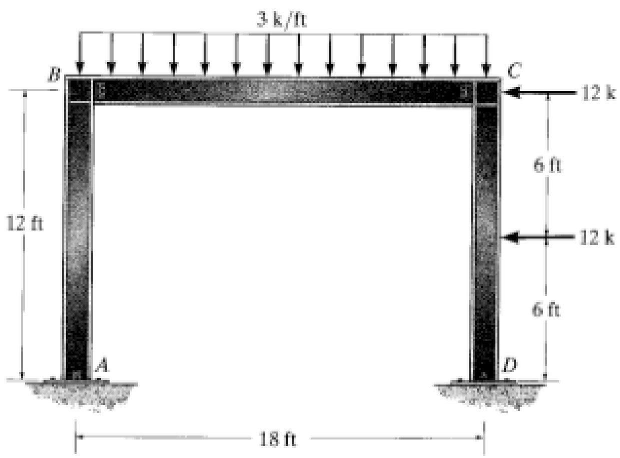
CE 371 - Section 002

HW No. 10

√**11–21.** Determine the moments at the ends of each member. Assume *A* and *D* are pins and *B* and *C* are fixed-connected joints *EI* is the same for all members.



Sol:

$FEM_{BC} = -wL^2/12 = -81$; $\quad$ $FEM_{CB} = wL^2/8 = 81$

$FEM_{CD} = -3PL/16 = -27$

$\Psi_{BC} = 0$

$\Psi_{AB} = \Psi_{CD} = \Psi$

$M_{BA} = 3EI(\theta_B + \Psi)/12$

$M_{BC} = 2EI(2\theta_B + \theta_C)/18 - 81$

$M_{CB} = 2EI(2\theta_C + \theta_B)/18 + 81$

$M_{CD} = 3EI(\theta_C + \Psi)12 - 27$

$M_{BA} + M_{BC} = 0$; $\quad$ $0.472EI\,\theta_B + 0.11EI\theta_C + 0.25\,EI\,\Psi = 81$ $\quad$ (Point 1)

$M_{CB} + M_{CD} = 0$; $\quad$ $0.11EI\theta_B + 0.472EI\theta_C + 0.25EI\,\Psi = -54$ $\quad$ (Point 1)

$+\Sigma M_B = 0$; $\quad$ $M_{BA} - 12V_A = 0$

$V_A = M_{BA}/12$ $\quad$ (Point 1)

$+\Sigma M_C = 0$; $\quad$ $M_{CD} + 12(6) - 12V_D = 0$

$V_D = M_{CD}/12 + 6$ $\quad$ (Point 1)

From FBD of frame:

$\overset{+}{\rightarrow}\Sigma F = 0;$ $V_A + V_D = 24$ (Point 1)

$M_{BA} + M_{CD} = 216$

$0.25EI\theta_B + 0.25EI\theta_C + 0.5EI\,\Psi = 243$ (Point 1)

$\theta_B = -137.077/EI$

$\theta_C = -510.923/EI$

$\Psi = 810/EI$

$M_{BA} = 168$k.ft Ans (Point 1)

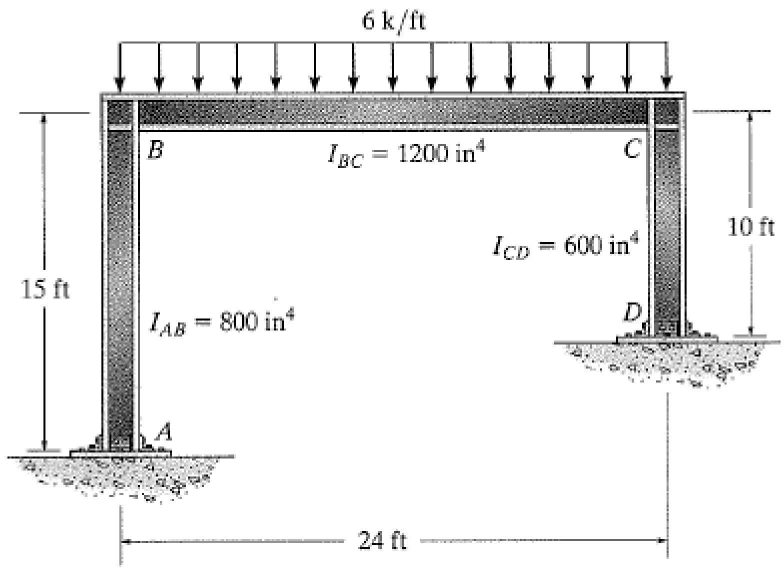
$M_{BC} = -168$k.ft Ans (Point 1)

$M_{CB} = -47.8$k.ft Ans (Point 1)

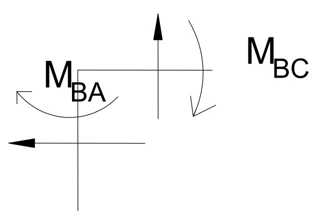
$M_{CD} = 47.8$k.ft Ans (Point 1)

11-22.

Using the slope deflection method, determine the moments at the ends of each member. Assume the supports at A and D are fixed. The moment of inertia of each member is shown on the figure. Let E = 29,000ksi.



Sol:



$FEM_{BC} = -wL^2/12 = -288$ $\qquad$ $FEM_{CB} = wL^2/12 = 288$

$\theta_A = \theta_D = \Psi_{BC} = 0$

$\Psi_{AB} = \Psi = \Delta/15$

$\Psi_{DC} = \Delta/10 = 1.5\Psi$

$M_N = 2EI(2\theta_N + \theta_F - 3\Psi)/L + FEM_N$

$M_{AB} = 2E(800)(\theta_B - 3\Psi)/15$

$M_{BA} = 2E(800)(2\theta_B - 3\Psi)/15$

$M_{BC} = 2E(1200)(2\theta_B + \theta_C)/24 - 288$

$M_{CB} = 2E(1200)(2\theta_C + \theta_B)/24 + 288$

$M_{CD} = 2E(600)(2\theta_C - 3(1.5)\Psi)/10$

$M_{DC} = 2E(600)(\theta_C - 3(1.5)\Psi)/10$

Moment equilibrium at B and C:

$M_{BA} + M_{BC} = 0;$ $413.3\theta B + 100E\theta C - 320\ \Psi = 288$ (1)

$M_{CB} + M_{CD} = 0;$ $100E\theta_B + 440E\theta_C - 540E\ \Psi = -288$ (2)

(Point 1)

From FBDs of members AB and CD

$\curvearrowleft + \Sigma M_B = 0;$ $M_{BA} + M_{AB} + V_A(15) = 0$ (Point 1)

$\curvearrowleft + \Sigma M_C = 0;$ $M_{CD} + M_{DC} + V_D(10) = 0$ (Point 1)

For entire frame,

$\Sigma F_X = 0;$ $V_A + V_D = 0$

Thus,

$M_{BA} + M_{AB} + 1.5(M_{CD} + M_{DC}) = 0$

$320E\ \theta_B + 540E\theta_C - 2260E\ \Psi = 0$ (3)

(Point 1)

Solving 1, 2 and 3,

$\theta_B = 0.84589/E$

$\theta_C = -0.99016/E$

$\Psi = -0.11681/E$

Thus,

$M_{AB} = 128$k.ft Ans (Point 1)

$M_{BA} = 218$k.ft Ans (Point 1)

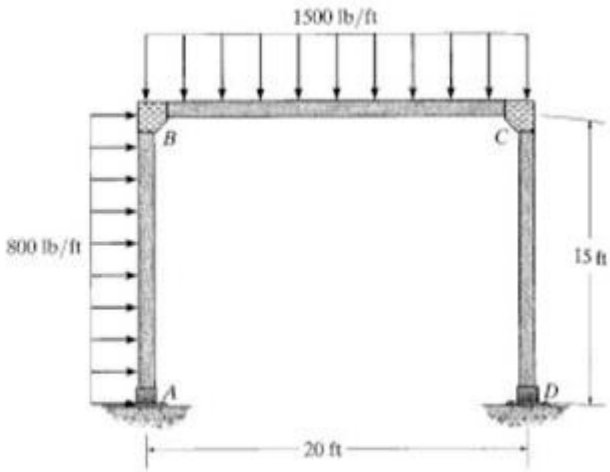
$M_{BC} = -218$k.ft Ans (Point 1)

$M_{CB} = 175$k.ft Ans (Point 1)

$M_{CD} = -175$k.ft Ans (Point 1)

$M_{DC} = -55.7$k.ft Ans (Point 1)

*11–24. Determine the moments acting at the ends of each member. EI is the same for all members. Assume all joints are fixed.



Prob. 11–24

Sol:

$FEM_{AB} = -wL^2/12 = -15;$ $FEM_{BA} = wL^2/12 = 15$

$FEM_{BC} = -wL^2/12 = -50$ $FEM_{CB} = wL^2/12 = 50$

$\theta_A = \theta_D = \Psi_{BC} = 0$

$\Psi_{AB} = \Psi_{CD} = \Psi$

$M_N = 2EI\,(2\theta_N + \theta_F - 3\Psi)/L + FEM_N$

$M_{AB} = 2EI\,(\theta_B - 3\Psi)/15 - 15$

$M_{BA} = 2EI\,(2\theta_B - 3\,\Psi)/15 + 15$

$M_{BC} = 2EI\,(2\theta_B + \theta_C)/20 - 50$

$M_{CB} = 2EI\,(2\theta_C + \theta_B)/20 + 50$

$M_{CD} = 2EI\,(2\theta_C - 3\,\Psi)/15$

$M_{DC} = 2EI\,(\theta_C - 3\,\Psi)/15$

$M_{BA} + M_{BC} = 0;$ $0.4667EI\,\theta_B + 0.1EI\,\theta_C - 0.4EI\,\Psi = 35$ (1) (Point 1)

$M_{CB} + M_{CD} = 0;$ $0.1EI\,\theta_B + 0.4667EI\theta_C - 0.4EI\,\Psi = -50$ (2) (Point 1)

$\curvearrowleft + \Sigma M_B = 0;$ $M_{BA} + M_{AB} + V_A(15) - 90 = 0$ (Point 1)

$\curvearrowleft + \Sigma M_C = 0;$ $M_{CD} + M_{DC} + V_D(15) = 0$ (Point 1)

$V_A + V_D = 12$

Thus,

$M_{BA} + M_{AB} + M_{CD} + M_{DC} = -90$

$0.4EI\,\theta_B + 0.4EI\theta_C - 1.6EI\,\Psi = -90$

$\theta_B = 156.818/EI$

$\theta_C = -75/EI$

$\Psi = 76.704/EI$

Thus,

$M_{AB} = -24.8$ k.ft (Point 1)

$M_{BA} = 26.1$ k.ft (Point 1)

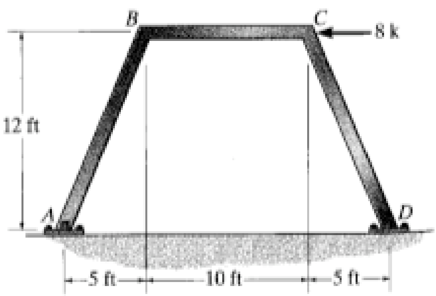
$M_{BC} = -26.1$ k.ft (Point 1)

$M_{CB} = 50.7$ k.ft (Point 1)

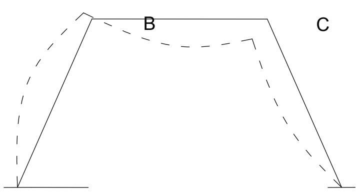
$M_{CD} = -50.7$ k.ft (Point 1)

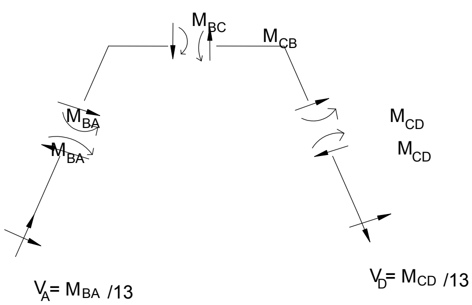
$M_{DC} = -40.7$ k.ft (Point 1)

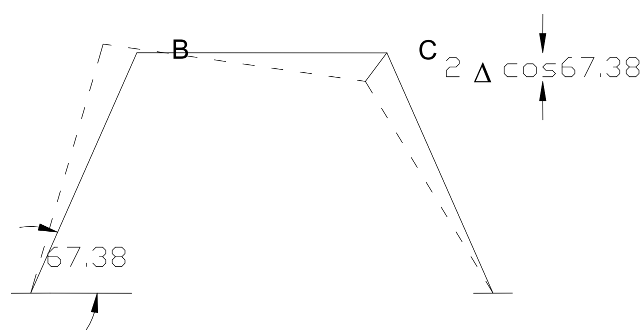
11–25. Determine the moment at each joint of the battered-column frame. The supports at *A* and *D* are pins. *EI* is constant.

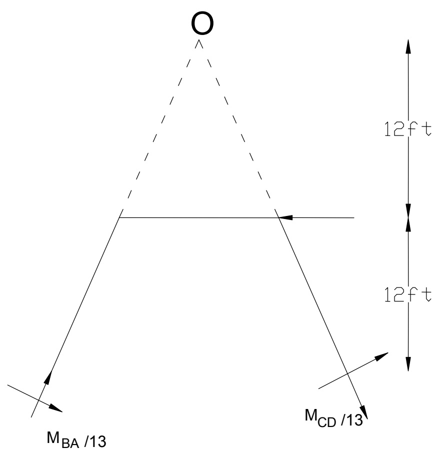


Sol:









$FEM_{BA} = FEM_{BC} = FEM_{CB} = FEM_{CD} = 0$

$\Psi_{AB} = \Psi_{DC} = \Delta/13;$ $\Psi_{BC} = 2\ \Delta Cos67.38°/10$

$\Psi_{AB} = \Psi_{DC} = \Psi_{BC}$

$M_N = 3EI(\theta_N - \Psi)/L + FEM_N$

$M_{BA} = 3EI\ (\theta_B + \Psi_{AB})/13 + 0$

$M_{BA} = 0.2308EI(\theta_B + \Psi_{AB})$

$M_N = 2EI\ (2\theta_N + \theta_F - 3\ \Psi)/L + FEM_N$

$M_{BC} = 2EI\ (2\theta_B + \theta C - 3\ \Psi_{AB})/10 + 0$

$M_{BC} = 0.2EI\ (2\theta_B + \theta_C - 3\Psi_{AB}) + 0$

$M_{CB} = 2EI\ (2\theta_C + \theta_B - 3\Psi_{AB})/10 + 0$

$M_{CB} = 0.2EI(2\theta_C + \theta_B - 3\Psi_{AB}) + 0$

$M_N = 3EI(\theta_N - \Psi)/L + FEM_N$

$M_{CD} = 3EI(\theta_C + \Psi_{AB})/L + 0$

$M_{CD} = 0.2308EI\ (\theta_C + \Psi_{AB})$

$M_{BA} + M_{BC} = 0$

$M_{CD} + M_{CB} = 0$

$+\Sigma M_O = 0;$ $M_{BA}(26)/13 + M_{CD}(26)/13 - 8(12) = 0$ (Points 2)

$2M_{BA} + 2M_{CD} - 96 = 0$

$\theta_B = \theta_C = 32/EI$

$\Psi_{AB} = 72/EI$

M_{BA} = 24k.ft	Ans	(Points 2)
M_{BC} = -24k.ft	Ans	(Points 2)
M_{CB} = -24k.ft	Ans	(Points 2)
M_{CD} = 24k.ft	Ans	(Points 2)