24. Using the moment-area theorems, determine the slope at point $B$ and the deflection at point $C$. $E$ and $I$ are constant over the length of the member. Assume that the support at $A$ is a roller and the support at $B$ is a pin.

Sol:

By symmetry of loading, $A_Y = B_Y = 10k$

From the $M/EI$ diagram,

$A_1 = A_2 = A_3 = A_4 = -250/EI$

$\theta_B:$

$\theta_B(20') = t_{A/B} = A_2(10/3) + A_3[10 + 2(10)/3]$

$\theta_B = [-250(10')/EI \times 3 - 250(10' + 20'/3)/EI]/20'$

$= 250/EI \quad (>)$

$t_{A/B}: \quad = \theta_B(20') = 250(20)/EI = 5000/EI\uparrow$

$t_{C/B}: \quad t_{C/B} = \theta_B*2(10)/3 = 250(20)/3*EI = 1667/EI\downarrow$
$\Delta' (\tan \beta \rightarrow \text{pt C}): \quad \frac{1}{2}(t_{A/B}) = \frac{2500}{EI}$

$\Delta_C = \Delta' - \frac{t_{C/B}}{E} = \frac{2500}{EI} - \frac{1667}{EI} = \frac{833}{EI}$

Summary:

$\theta_B = \frac{250}{EI} \quad \text{(Ans)}$  \quad \text{(Points 5)}

$\Delta_C = \frac{833}{EI} \quad \text{(Ans)}$  \quad \text{(Points 5)}
25. Using the moment-area theorems, determine the slope just to the left and just to the right of the hinge at B. Also determine the deflection at D. Assume the beam is fixed at A and that C is a roller. E and I are constant over the length of the structure.

Sol:

Cut @ hinge: BCD

ΣM_B = 0 = 5kN.m - C_Y (5m)
C_Y = 1kN↑

Full Structure:

+ΣF_Y = 0 = -A_Y + 1
A_Y = 1kN↓

Ψ ΣM_A = 0 = -M_A - 5kN.m + 1kN(10m)
M_A = 5kN.m Ψ

A_1 = 5^2/2*EI = 12.5/EI
A_2 = 12.5/EI
A_3 = 25/EI

θ_{BL}: θ_{BL} = θ_{B/A} = A_1 = 12.5/EI < Ψ

θ_C: θ_C = (Δ_B + t_{B/C})/5m = [A_1(2*5/3) + A_2(2*5/3)]/5m
\[
\theta_C = 12.5(2)/(3*EI) + 12.5(2)/(3*EI) = 16.67EI
\]

\[
\theta_{BR} = \theta_C - \theta_{B/C} = 16.67/EI - 12.5/EI = 4.17/EI
\]

\[
\Delta_D = \theta_C(5m) + t_{D/C} = 16.67(5m)/EI + A_3(5m)/2
\]

\[
\Delta_D = 16.67(5)/(EI) + 25(5)/(2*EI) = 145.85/EI
\]

Summary

\[
\theta_L = 12.5/EI < \theta \quad \text{Ans (Points 4)}
\]

\[
\theta_{BR} = 4.17/EI \quad \text{Ans (Points 4)}
\]

\[
\Delta_D = 145.85/EI \quad \text{Ans (Points 2)}
\]
26. Use the moment-area theorems and determine the value of \( a \) so that the slope at \( A \) is equal to zero. \( EI \) is constant.

\[ \tan A = (\theta_{AC})_1 = \frac{1}{2}(PL/4EI)(L/2) = PL^2/16EI \]
\[ (t_{B/A})_2 = \frac{1}{2}(-Pa/EI)(L)(2L/3) = Pa^2/3EI \]
\[ (\theta_A)_2 = (t_{B/A})_2/2 = (-PaL^2/3EI)/L = PaL/3EI \]

Require,
\[ \theta_A = 0 = (\theta_A)_1 - (\theta_A)_2 \]
\[ 0 = PL^2/16EI - PaL/3EI \]
\[ a = 3L/16 \quad \text{Ans} \quad (\text{Points 10}) \]
27. Using the moment-area theorems, determine the horizontal and vertical components of displacement at C. Let $E = 29,000$ ksi and $I = 80 \text{ in}^4$ for each member.

Sol:

$$-\sum F_X = 0; \quad A_X - 0.2(8) = 0$$

$$A_X = 1.6k$$

$$\Psi \sum M_A = 0; \quad -1.6(4) + M_A = 0$$

$$M_A = 6.4k\text{ft}$$

$$A_1 = 64/EI \text{ kft}^2$$

$$A_2 = 6.4(8)/3EI = 17.07/EI$$

$$\theta_B = A_1 = 64/EI < \theta$$

$$\Delta B_Y = \Delta C_Y = A_1 (5^\circ) = 64(5)/EI = 320/EI$$
$\theta_{CB} = A_2 = 17.07/EI$ 

$t_{CB} = A_2(3/4)(8) = 17.07(3)(2)/EI = 102.42/EI$ 

$\theta_B(8) = 64(8)/EI = 512/EI$ 

$\Delta C_X = (102.42 + 512)/EI = 614.42/EI$ 

$EI = 29000\text{ksi (80in}^4) = 2.32 \times 10^6\text{k.in}^2$ 

$\Delta C_Y = 320(1728)/2.32 \times 10^6 \text{in} = 0.24''$ 

$\Delta C_X = 614.42(1728)/2.32 \times 10^6 = 0.46''$ 

Summary:

$\Delta C_Y = 0.24''$ \hspace{1cm} \text{Ans (Points 5)} 

$\Delta C_X = 0.46''$ \hspace{1cm} \text{Ans (Points 5)}
28. Using the moment-area theorems, determine the slope at A and the deflection of B. The moment of inertia for each member is indicated in the figure, and $E = 29,000$ ksi. Assume A is a roller support and D is a pin support.

Sol:

$+\Sigma F_Y: \quad A_Y + D_Y = 5(8)$

By Symmetry: \quad $A_Y = D_Y = 5(4) = 20$k

$M/EI @centre-line: \quad 5*8^2/8*EI = 40/EI$ kft

$A1 = A2 = 40(4)(2)/3*EI = 106.7/EI$
\[ \theta_B = \frac{A_1}{106.7/EI} \]

\[ \theta_C = \frac{106.7}{E \text{I}_{BC}} > \theta_D \]

\[ \Delta_C = \theta_D (12) = 106.7(12)/E \text{I}_{BC} = 1280 \text{ kft}^3/E \text{I}_{BC} \text{ kin}^2 \]

\[ \Delta_B = 1280(1728) / (29000*900) = 0.085" \]

\[ \theta_A = \Delta_B / 12" = 0.085"/12(12") = 0.59 \times 10^{-3} \text{ rad} \]

Summary:

\[ \Delta_B = 0.085" \quad \text{Ans (Points 5)} \]

\[ \theta_A = 0.59 \times 10^{-3} \text{ rad} < \theta \text{ or } 33.7 \times 10^{-3} \text{ (degrees)} \quad \text{Ans (Points 5)} \]