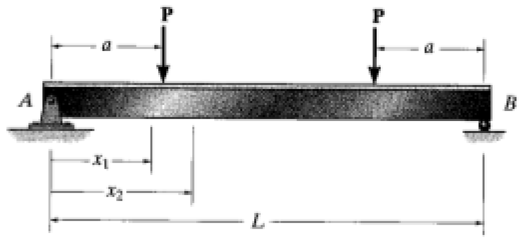
CE 371 - Section 02

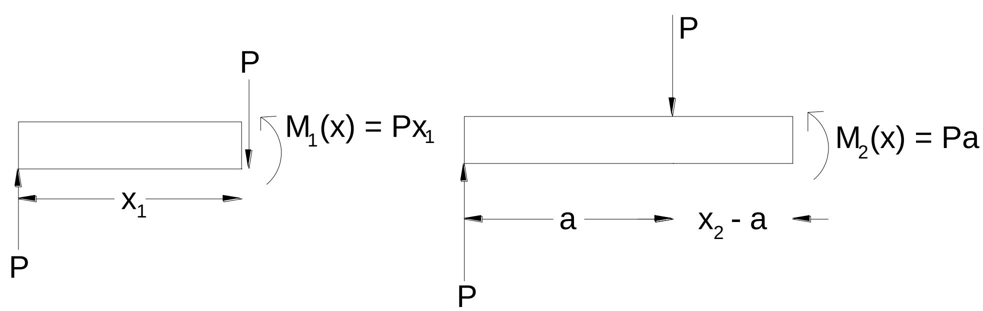
Homework No. 6 - Solutions

8-3

Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the slope at A and the maximum deflection. EI is constant



Sol:



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = Px_1$

$EI\, d^2v_1/dx_1^2 = Px_1$

$EI\, dv_1/dx_1 = Px_1^2/2 + C_1$ (1)

$EIv_1 = Px_1^2/6 + C_1x_1 + C_2$ (2)

For $M_2(x) = Pa$

$EId^2v_2/dx_2^2 = Pa$

$EIdv_2/dx_2 = Pax_2 + C_3$ (3)

$EIv_2 = Pax_2^2/2 + C_3x_2 + C_4$ (4)

Boundary Conditions

$v_1 = 0$ at $x = 0$

From eq. (2)

$C_2 = 0$

Due to symmetry:

$dv_2/dx_2 = 0$ at $x_2 = L/2$

From eq.(3)

$0 = PaL/2 + C_3$

$C_3 = -PaL/2$

Continuity conditions:

$v_1 = v_2$ at $x_1 = x_2 = a$

$Pa^3/6 + C_1a = Pa^3/2 – Pa^2L/2 + C_4$

$C_1a - C_4 = Pa^3/3 – Pa^2L/2$ (5)

$dv_1/dx_1 = dv_2/dx_2$ at $x_1 = x_2 = a$

$Pa^2/2 + C_1 = Pa^2 – PaL/2$

$C_1 = Pa^2/2 – PaL/2$

Subtitute C_1 into eq. (5)

$C_4 = Pa^3/6$

$dv_1/dx_1 = P(x_1^2 + a^2 –aL) /2EI$

$\theta_A = dv_1/dx_1\Big|_{x1=0}$ $= Pa(a – L)/2EI$ **Ans** **(Points 3)**

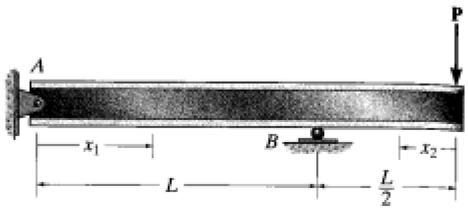
$v_1 = Px_1\{(x_1^2 + 3a(a –L)\}/6EI$ **Ans** **(Points 3)**

$v_2 = Pa\{(3x(x – L) + a^2\}/6EI$ **Ans** **(Points 3)**

$v_{max} = v_2\Big|_{x= L/2}$ $= Pa(4a^2 – 3L^2)/24EI$ **Ans** **(Point 1)**

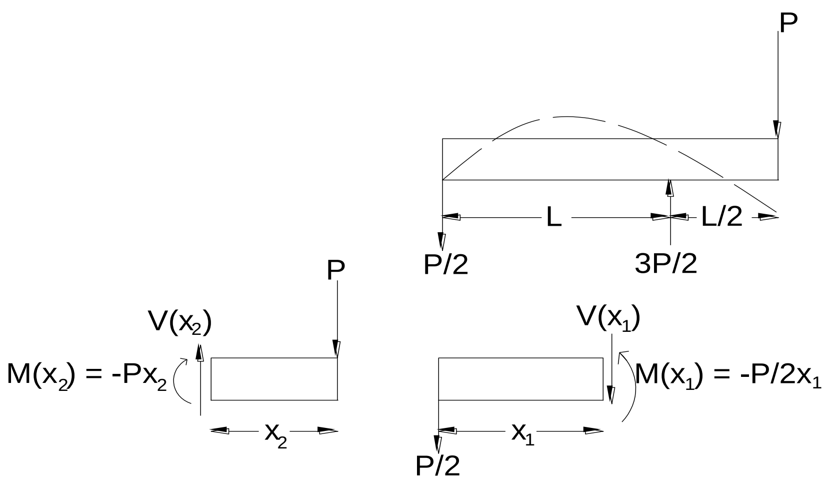
8-4

Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify beam's maximum deflection. EI is constant



Prob. 8–4

Sol:



Slope and Elastic Curve

$EId^2v/dx^2 = M(x)$

For $M(x_1) = -Px_1/2$,

$EId^2v_1/dx^2{}_1 = -\ Px_1/2$

$EIdv_1/dx_1 = -Px_1^2/4 + C_1$ (1)

$EIv_1 = -Px_1^3/12 + C_1x_1 + C_2$ (2)

For $M(x_2) = -Px_2$,

$EId^2v_2/dx_2^2 = -Px_2$

$EIdv_2/dx_2 = -Px_2^2/2 + C_3$ (3)

$EIv_2 = -Px_2^3/6 = C_3x_2 + C_4$ (4)

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$

From eq.(2), $C_2 = 0$

$v_1 = 0$ at $x_1 = L$

From eq. (2)

$0 = -PL^3/12 + C_1L$ $C_1 = PL^2/12$

$v_2 = 0$ at $x_2 = L/2$

From eq. (4),

$0 = -PL^3/48 + LC_3/2 + C_4$ (5)

Continuity Conditions:

At $x_1 = L$ and $x_2 = L/2$, $dv_1/dx_1 = -dv_2/dx_2$

From eqs (1) and (3),

$-PL^2/4 + PL^2/12 = -(PL^2/8 + C_3)$

$C_3 = 7PL^2/24$

From eq. (5)

$C_4 = -PL^3/8$

The Slope:

Substitute the value of C_1 into eq. (1)

$dv_1/dx_1 = P(L^2 – 3x_1^2)/12EI$

$dv_1/dx_1 = 0 = P(L^2 – 3x_1^2)/12EI$

$x_1 = L/\sqrt{3}$

The Elastic Curve:

Substitute the values of C_1, C_2, C_3 and C_4 into eqs (2) and (4), respectively,

$v_1 = Px_1(-x_1^2 + L^2)/12EI$ **Ans** **(Points 4)**

$v_D = v_1\big|_{x1 = L/\sqrt{3}} = P(L/\sqrt{3})(-L^2/3 + L^2)/12EI = 0.0321\ PL^3/EI$

$v_2 = P(-4x_2^3 + 7L^2x_2 – 3L^3)/24EI$ **Ans** **(Points 4)**

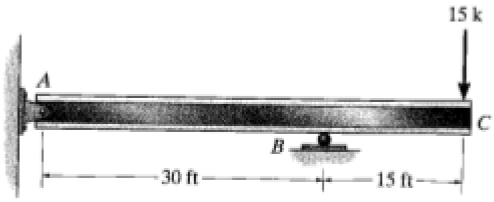
$v_c = v_2\big|_{x2 = 0} = -PL^3/8EI$

Hence,

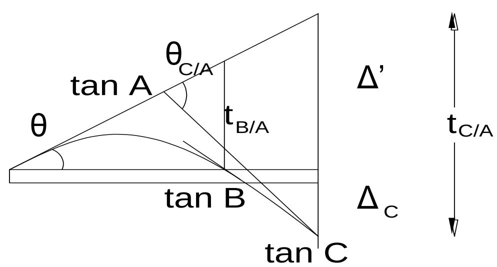
$v_{max} = v_c = PL^3/8EI$ **Ans** **(Points 2)**

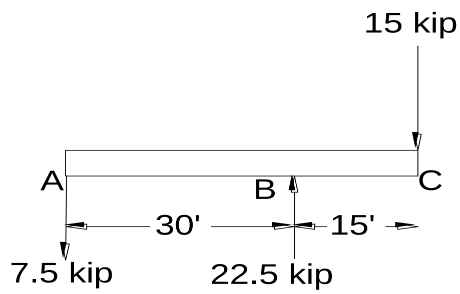
8-12

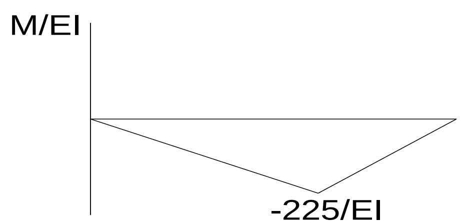
Use the moment- area theorems to determine the slope and deflection at C. EI is constant



Sol:







$\theta_A = \left|t_{B/A}\right|/30$

$t_{B/A} = ½\ (-225/EI)(30)(10) = -33750/EI$

$\theta_A = 1125/EI$

$\theta_{C/A} = ½\ (-225/EI)(30) + ½\ (-225/EI)(15) = -5062.5/EI = 5062.5/EI$

$\theta_C = \theta_{C/A} + \theta_A$

$\theta_C = 5062.5/EI - 1125/EI = 3937.5/EI$ **Ans** **(Points 5)**

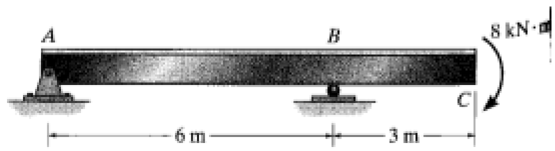
$\Delta_C = \left| t_{C/A} \right| - 45/30 \left| t_{B/A} \right|$

$t_{C/A}$ = ½ (-225/EI)(30(25) + ½ (-225/EI)(15)(10) = -101250/EI

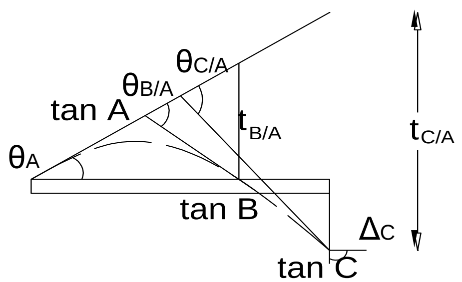
Δ_C = 101250/EI – 45/30(33750/EI) = 50625/EI **Ans** **(Points 5)**

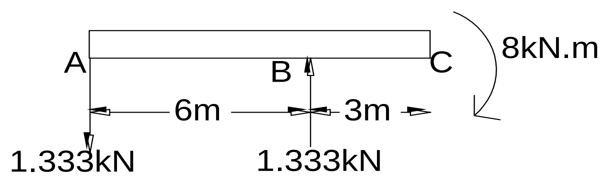
8-20

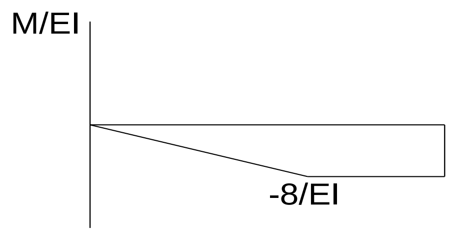
Use the moment- area theorems and determine the deflection at C and the slope of the beam at A, B and C. EI is constant.



Sol:







$t_{B/A}$ = ½ (-8/EI)(6)(2) = -48/EI

$t_{C/A}$ = ½(-8/EI)(6)(3+2) + (-8/EI)(3)(1.5) = -156/EI

$\Delta_C = |t_{C/A}| - 9/6|t_{B/A}|$ = 156/EI – 9(48)/6EI = 84/EI **Ans** **(Points 3)**

$\theta_A = |t_{B/A}|$ /6 = 8/EI **Ans** **(Points 3)**

$\theta_{B/A}$ = ½ (-8/EI)(6) = -24/EI = 24/EI

$\theta_B = \theta_{B/A} + \theta_A$

θ_B = 24/EI – 8/EI = 16/EI **Ans** **(Points 2)**

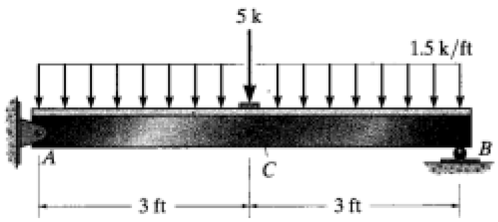
$\theta_{C/A}$ = ½ (-8/EI)(6) + (-8/EI)(3) = -48/EI = 48/EI

$\theta_C = \theta_{C/A} + \theta_A$

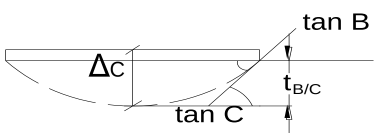
θ_C = 48/EI – 8/EI = 40/EI **Ans** **(Points 2)**

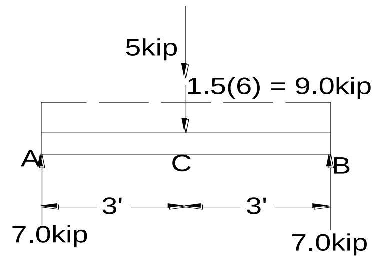
8-24

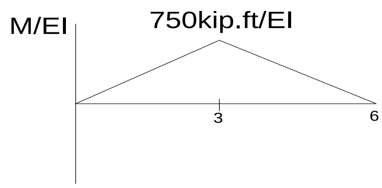
Use the moment- area theorems and determine the slope at B and the displacement at C. The member is an A-36 steel structure Tee for which I = 76.8in^4.

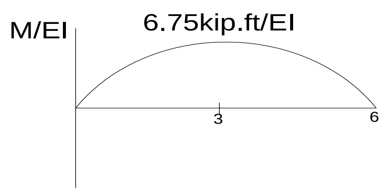


Sol:









Moment Area Theorems:

Due to symmetry, the slope at midspan C is zero. Hence the slope at B is

$\theta_B = \left|\theta_{B/C}\right| = ½ (7.50/EI) (3) + 2/3(6.75/EI) (3)$

$= 24.75 kip.ft^2/EI$

$= 24.75(144)/ \{29(10^3) (76.8)\}$

$= 0.00160$ rad **Ans** **(Points 5)**

The displacement at C is

$$\Delta_C = \left| t_{A/C} \right| = \tfrac{1}{2}(7.50/EI)(3)(2/3)(3) + 2/3(6.75/EI)(3)(5/8)(3)$$

$$= 47.8125 \text{ kip.ft}^3/EI$$

$$= 47.8125(1728)/\{29(10^3)(76.8)\}$$

$$= 0.0371 \text{ in} \downarrow$$

Ans **(Points 5)**