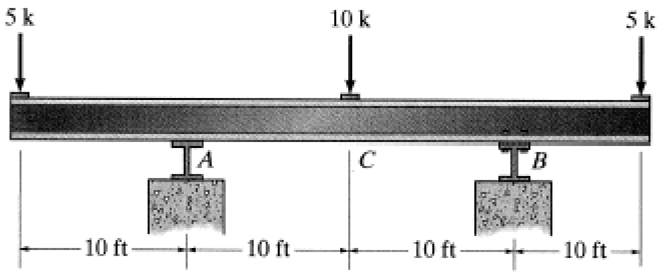
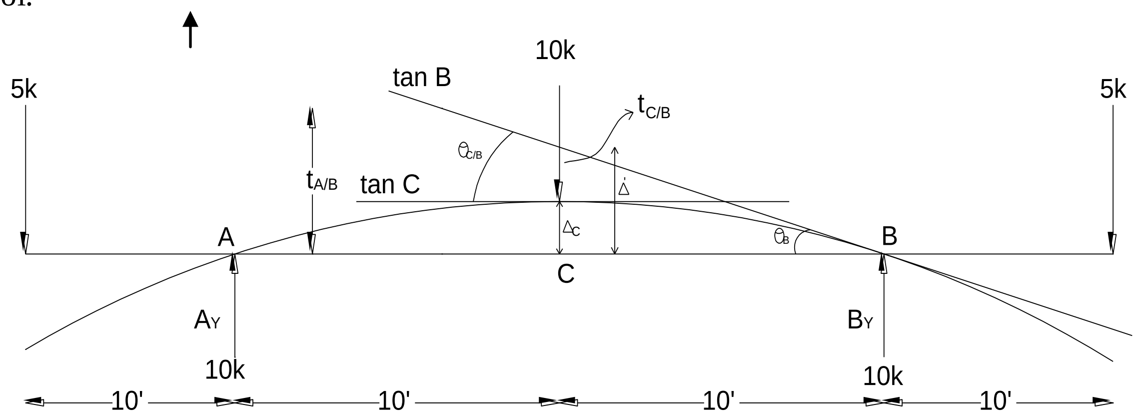
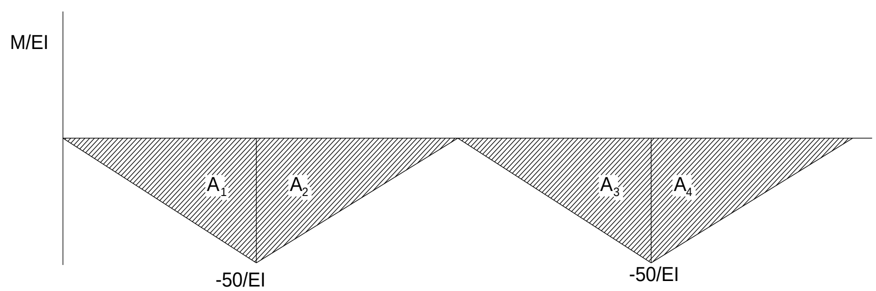
CE 371.01 – Structural Analysis I
Homework #7

24. Using the moment-area theorems, determine the slope at point *B* and the deflection at point *C*. *E* and *I* are constant over the length of the member. Assume that the support at *A* is a roller and the support at *B* is a pin.



Sol:





By symmetry of loading, $A_Y = B_Y = 10k$

From the M/EI diagram,

$A_1 = A_2 = A_3 = A_4 = -250/EI$

θ_B: $\theta_B(20') = t_{A/B} = A_2(10/3) + A_3[10 + 2(10)/3]$

$\theta_B = [-250(10')/EI*3 - 250(10' + 20'/3)/EI]/20'$

$= 250/EI$ ↻ >

$t_{A/B}$: $= \theta_B(20') = 250(20)/EI = 5000/EI$ ↑

$t_{C/B}$: $t_{C/B}$ ↓ $= \theta_B*2(10)/3 = 250(20)/3*EI = 1667/EI$ ↓

Δ' (tanB⟶ pt C): $=1/2(t_{A/B}) = 2500/EI$ ↑

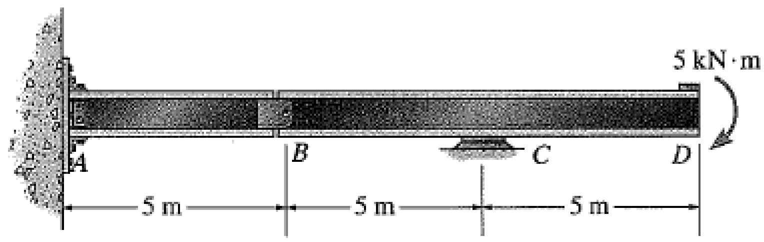
$\Delta_C = \Delta'$↑ $- t_{C/B}$↓ $= 2500/EI – 1667/EI = 833/EI$↑

Summary:

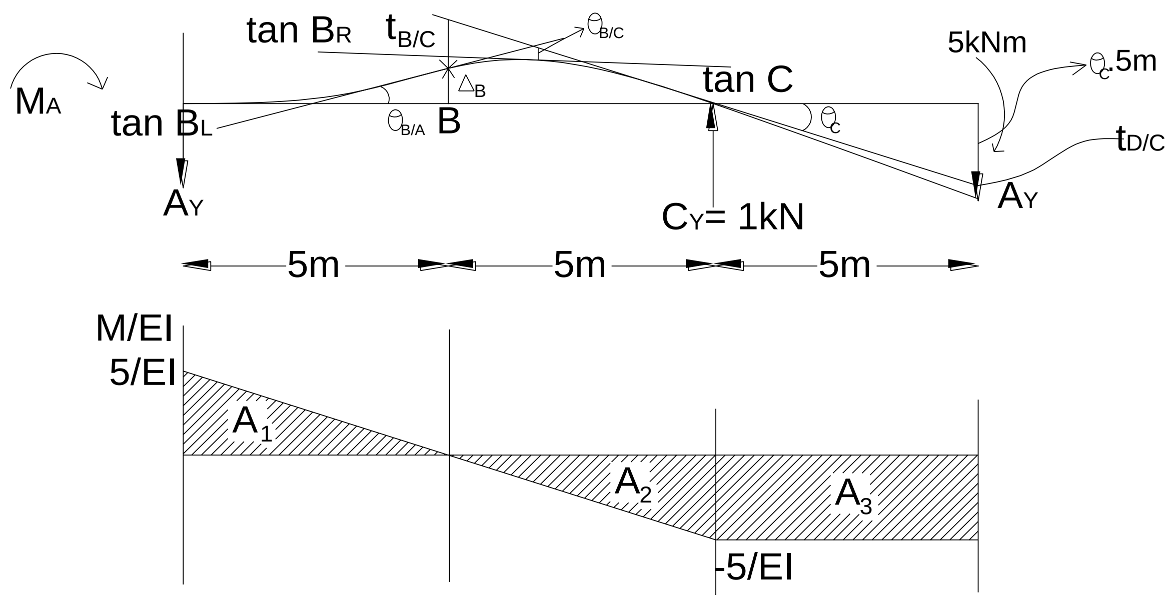
$\theta_B = 250/EI$ ↷ > **Ans** **(Points 5)**

$\Delta_C = 833/EI$ ↑ **Ans** **(Points 5)**

25. Using the moment-area theorems, determine the slope just to the left and just to the right of the hinge at *B*. Also determine the deflection at *D*. Assume the beam is fixed at *A* and that *C* is a roller. *E* and *I* are constant over the length of the structure.



Sol:



Cut @ hinge: BCD

↺ $\Sigma M_B = 0 = 5\text{kN.m} - C_Y(5\text{m})$

$C_Y = 1\text{kN}\uparrow$

Full Structure:

$+\Sigma F_Y = 0 = -A_Y + 1$

$A_Y = 1\text{kN}\downarrow$

↻ $\Sigma M_A = 0 = - M_A - 5\text{kN.m} + 1\text{kN}(10\text{m})$

$M_A = 5\text{kN.m}$ ↺

$A_1 = 5^2/2{*}EI = 12.5/EI$

$A_2 = 12.5/EI$

$A_3 = 25/EI$

θ_{BL}: $\theta_{BL} = \theta_{B/A}$ ↻ $= A_1 = 12.5/EI <$ ↻

θ_C: $\theta_C = (\Delta_B + t_{B/C})/5m = [A_1(2{*}5/3) + A_2(2{*}5/3)]/5m$

θ_C = 12.5(2)/(3*EI) + 12.5(2)/(3*EI) = 16.67EI↻>

θ_{BR}: θ_{BR} = θ_C ↻> - $\theta_{B/C}$↺> = 16.67/EI – A2 = 16.67/EI – 12.5/EI = 4.17/EI↻ >

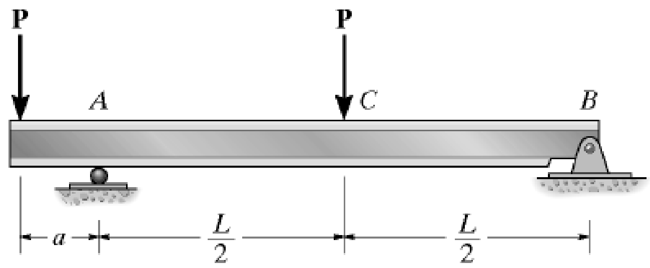
$\Delta_{D:}$ Δ_D = θ_C↓(5m) + $t_{D/C}$↓ = 16.67(5m)/EI + A_3 (5m)/2

Δ_D = 16.67(5)/(EI) + 25(5)/(2*EI) = 145.85/EI↓

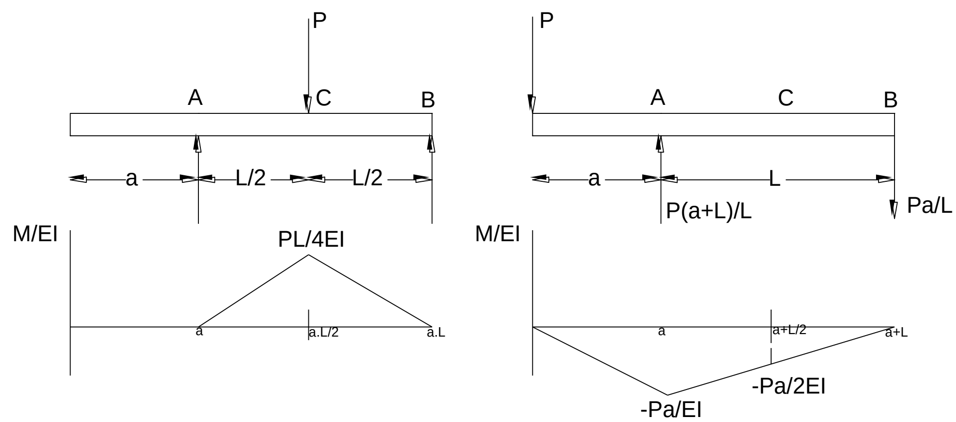
Summary

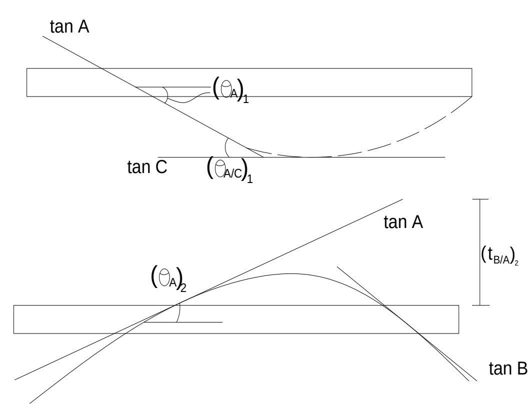
θ_L = 12.5/EI < ↺	**Ans**	**(Points 4)**
θ_{BR} = 4.17/EI ↻ >	**Ans**	**(Points 4)**
Δ_D = 145.85/EI↓	**Ans**	**(Points 2)**

26. Use the moment-area theorems and determine the value of *a* so that the slope at *A* is equal to zero. *EI* is constant.



Sol:





Moment Area Theorems:

$\theta_A = (\theta_{A/C})_1 = ½(PL/4EI)(L/2) = PL^2/16EI$

$(t_{B/A})_2 = ½(-Pa/EI)(L)(2L/3) = Pa^2/3EI$

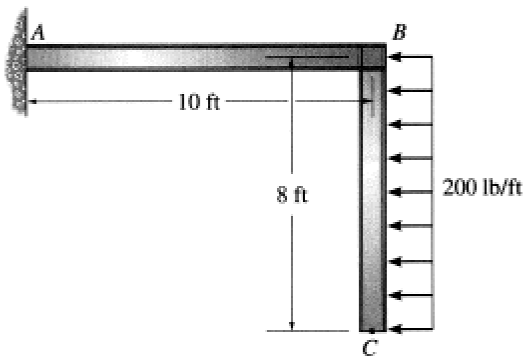
$(\theta_A)_2 = |(t_{B/A})_2|/2 = (-PaL^2/3EI)/L = PaL/3EI$

Require, $\theta_A = 0 = (\theta_A)_1 - (\theta_A)_2$

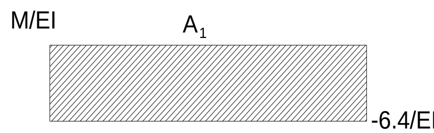
$0 = PL^2/16EI - PaL/3EI$

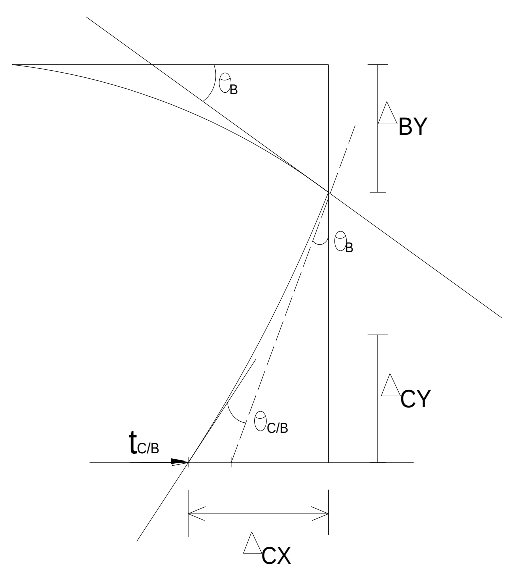
$a = 3L/16$ **Ans** **(Points 10)**

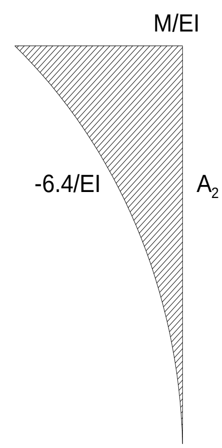
27. Using the moment-area theorems, determine the horizontal and vertical components of displacement at C. Let $E = 29{,}000$ ksi and $I = 80\ in^4$ for each member.



Sol:







$\overset{+}{\rightarrow}\Sigma F_X = 0;$ $A_X - 0.2(8) = 0$

$A_X = 1.6k \rightarrow$

$\circlearrowright \Sigma M_A = 0;$ $-1.6(4) + M_A = 0$

$M_A = 6.4k.ft \circlearrowright$

$A_1 = 64/EI\ kft^2$

$A_2 = 6.4(8)/3EI = 17.07/EI$

$\theta_B = A_1 = 64/EI <$ ↶

$\Delta B_Y = \Delta C_Y = A_1\,(5') = 64(5)/EI = 320/EI \downarrow$

$\theta_{C/B} = A_2 = 17.07/EI$ ↻

$t_{C/B} = A_2(3/4)(8) = 17.07(3)(2)/EI = 102.42/EI$ ←

$\theta_B(8) = 64(8)/EI = 512/EI$ ←

$\Delta C_X = (102.42 + 512)/EI = 614.42/EI$ ←

$EI = 29000ksi\ (80in^4) = 2.32 \times 10^6 k.in^2$

$\Delta C_Y = 320(1728)/2.32 \times 10^6$ in = 0.24” ↓

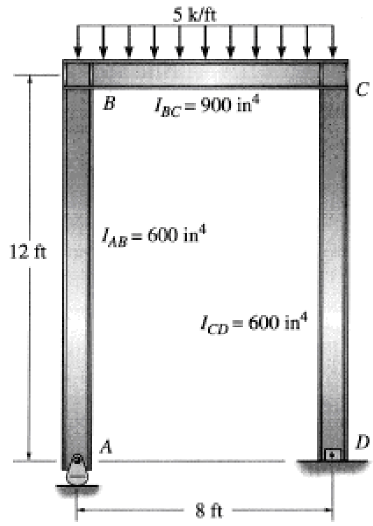
$\Delta C_X = 614.42(1728)/2.32 \times 10^6 = 0.46$” ←

Summary:

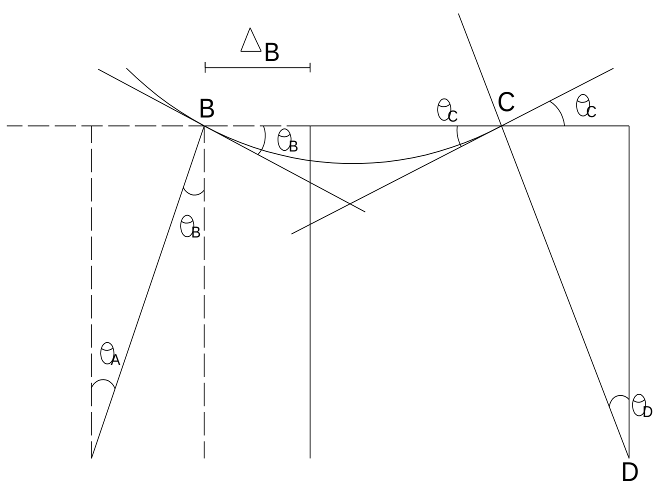
$\Delta C_Y = 0.24$”↓ **Ans** **(Points 5)**

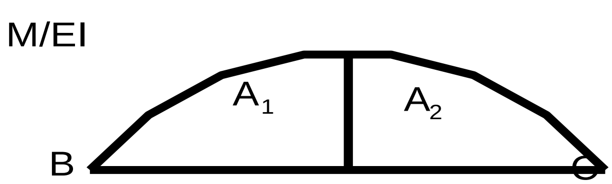
$\Delta C_X = 0.46$”← **Ans** **(Points 5)**

28. Using the moment-area theorems, determine the slope at *A* and the deflection of *B*. The moment of inertia for each member is indicated in the figure, and *E* = 29,000 ksi. Assume *A* is a roller support and *D* is a pin support.



Sol:





$+\Sigma F_Y$; $A_Y + D_Y - 5(8)$

By Symmetry: $A_Y = D_Y = 5(4) = 20k\uparrow$

M/EI @centre-line: $5*8^2/8*EI = 40/EI$ kft

$A1 = A2 = 40(4)(2)/3*EI = 106.7/EI$

$\theta_B = A_1 = 106.7/EI$ ↻

$\theta_C = 106.7/EI_{BC}$ ↺ > = θ_D

$\Delta_C = \theta_D\,(12) = \Delta_B = 106.7(12)/EI_{BC} = 1280\ kft^3/EI_{BC}\ kin^2$

$\Delta_B = 1280(1728) / (29000{*}900) = 0.085''$ ←

$\theta_A = \Delta_B/12' = 0.085''/12(12'') = 0.59 \times 10^{-3}$ rad ↻ >

Summary:

$\Delta_B = 0.085''$ ← **Ans (Points 5)**

$\theta_A = 0.59 \times 10^{-3}$ rad < ↻ or 33.7×10^{-3} (degrees) **Ans (Points 5)**