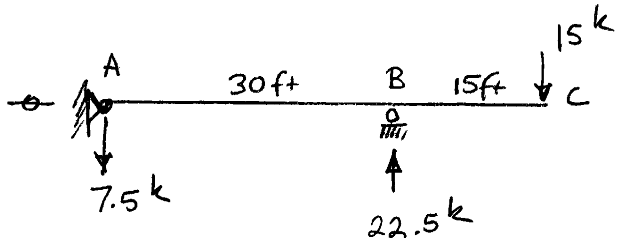
CE 371 – Section 01
HW 7

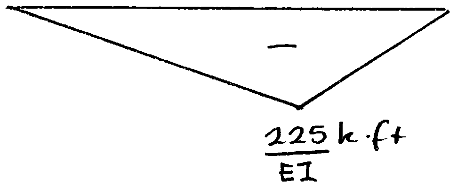
29.



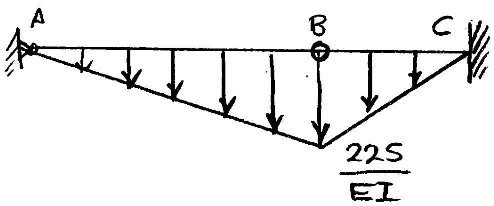
EI const

θ_C , Δ_C ?

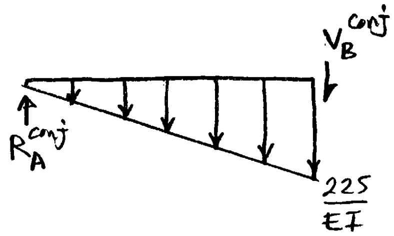
$\left[\frac{M}{EI}\right]$



<u>conjugate beam:</u>



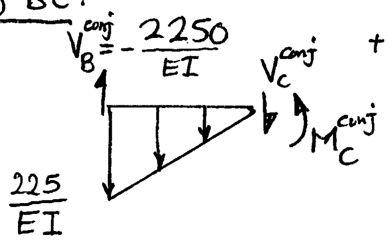
<u>FBD AB:</u>



$\curvearrowright + \sum M_{about\,A} = 0 : \quad \frac{1}{2} \cdot \frac{225}{EI} \cdot 30' \cdot 20' + V_B^{conj} \cdot 30 = 0$

$\Rightarrow V_B^{conj} = -\frac{2250}{EI}$

<u>FBD BC:</u>



$+\uparrow \sum F_{vert} = 0 : \quad -\frac{1}{2} \frac{225}{EI} \cdot 15 - \frac{2250}{EI} - V_C^{conj} = 0$

$\Rightarrow V_C^{conj} = -\frac{3938}{EI} = \theta_C$ //

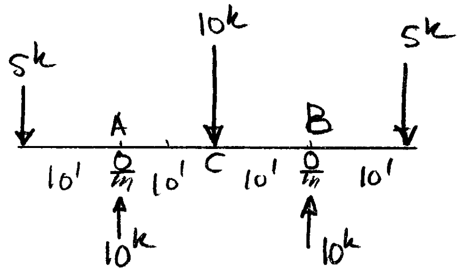
↑ means θ_C is CW, i.e. ↻

$\curvearrowright + \sum M_{about\ C} = 0 \quad : \quad -\frac{1}{2}\,\frac{225}{EI}\,15'(10') + \left(-\frac{2250}{EI} \times 15\right) - M_C^{conj} = 0$

$M_C^{conj} = -\frac{50{,}625}{EI} = \Delta_C$

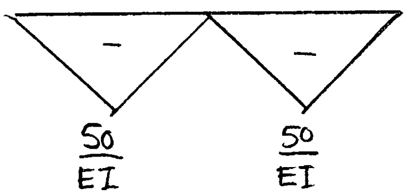
means Δ_C is $\downarrow$

30.

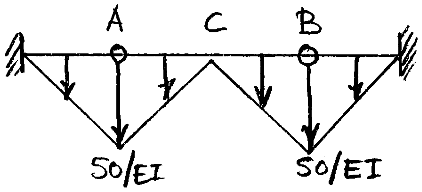


EI const.

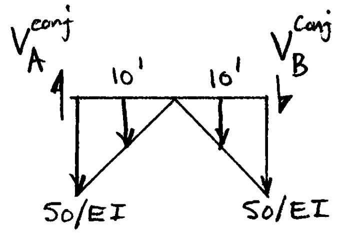
$\left[\frac{M}{EI}\right]$



Conj. beam



FBD ACB:



$\circlearrowleft\!+ \sum M_{about\ B} = 0$:

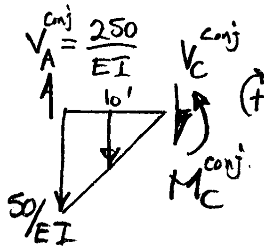
$$V_A^{conj} \cdot 20' - \frac{1}{2}\frac{50}{EI}\cdot 10' \cdot \left(10' + \frac{2}{3}10'\right) - \frac{1}{2}\frac{50}{EI}10' \cdot \frac{10'}{3} = 0$$

$$V_A^{conj} = \frac{250\ k \cdot ft^2}{EI} = \theta_A \text{ (CCW)}$$

$$\sum F_{vert} = 0 \ ; \Rightarrow V_B^{conj} = -\frac{250\ k \cdot ft^2}{EI} = \theta_B$$

θ_B is CW

FBD AC:

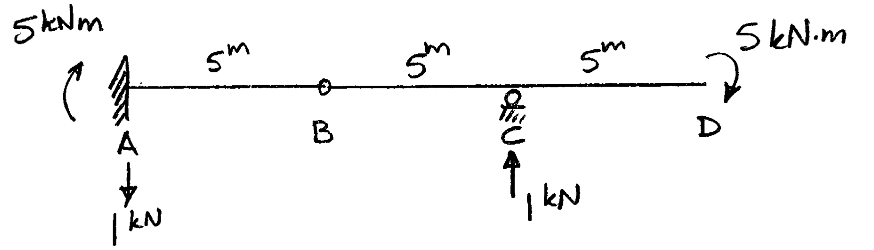


$\circlearrowleft\!+ \sum M_{about\ C} = 0$;

$$\frac{250}{EI} \cdot 10' - \frac{1}{2}\frac{50}{EI} \cdot 10' \cdot \frac{2}{3}10' - M_C^{conj} = 0$$

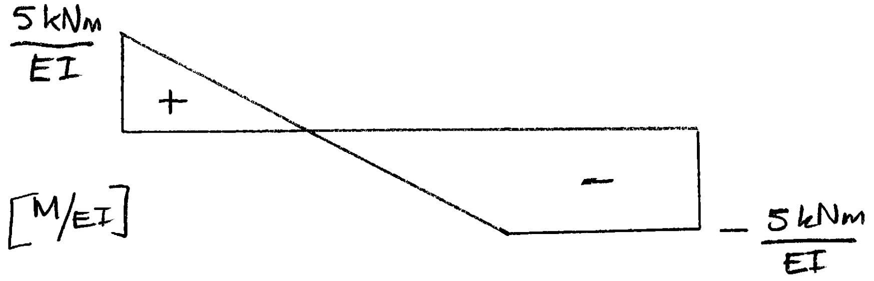
$$\Rightarrow M_C^{conj} = \frac{833\ k \cdot ft^3}{EI} = \Delta_C \uparrow$$

31.

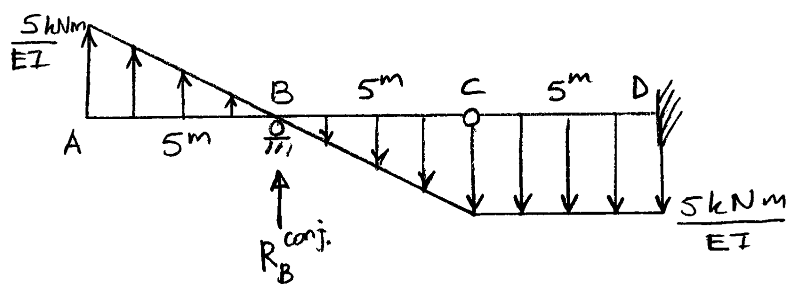


EI Const.

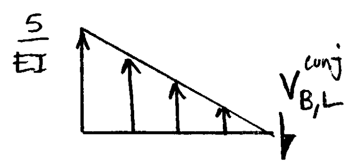
$\theta_{B,L}$, $\theta_{B,R}$, Δ_D



Conj. beam



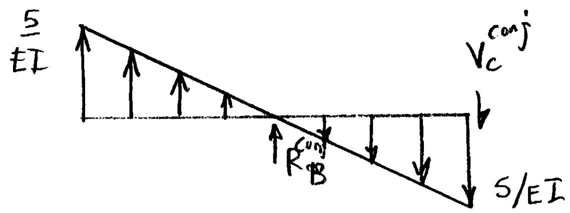
FBD AB (just to the left of support @ B)



$+\uparrow \sum F_{vert} = 0$: $\quad \frac{1}{2}\frac{5}{EI} \cdot 5^m - V_{B,L}^{conj} = 0$

$$\Rightarrow V_{B,L}^{conj} = \frac{12.5 \text{ kNm}^2}{EI} = \theta_{B,L} \text{ (CCW)}$$

FBD ABC (left of hinge)



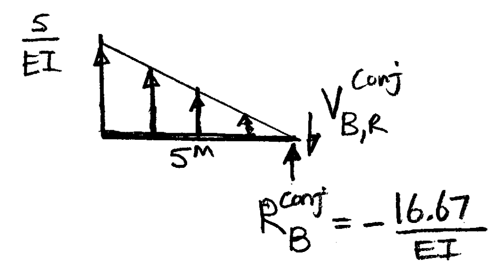
$\curvearrowright + \sum M_{about\ C} = 0$

$$\frac{1}{2}\frac{5}{EI} \cdot 5^m \left(5^m + \frac{2}{3}5^m\right) + R_B \cdot 5^m - \frac{1}{2}\frac{5}{EI} \cdot 5 \cdot \frac{1}{3} 5^m = 0$$

$$\Rightarrow R_B = -\frac{16.67 \text{ kNm}^2}{EI}$$

$$\sum F_{vert} = 0 \; : \; \Rightarrow \; V_c^{conj} = -\frac{16.67 \, kNm^2}{EI}$$

FBD AB (just to the right of support @ B)



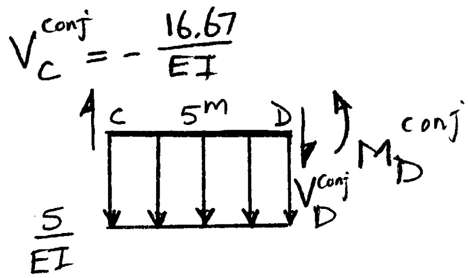
$$+\uparrow \sum F_{vert} = 0$$

$$\frac{1}{2} \frac{5}{EI} 5^m + \left(-\frac{16.67}{EI}\right) - V_{B,R}^{conj} = 0$$

$$\Rightarrow V_{B,R}^{conj} = -\frac{4.167 \, kNm^2}{EI} = \theta_{B,R}$$

(↑ CW)

FBD CD

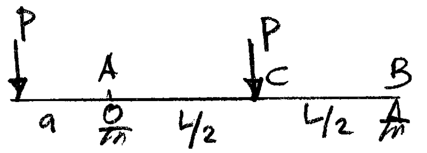


$$\circlearrowleft + \sum M_{about\,D} = 0 \; : \quad \left(-\frac{16.67}{EI}\right) 5^m - \frac{5}{EI} \cdot 5^m \times (2.5^m) - M_D^{conj} = 0$$

$$M_D^{conj} = -\frac{145.83 \, kNm^3}{EI} = \Delta_D \downarrow$$

(↑ implies downward disp. in the actual beam)

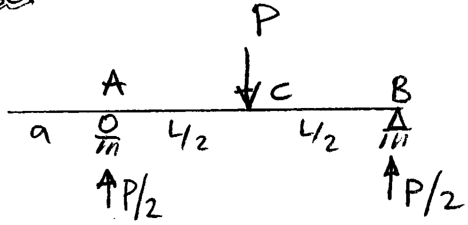
32)



EI const. want $\theta_A = 0$

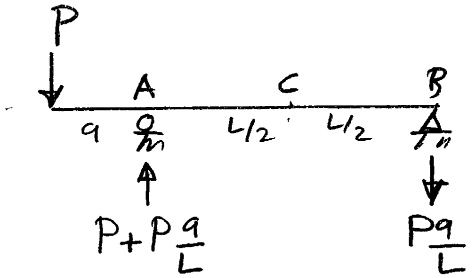
let's use principle of superposition and consider each load separately.

subcase 1

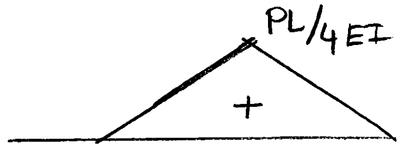


+

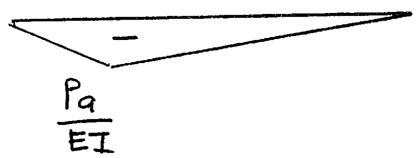
subcase 2



$\left[\frac{M}{EI}\right]$

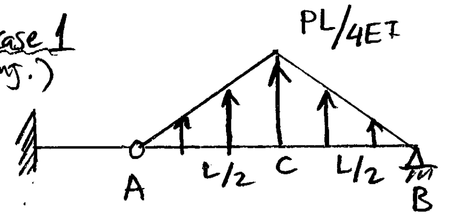


$\left[\frac{M}{EI}\right]$

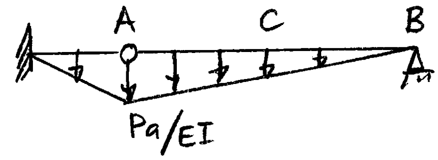


we can consider conjugate beam for each subcase

subcase 1 (conj.)

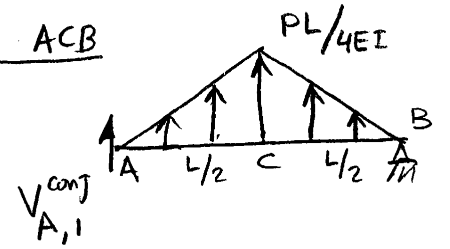


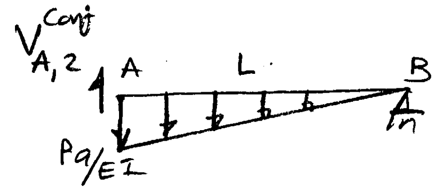
subcase 2 (conj.)



$\theta_A = 0 \Rightarrow V_A^{conj} = 0$

FBD ACB





$\circlearrowright + \sum M_{about\ B} = 0$

$\Rightarrow V_{A,1}^{conj} = -\frac{PL^2}{16EI} = \theta_{A,1}$ ↻

$\circlearrowright + \sum M_{about\ B} = 0$

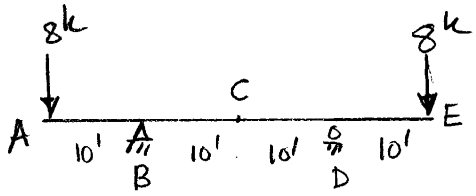
$V_{A,2}^{conj} = \frac{PaL}{3EI} = \theta_{A,2}$ ↺

$\theta_A = 0$

$$\Rightarrow \quad -\frac{PL^2}{16EI} + \frac{PaL}{3EI} = 0$$

$$\Rightarrow \quad a = \frac{3}{16} L$$

33)

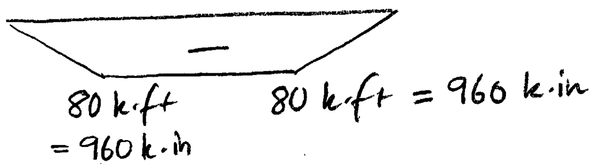


$I_{AB} = I_{DE} = 400 \text{ in}^4$

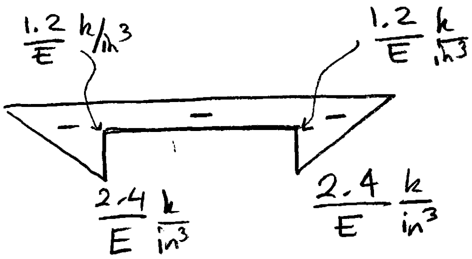
$I_{BD} = 800 \text{ in}^4$

$E = 29{,}000 \text{ ksi}$

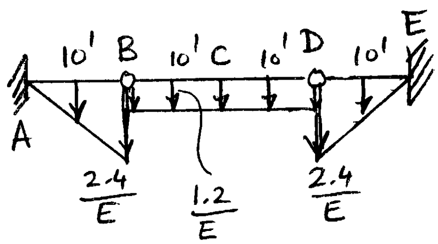
[M]



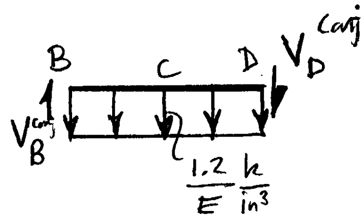
$\left[\frac{M}{EI}\right]$



conj. beam



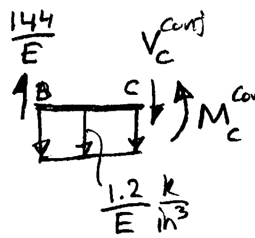
FBD BCD



$V_B^{conj} = -V_D^{conj} = \frac{144}{E} \frac{k}{in^2}$

$\Rightarrow \theta_B = \frac{144}{E} \frac{k}{in^2} = \frac{144}{29{,}000} \approx 5 \times 10^{-3} \text{ rad}$

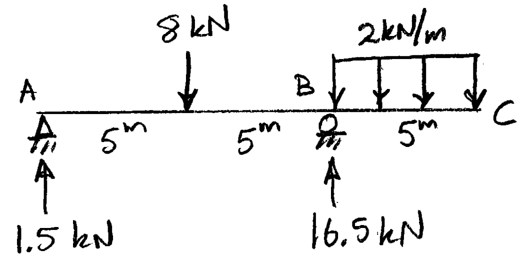
FBD BC



$\circlearrowleft \sum M_{about\ C} = 0; \quad \frac{144}{E}(10 \times 12) - \frac{1.2}{E}(10 \times 12)(5 \times 12) - M_C^{conj} = 0$

$\Rightarrow M_C^{conj} = \frac{8640}{E} \frac{k}{in} = 0.3 \text{ in} = \Delta_C \uparrow$

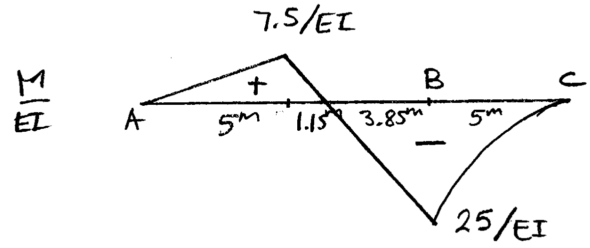
34)



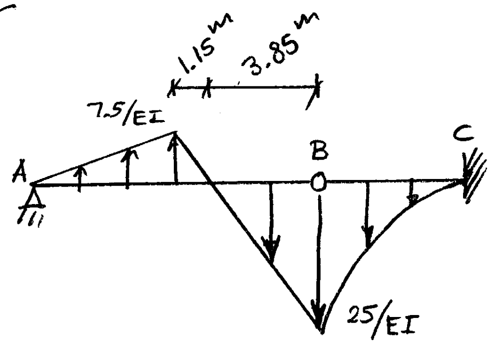
$E = 200$ GPa

$I = 70 \times 10^6 \ mm^4$

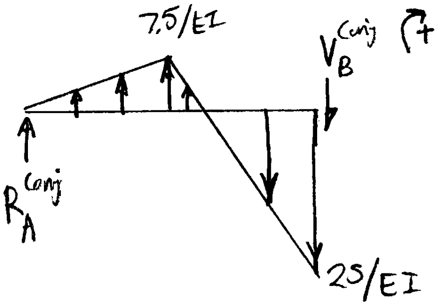
$\theta_c = ?$



conj. beam



FBD AB



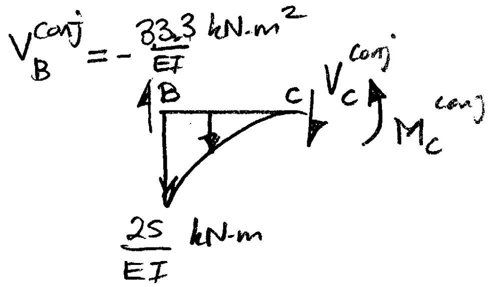
$\curvearrowright + \sum M_{about\ A} = 0$

$$-\frac{1}{2} \times \frac{7.5}{EI} \times 5 \times \left(\frac{2}{3} \times 5\right) - \frac{1}{2} \times \frac{7.5}{EI} \times (1.15)\left(5 + \frac{1}{3} \times 1.15\right)$$

$$+ \frac{1}{2} \frac{25}{EI} \times (3.85)\left(5 + 1.15 + \frac{2}{3} \times 3.85\right) + V_B^{conj} \times 10 = 0$$

$$V_B^{conj} = -\frac{33.3}{EI} \ kNm^2$$

FBD BC



$+\uparrow \sum F_{vert} = 0 : \quad -\frac{33.3}{EI} - \frac{1}{3}\, 25 \times 5 - V_c^{conj} = 0$

$$V_c^{conj} = -\frac{75}{EI}\ kN.m^2 = \theta_c$$

converting to matching units

$$\theta_c = -\frac{75}{200 \times 10^6 \times 70 \times 10^6 \times (1{,}000)^4} = -5.36 \times 10^{-3}\ rad$$

CW