 & \Sigma M_y = 0 & \Sigma M_z = 0 \end{array} \qquad (2\text{–}1)$$

The principal load-carrying portions of most structures, however, lie in a single plane, and since the loads are also coplanar, the above requirements for equilibrium reduce to

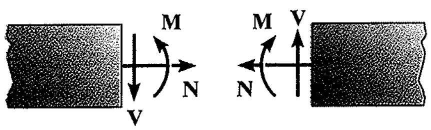
$$\boxed{\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M_O &= 0 \end{aligned}} \qquad (2\text{–}2)$$

Here ΣF_x and ΣF_y represent, respectively, the algebraic sums of the x and y components of all the forces acting on the structure or one of its members, and ΣM_O represents the algebraic sum of the moments of these force components about an axis perpendicular to the x–y plane (the z axis) and passing through point O.

Whenever these equations are applied, *it is first necessary to draw a free-body diagram of the structure or its members.* If a member is selected, it must be *isolated* from its supports and surroundings and its outlined shape drawn. All the forces and couple moments must be shown that act *on the member*. In this regard, the types of reactions at the supports can be determined using Table 2–1. Also, recall that forces common to two members act with equal magnitudes but opposite directions on the respective free-body diagrams of the members.

If the *internal loadings* at a specified point in a member are to be determined, the *method of sections* must be used. This requires that a "cut" or section be made perpendicular to the axis of the member at the point where the internal loading is to be determined. A free-body diagram of either segment of the "cut" member is isolated and the internal loads are then determined from the equations of equilibrium applied to the segment. In general, the internal loadings acting at the cut section of the member will consist of a normal force **N**, shear force **V**, and bending moment **M**, as shown in Fig. 2–17.

We will cover the principles of statics that are used to determine the external reactions on structures in Sec. 2–5. Internal loadings in structural members will be discussed in Chapter 4.



internal loadings

Fig. 2–17

2–4 Determinacy and Stability

Before starting the force analysis of a structure, it is necessary to establish the determinacy and stability of the structure.

Determinacy. The equilibrium equations provide both the *necessary and sufficient* conditions for equilibrium. When all the forces in a structure can be determined strictly from these equations, the structure is referred to as *statically determinate*. Structures having more unknown forces than available equilibrium equations are called *statically indeterminate*. As a general rule, a structure can be identified as being either statically determinate or statically indeterminate by drawing free-body diagrams of all its members, or selective parts of its members, and then comparing the total number of unknown reactive force and moment components with the total number of available equilibrium equations.* For a coplanar structure there are at most *three* equilibrium equations for each part, so that if there is a total of n parts and r force and moment reaction components, we have

$$\begin{aligned} r &= 3n, \text{ statically determinate} \\ r &> 3n, \text{ statically indeterminate} \end{aligned} \tag{2–3}$$

In particular, if a structure is *statically indeterminate*, the additional equations needed to solve for the unknown reactions are obtained by relating the applied loads and reactions to the displacement or slope at different points on the structure. These equations, which are referred to as *compatibility equations*, must be equal in number to the *degree of indeterminacy* of the structure. Compatibility equations involve the geometric and physical properties of the structure and will be discussed further in Chapter 10.

We will now consider some examples to show how to classify the determinacy of a structure. The first example considers beams; the second example, pin-connected structures; and in the third we will discuss frame structures. Classification of trusses will be considered in Chapter 3.

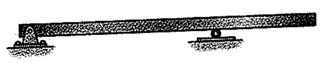
*Drawing the free-body diagrams is not strictly necessary, since a "mental count" of the number of unknowns can also be made and compared with the number of equilibrium equations.

EXAMPLE 2-3

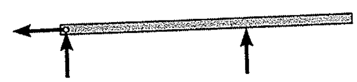
Classify each of the beams shown in Fig. 2–18*a* through 2–18*d* as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The beams are subjected to external loadings that are assumed to be known and can act anywhere on the beams.

Solution

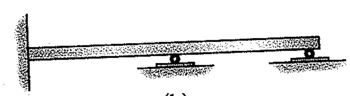
Compound beams, i.e., those in Fig. 2–18*c* and 2–18*d*, which are composed of pin-connected members must be disassembled. Note that in these cases, the unknown reactive forces acting between each member must be shown in equal but opposite pairs. The free-body diagrams of each member are shown. Applying $r = 3n$ or $r > 3n$, the resulting classifications are indicated.



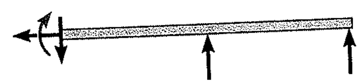
(a)



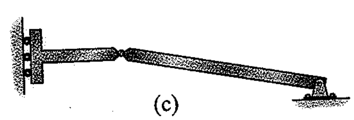
$r = 3,\ n = 1,\ 3 = 3(1)$ Statically determinate *Ans.*



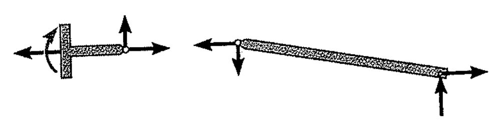
(b)



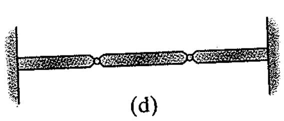
$r = 5,\ n = 1,\ 5 > 3(1)$ Statically indeterminate to the second degree *Ans.*



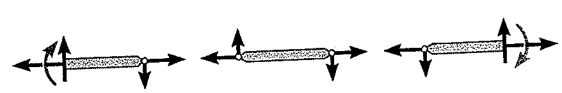
(c)



$r = 6,\ n = 2,\ 6 = 3(2)$ Statically determinate *Ans.*



(d)



$r = 10,\ n = 3,\ 10 > 3(3)$ Statically indeterminate to the first degree *Ans.*

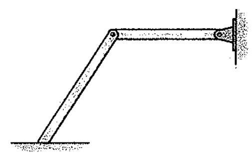
Fig. 2–18

EXAMPLE 2-4

Classify each of the pin-connected structures shown in Fig. 2–19*a* through 2–19*d* as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The structures are subjected to arbitrary external loadings that are assumed to be known and can act anywhere on the structures.

Solution

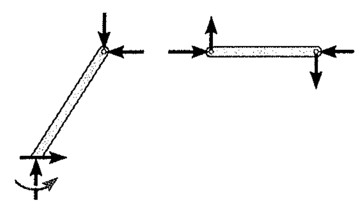
Classification of pin-connected structures is similar to that of beams. The free-body diagrams of the members are shown. Applying $r = 3n$ or $r > 3n$, the resulting classifications are indicated.

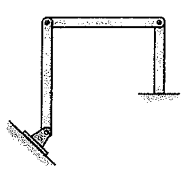


(a)

$r = 7, n = 2, 7 > 6$

Statically indeterminate to the first degree *Ans.*

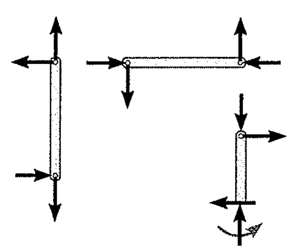


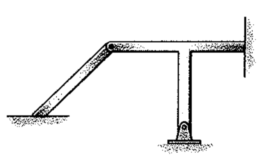


(b)

$r = 9, n = 3, 9 = 9,$

Statically determinate *Ans.*

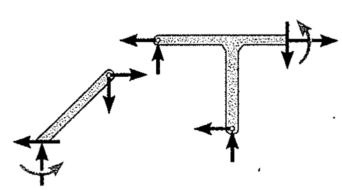


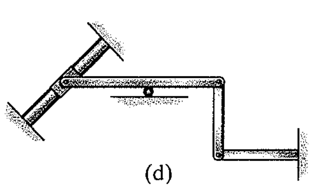


(c)

$r = 10, n = 2, 10 > 6,$

Statically indeterminate to the fourth degree *Ans.*





(d)

$r = 9, n = 3, 9 = 9,$

Statically determinate *Ans.*

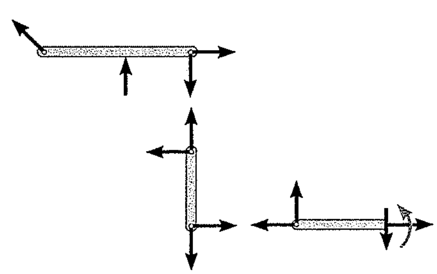
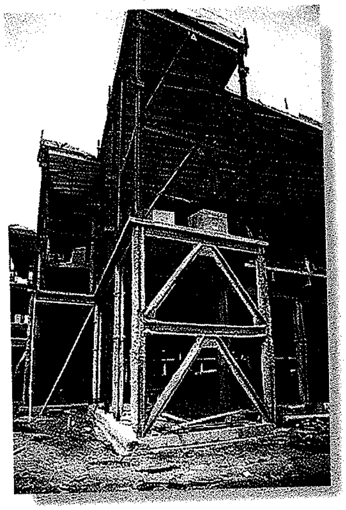


Fig. 2–19



The K-bracing on this frame provides stability, that is lateral support from wind and vertical support of the floor girders. Notice the use of concrete grout, which is applied to insulate the steel to keep it from losing its strength in the event of a fire.

In general, then, a structure will be geometrically unstable—that is, it will move slightly or collapse—if there are fewer reactive forces than equations of equilibrium; or if there are enough reactions, instability will occur if the lines of action of the reactive forces intersect at a common point or are parallel to one another. If the structure consists of several members or components, local instability of one or several of these members can generally be determined *by inspection*. If the members form a collapsible mechanism, the structure will be unstable. We will now formulize these statements for a *coplanar structure* having n members or components with r unknown reactions. Since three equilibrium equations are available for each member or component, we have

$r < 3n$	unstable	
$r \geq 3n$	unstable if member reactions are concurrent or parallel or some of the components form a collapsible mechanism	(2–4)

If the structure is unstable, *it does not matter* if it is statically determinate or indeterminate. In all cases such types of structures must be avoided in practice.

The following examples illustrate how structures or their members can be classified as stable or unstable. Structures in the form of a truss will be discussed in Chapter 3.

EXAMPLE 2-6

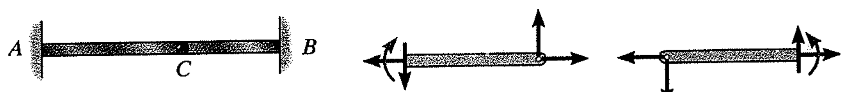
Classify each of the structures in Fig. 2–24*a* through 2–24*e* as stable or unstable. The structures are subjected to arbitrary external loads that are assumed to be known.

Solution

The structures are classified as indicated.

Fig. 2–24

The member is *stable* since the reactions are nonconcurrent and non-parallel. It is also statically determinate. *Ans.*



(b)

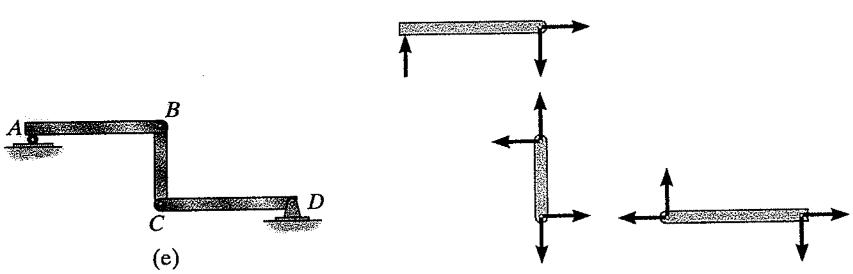
The compound beam is *stable*. It is also indeterminate to the second degree. *Ans.*

(c)

The member is *unstable* since the three reactions are concurrent at B. *Ans.*

(d)

The beam is *unstable* since the three reactions are all parallel. *Ans.*



(e)

The structure is *unstable* since $r = 7$, $n = 3$, so that, by Eq. 2–4, $r < 3n, 7 < 9$. Also, this can be seen by inspection, since AB can move horizontally without restraint. *Ans.*

PROCEDURE FOR ANALYSIS

The following procedure provides a method for determining the *joint reactions* for structures composed of pin-connected members.

Free-Body Diagrams

- Disassemble the structure and draw a free-body diagram of each member. Also, it may be convenient to supplement a member free-body diagram with a free-body diagram of the *entire structure*. Some or all of the support reactions can then be determined using this diagram.
- Recall that reactive forces common to two members act with equal magnitudes but opposite directions on the respective free-body diagrams of the members.
- All two-force members should be identified. These members, regardless of their shape, have no external loads on them, and therefore their free-body diagrams are represented with equal but opposite collinear forces acting on their ends.
- In many cases it is possible to tell by inspection the proper arrowhead sense of direction of an unknown force or couple moment; however, if this seems difficult, the directional sense can be assumed.

Equations of Equilibrium

- Count the total number of unknowns to make sure that an equivalent number of equilibrium equations can be written for solution. Except for two-force members, recall that in general three equilibrium equations can be written for each member.
- Many times, the solution for the unknowns will be straightforward if the moment equation $\Sigma M_O = 0$ is applied about a point (O) that lies at the intersection of the lines of action of as many unknown forces as possible.
- When applying the force equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$, orient the x and y axes along lines that will provide the simplest reduction of the forces into their x and y components.
- If the solution of the equilibrium equations yields a *negative* magnitude for an unknown force or couple moment, it indicates that its arrowhead sense of direction is *opposite* to that which was assumed on the free-body diagram.

EXAMPLE 2-7

Determine the reactions on the beam shown in Fig. 2–27*a*.

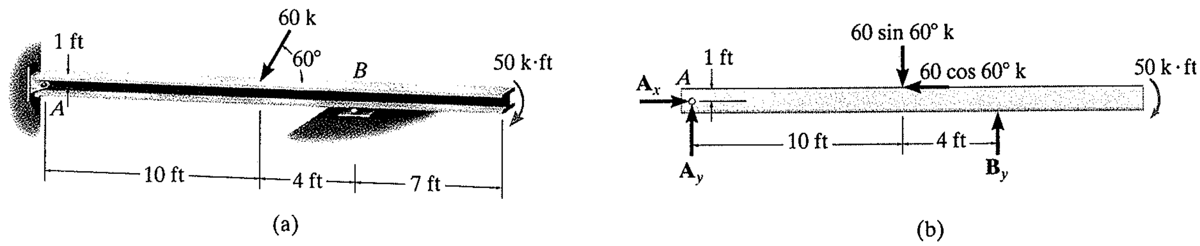


Fig. 2–27

Solution

Free-Body Diagram. As shown in Fig. 2–27*b*, the 60-k force is resolved into *x* and *y* components. Furthermore, the 7-ft dimension line is not needed since a couple moment is a *free vector* and can therefore act anywhere on the beam for the purpose of computing the external reactions.

Equations of Equilibrium. Applying Eqs. 2–2 in a sequence, using previously calculated results, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 60 \cos 60° = 0 \qquad A_x = 30.0 \text{ k}$ *Ans.*

$\circlearrowleft + \Sigma M_A = 0; \quad -60 \sin 60°(10) + 60 \cos 60°(1) + B_y(14) - 50 = 0 \qquad B_y = 38.5 \text{ k}$ *Ans.*

$+\uparrow \Sigma F_y = 0; \quad -60 \sin 60° + 38.5 + A_y = 0 \qquad A_y = 13.4 \text{ k}$ *Ans.*

EXAMPLE 2-8

Determine the reactions on the beam in Fig. 2–28*a*.

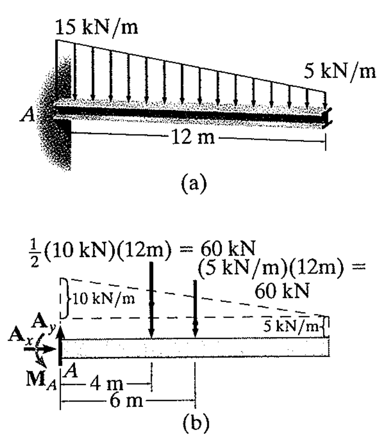


Fig. 2–28

Solution

Free-Body Diagram. As shown in Fig. 2–28*b*, the trapezoidal distributed loading is segmented into a triangular and uniform load. The *areas* under the triangle and rectangle represent the *resultant* forces. These forces act through the centroid of their corresponding areas.

Equations of Equilibrium

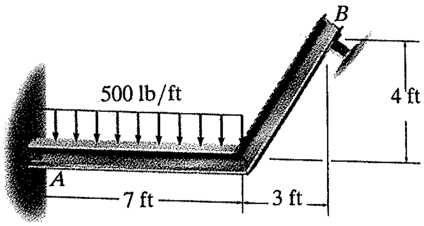
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$ *Ans.*

$+\uparrow \Sigma F_y = 0; \quad A_y - 60 - 60 = 0 \qquad A_y = 120 \text{ kN}$ *Ans.*

$\circlearrowleft + \Sigma M_A = 0; \quad -60(4) - 60(6) + M_A = 0 \qquad M_A = 600 \text{ kN} \cdot \text{m}$ *Ans.*

EXAMPLE 2-9

Determine the reactions on the beam in Fig. 2–29*a*. Assume *A* is a pin and the support at *B* is a roller (smooth surface).

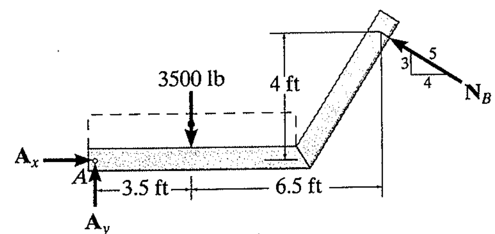


(a)

Fig. 2–29

Solution

Free-Body Diagram. As shown in Fig. 2–29*b*, the support ("roller") at *B* exerts a *normal force* on the beam at its point of contact. The line of action of this force is defined by the 3–4–5 triangle.



(b)

Equations of Equilibrium. Resolving $\mathbf{N}_B$ into x and y components and summing moments about A yields a direct solution for N_B. Why? Using this result, we can then obtain A_x and A_y.

$\circlearrowleft + \Sigma M_A = 0;\quad -3500(3.5) + \left(\frac{4}{5}\right)N_B(4) + \left(\frac{3}{5}\right)N_B(10) = 0$ *Ans.*

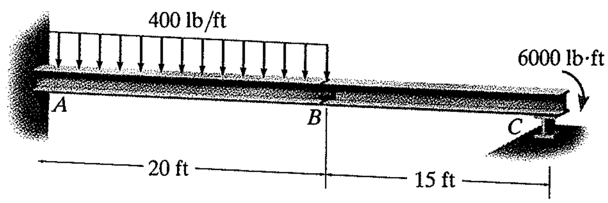
$$N_B = 1331.5 \text{ lb} = 1.33 \text{ k}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x - \frac{4}{5}(1331.5) = 0 \qquad A_x = 1.07 \text{ k}$ *Ans.*

$+\uparrow \Sigma F_y = 0;\quad A_y - 3500 + \frac{3}{5}(1331.5) = 0 \qquad A_y = 2.70 \text{ k}$ *Ans.*

EXAMPLE 2-10

The compound beam in Fig. 2–30*a* is fixed at A. Determine the reactions at A, B, and C. Assume that the connection at B is a pin and C is a roller.

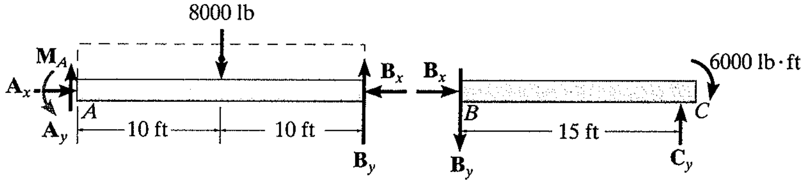


(a)

Fig. 2–30

Solution

Free-Body Diagrams. The free-body diagram of each segment is shown in Fig. 2–30*b*. Why is this problem statically determinate?



(b)

Equations of Equilibrium. There are six unknowns. Applying the six equations of equilibrium, using previously calculated results, we have

Segment BC:

$\circlearrowleft + \Sigma M_C = 0;\quad -6000 + B_y(15) = 0 \quad B_y = 400 \text{ lb}$ *Ans.*

$+\uparrow \Sigma F_y = 0;\quad -400 + C_y = 0 \quad C_y = 400 \text{ lb}$ *Ans.*

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad B_x = 0$ *Ans.*

Segment AB:

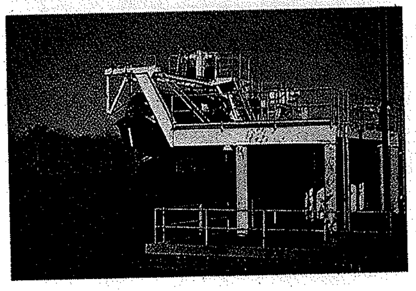
$\circlearrowleft + \Sigma M_A = 0;\quad M_A - 8000(10) + 400(20) = 0$

$M_A = 72.0 \text{ k} \cdot \text{ft}$ *Ans.*

$+\uparrow \Sigma F_y = 0;\quad A_y - 8000 + 400 = 0 \quad A_y = 7.60 \text{ k}$ *Ans.*

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x - 0 = 0 \quad A_x = 0$ *Ans.*

EXAMPLE 2-11



The side girder shown in the photo supports the boat and deck. An idealized model of this girder is shown in Fig. 2–31*a*, where it can be assumed *A* is a roller and *B* is a pin. Using a local code the anticipated deck loading transmitted to the girder is 6 kN/m. Wind exerts a *resultant* horizontal force of 4 kN as shown, and the mass of the boat that is supported by the girder is 23 Mg. The boat's mass center is at *G*. Determine the reactions at the supports.

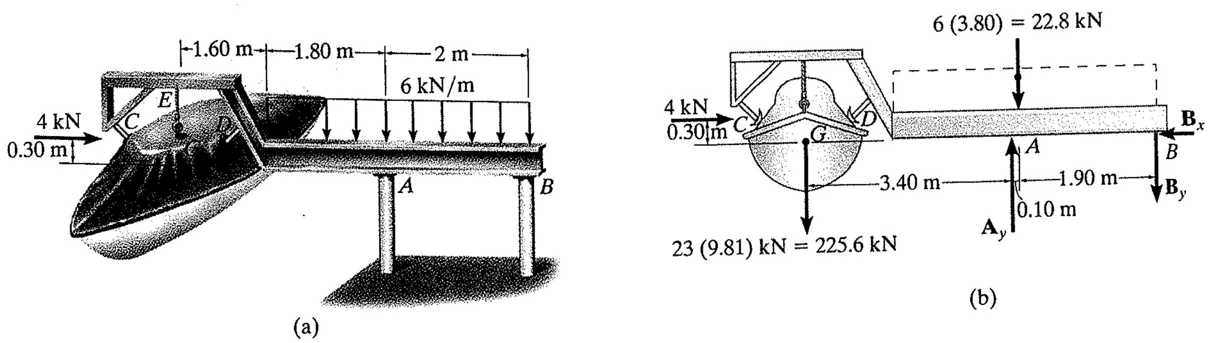


Fig. 2–31

Solution

Free-Body Diagram. Here we will consider the boat and girder as a single system, Fig. 2–31*b*. As shown, the distributed loading has been replaced by its resultant.

Equations of Equilibrium. Applying Eqs. 2–2 in sequence, using previously calculated results, we have

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 4 - B_x = 0$$

$$B_x = 4 \text{ kN} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_B = 0; \quad 22.8(1.90) - A_y(2) + 225.6(5.40) - 4(0.30) = 0$$

$$A_y = 630.2 \text{ kN} = 630 \text{ kN} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 630.2 - B_y - 22.8 - 225.6 = 0$$

$$B_y = 382 \text{ kN} \qquad \textit{Ans.}$$

Note: If the girder alone had been considered for this analysis then the normal forces at the shoes *C* and *D* would have to first be calculated using a free-body diagram of the boat. (These forces exist if the cable pulls the boat snug against them.) Equal but opposite normal forces along with the cable force at *E* would then act on the girder when its free-body diagram is considered. The same results would have been obtained; however, by considering the boat-girder system, these normal forces and the cable force become internal and do not have to be considered.

EXAMPLE 2-12

Determine the horizontal and vertical components of reaction at the pins A, B, and C of the two-member frame shown in Fig. 2–32a.

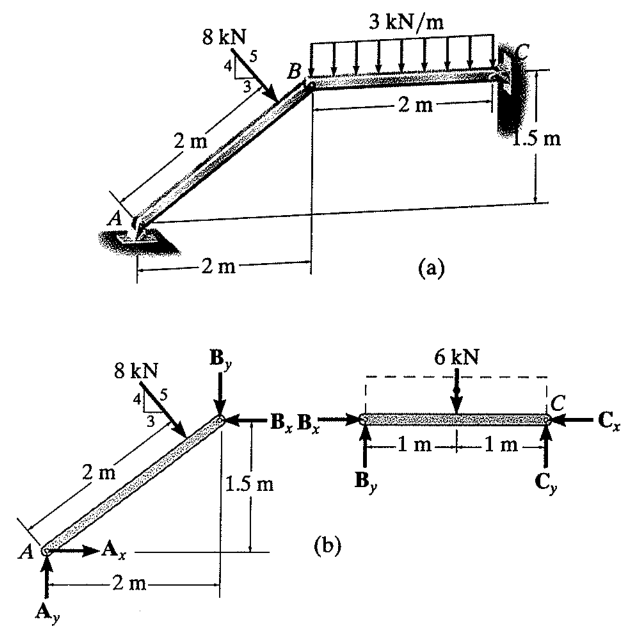


Fig. 2–32

Solution

Free-Body Diagrams. The free-body diagram of each member is shown in Fig. 2–32b.

Equations of Equilibrium. Applying the six equations of equilibrium in the following sequence allows a direct solution for each of the six unknowns.

Member BC:

$\curvearrowleft + \Sigma M_C = 0;\quad -B_y(2) + 6(1) = 0 \qquad B_y = 3\text{ kN}$ *Ans.*

Member AB:

$\curvearrowleft + \Sigma M_A = 0;\quad -8(2) - 3(2) + B_x(1.5) = 0 \qquad B_x = 14.7\text{ kN}$ *Ans.*

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x + \frac{3}{5}(8) - 14.7 = 0 \qquad A_x = 9.87\text{ kN}$ *Ans.*

$+\uparrow \Sigma F_y = 0;\quad A_y - \frac{4}{5}(8) - 3 = 0 \qquad A_y = 9.40\text{ kN}$ *Ans.*

Member BC:

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 14.7 - C_x = 0 \qquad C_x = 14.7\text{ kN}$ *Ans.*

$+\uparrow \Sigma F_y = 0;\quad 3 - 6 + C_y = 0 \qquad C_y = 3\text{ kN}$ *Ans.*