

1/12/15

①

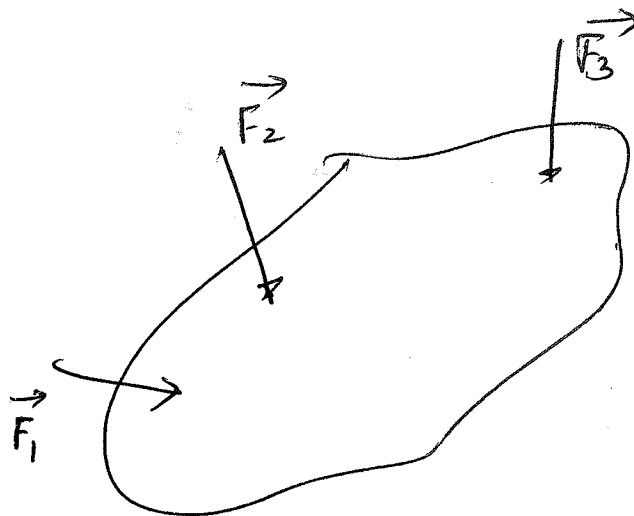
CE 270

Mechanics of Materials

$$\vec{F} = m \times \vec{a}$$

$$\vec{a} \rightarrow 0$$

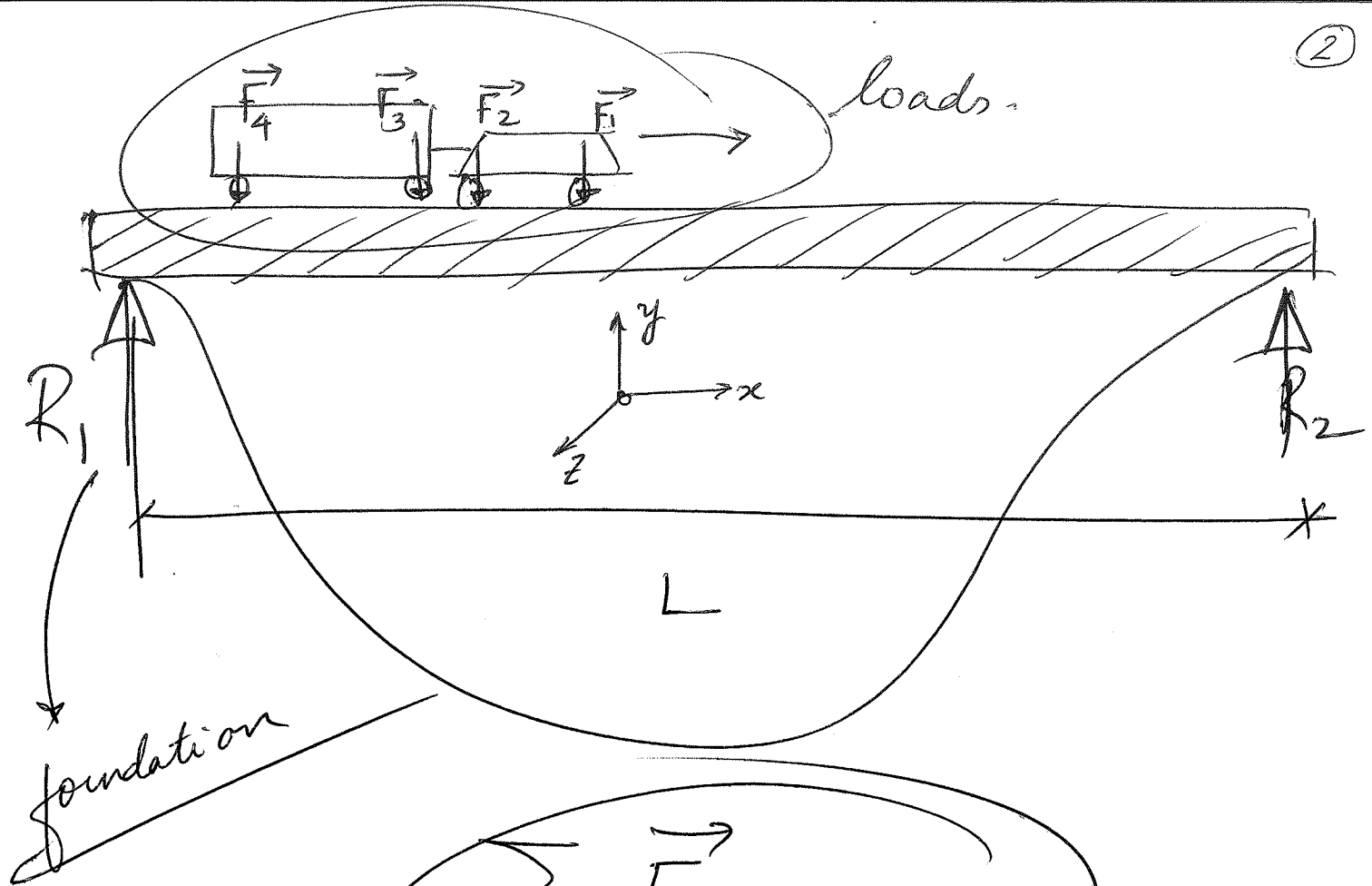
$$\vec{F} = 0$$



Static Equilibrium!

$$\sum \vec{F} = 0$$

(2)

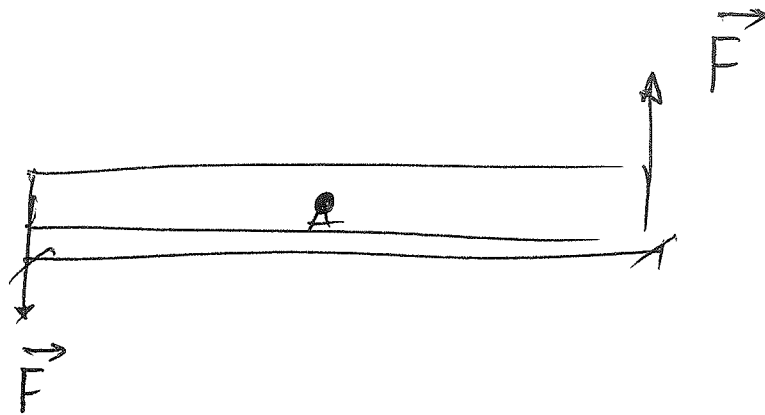


$$\sum \vec{F} = 0$$

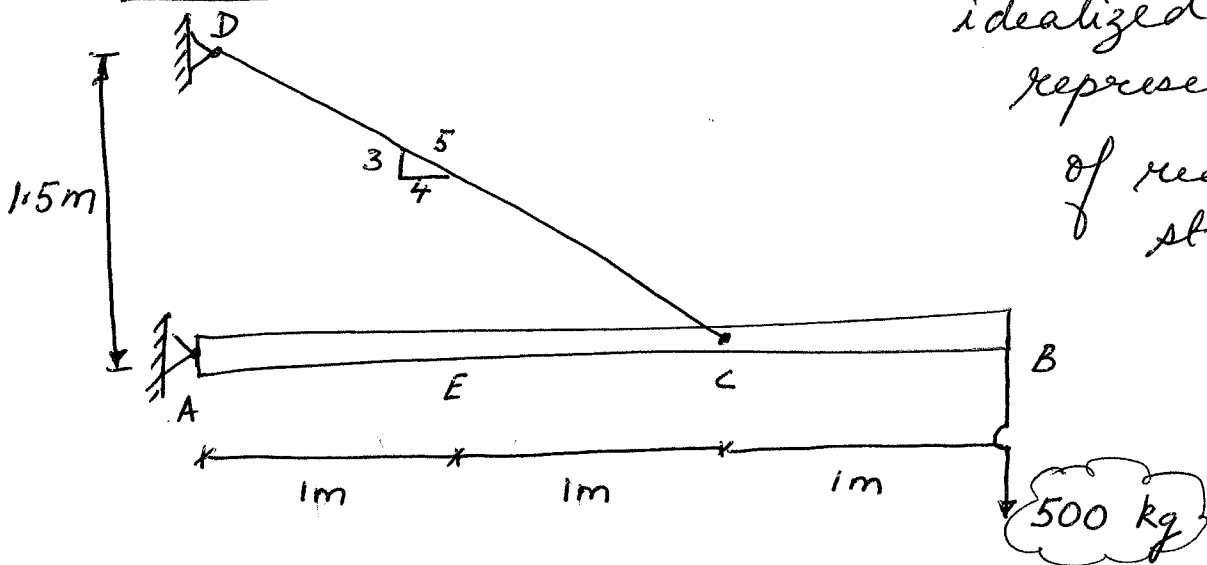
$$\rightarrow \sum \vec{F}_x = 0$$

$$\uparrow \sum \vec{F}_y = 0$$

$$\curvearrowright \sum M_o = 0$$



1/14/15

Example:

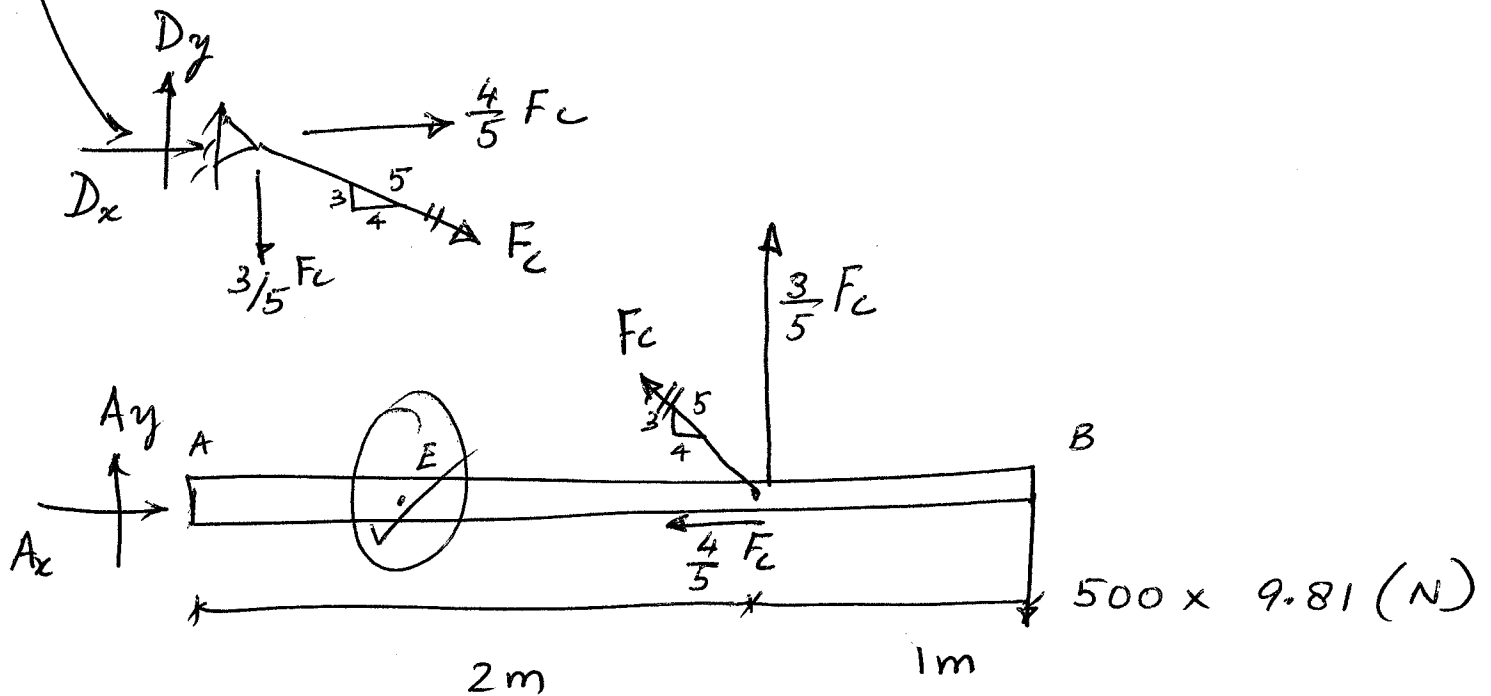
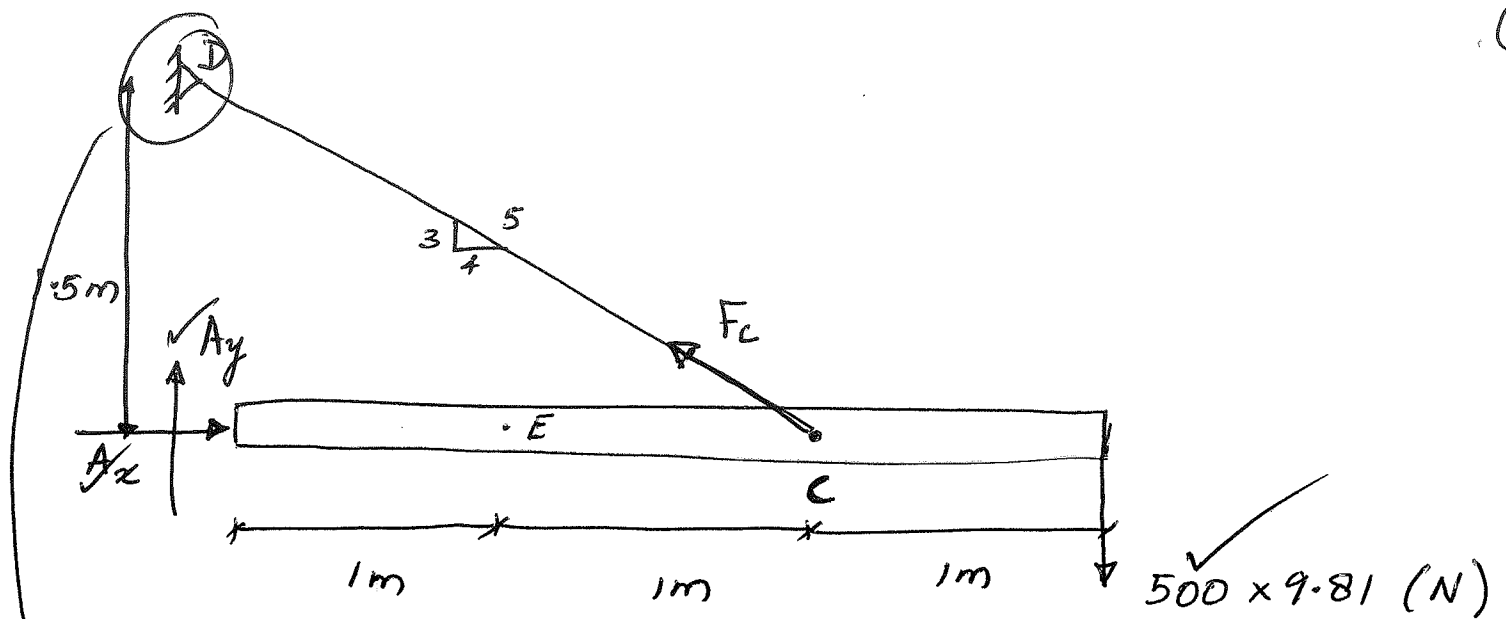
idealized
representation
of real
structure.

① Units are in SI

$$500 \text{ kg} \times 9.81 \text{ m/s}^2 = 500 \times 9.81 \text{ kg m/s}^2$$

N

② Construct the FBD of the problem!
Shows all the forces & all relevant
dimensions of the problem.
while being careful with units!



$$\rightarrow \sum F_x = 0 \quad \therefore A_x - \frac{4}{5} F_c = 0$$

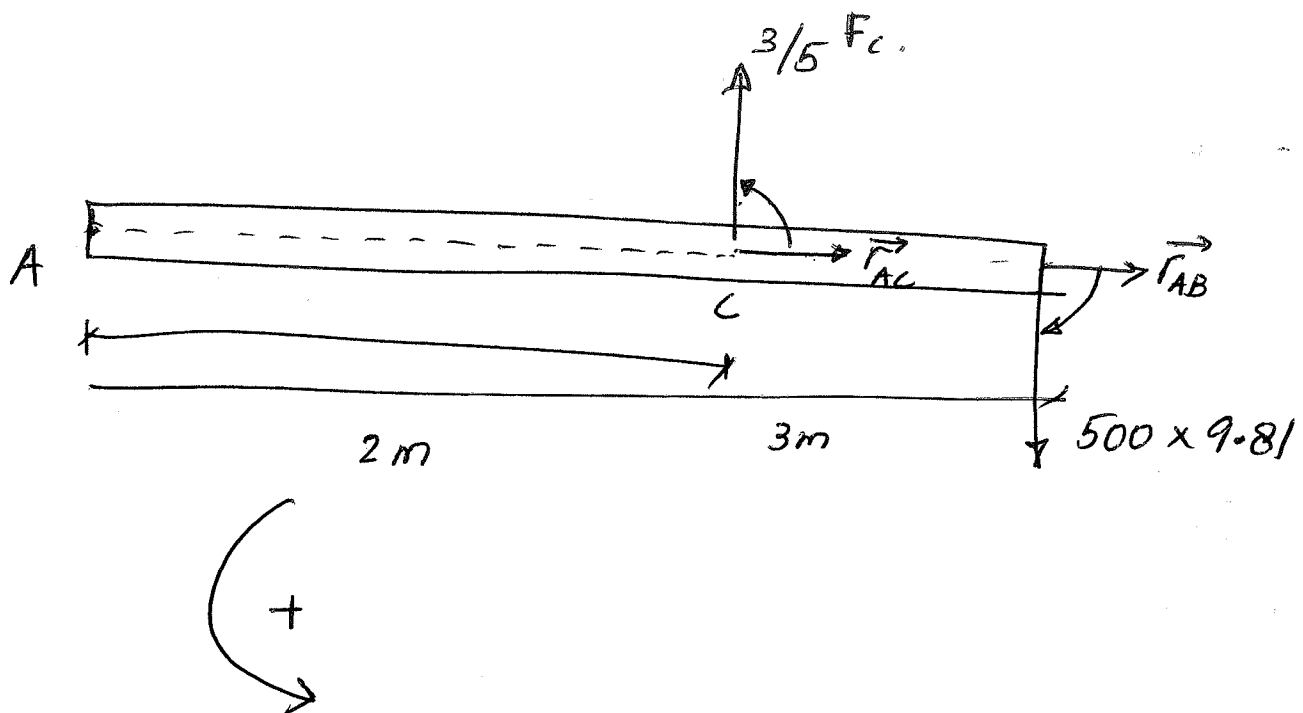
$$\uparrow \sum F_y = 0 \quad \therefore A_y + \frac{3}{5} F_c - 500 \times 9.81 \text{ (N)} = 0$$

$$\curvearrowright \sum M_A = 0 \quad \therefore 2\text{m} \times \frac{3}{5} F_c - 3\text{m} \times 500 \times 9.81 = 0$$

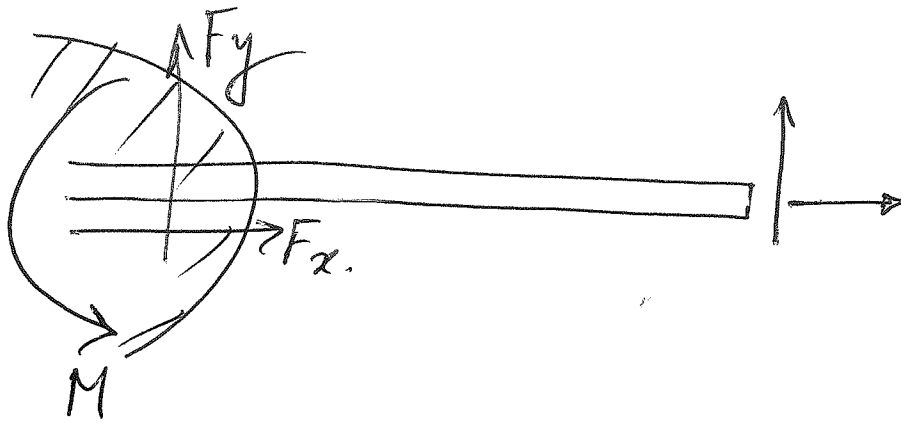
$$A_x = 9810 \text{ N} \quad A_y = -2452.5 \text{ N}$$

$$F_c = 12,262.5 \text{ N}$$

5



(6)



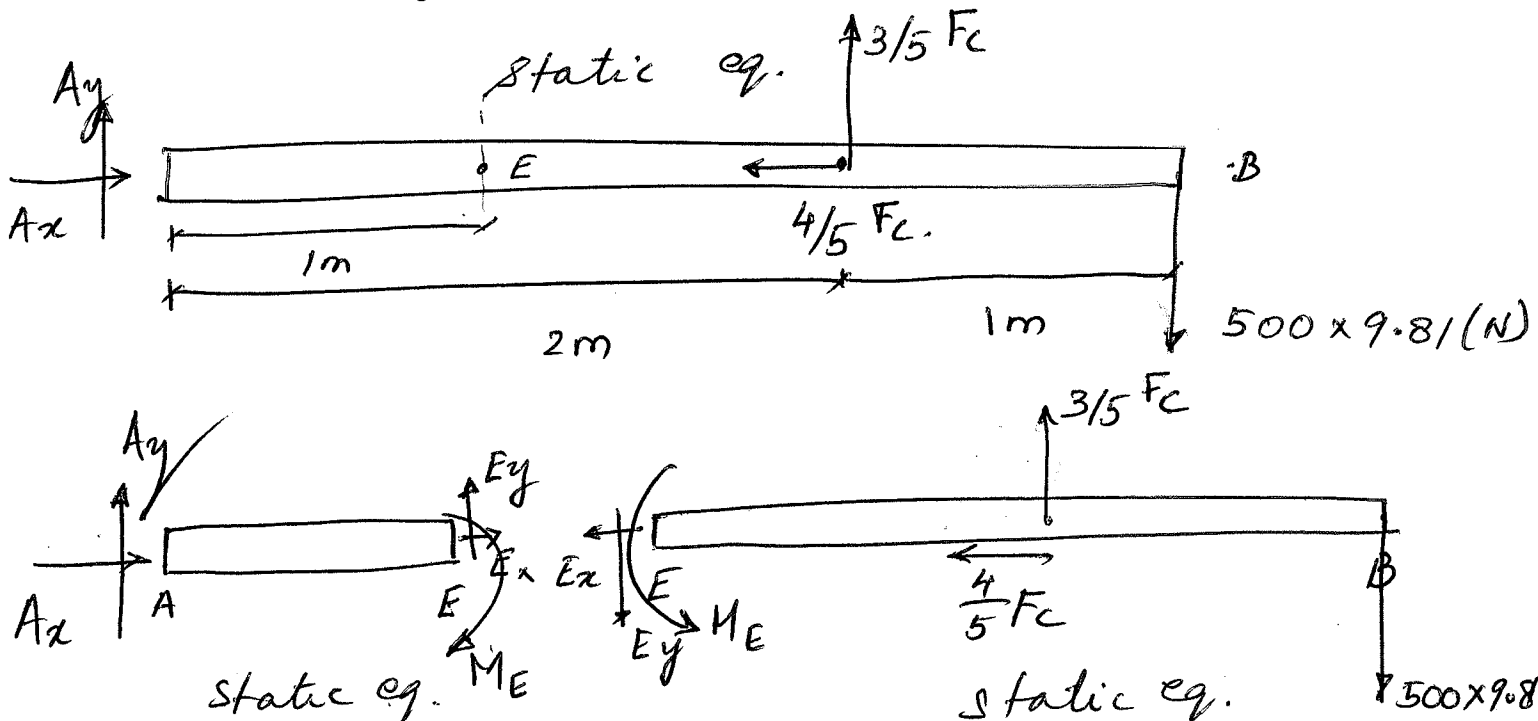
Equations of Equilibrium

Reaction & Supports

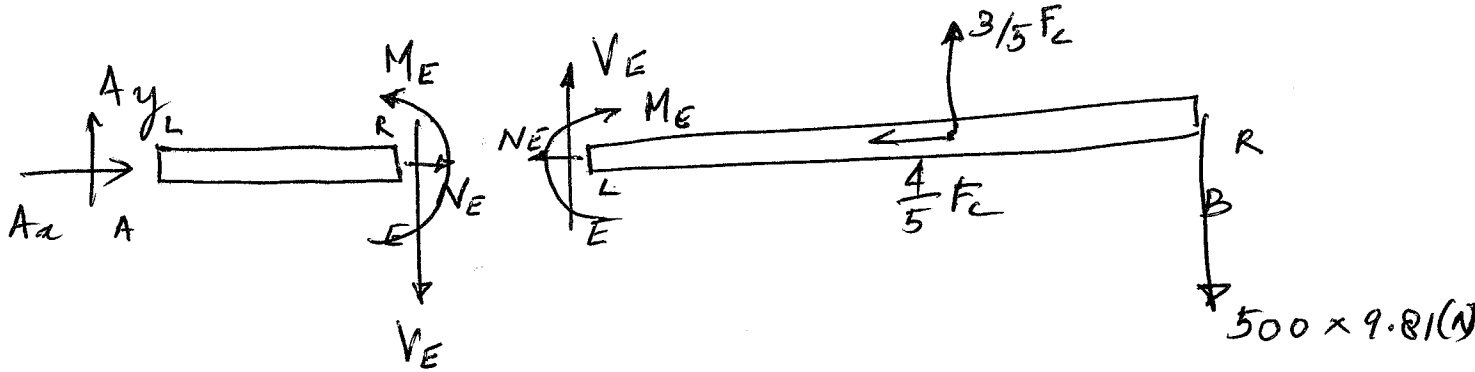
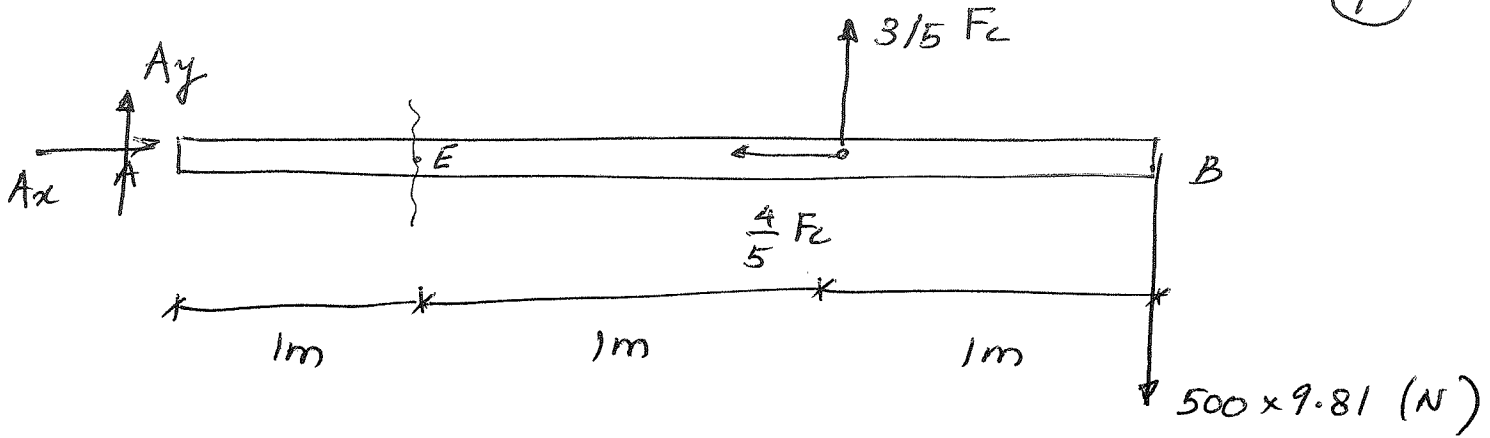
FBD & write solution for reactions!

What are internal forces?

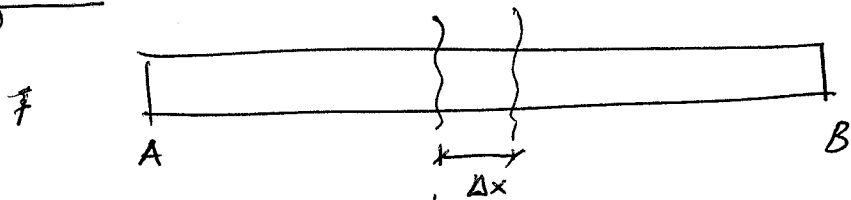
Why do I want to calculate them



7



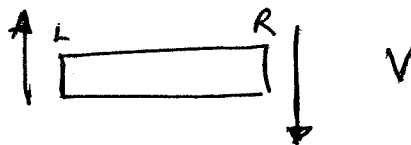
Internal forces:



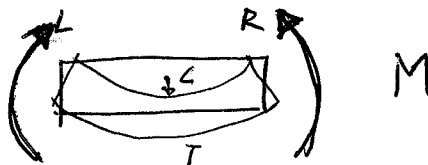
Sign conventions



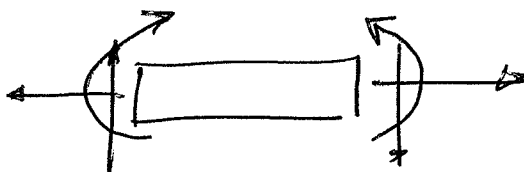
Axial Force
 $+N \rightarrow$ axial tension
 $-N \rightarrow$ compression

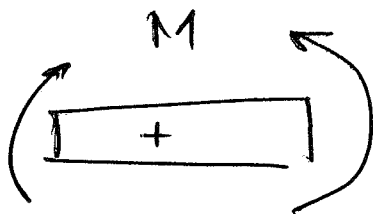
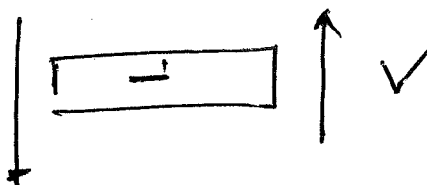
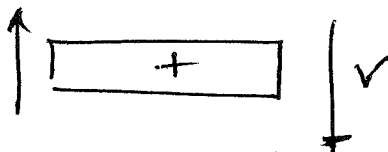


Shear Force
 $+V \rightarrow$ rotate clockwise
 $-V \rightarrow$ rotate counter clockwise

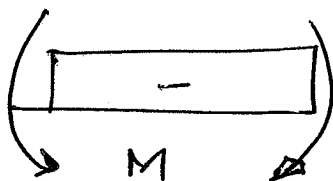


Bending Moment
 $+M \rightarrow$ compression on top
 $-M \rightarrow$ tension on top



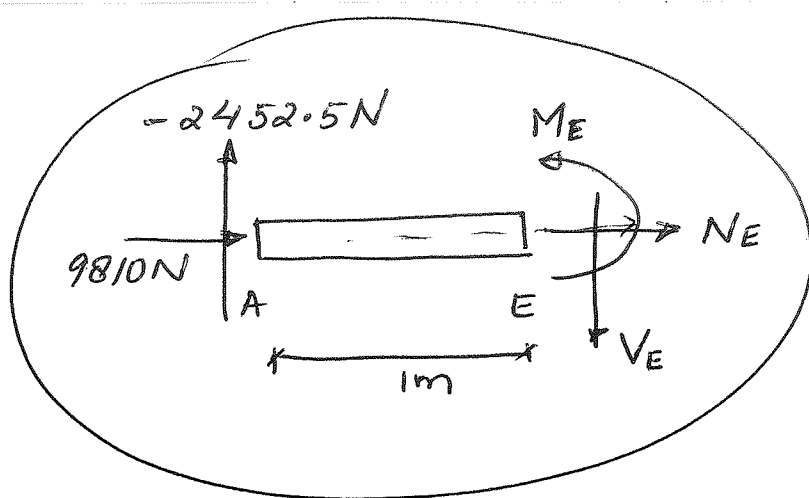


 sagging



 hogging

(9)



State of static equilibrium.

$$\rightarrow \sum \bar{F}_x = 0 \quad \therefore N_E + 9810 \text{ N} = 0$$

$$\uparrow \sum F_y = 0 \quad \therefore -2452.5 - V_E = 0$$

$$\curvearrowright \sum M_A = 0 \quad \therefore -V_E \times 1\text{m} + M_E = 0$$

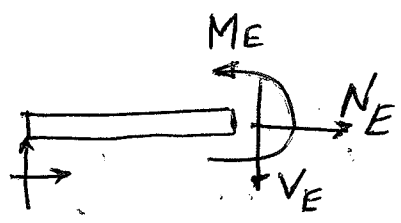
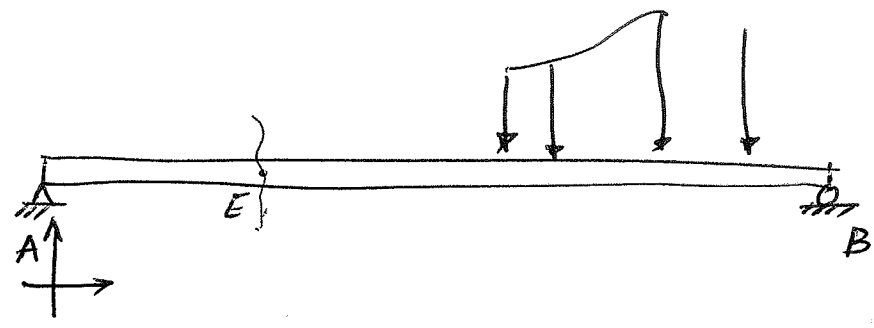
$$\therefore N_E = -9810 \text{ N} \quad (c)$$

$$V_E = -2452.5 \text{ N} \quad \downarrow \boxed{} \uparrow$$

$$M_E = V_E \times 1\text{m}$$

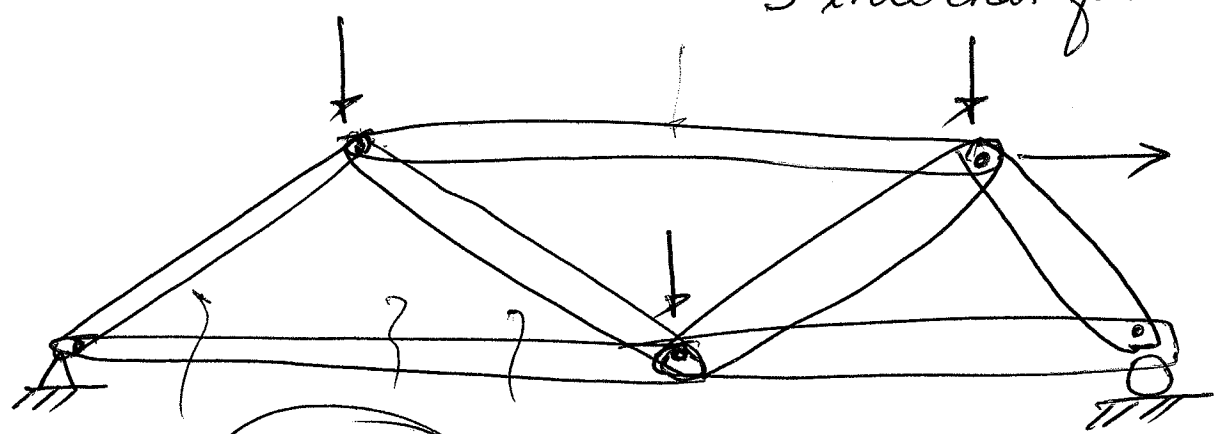
$$= -2452.5 \text{ N-m} \quad \left(\boxed{} \right)$$

- TRUSS $\rightarrow$ Structure
- STRESS

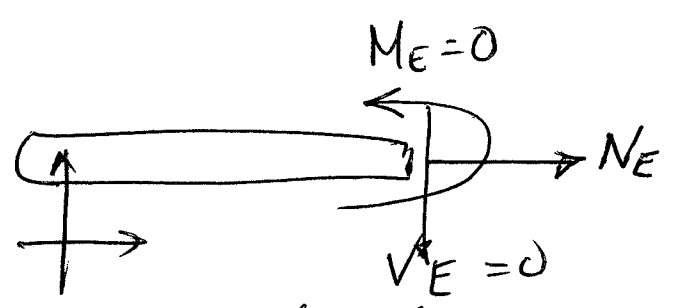


For any general structural member,

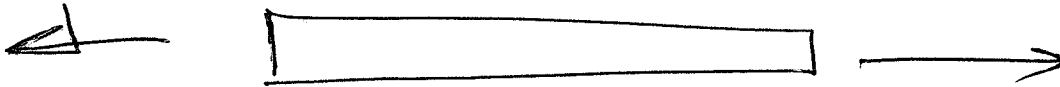
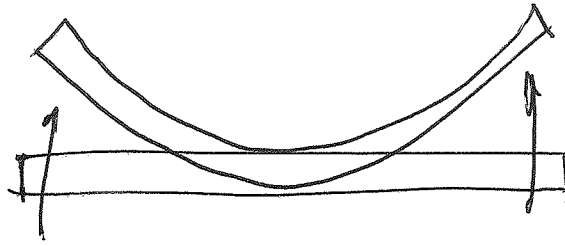
3 internal forces N_E
 V_E
 M_E



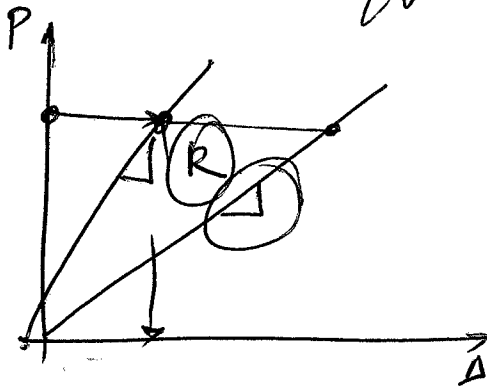
TRUSS



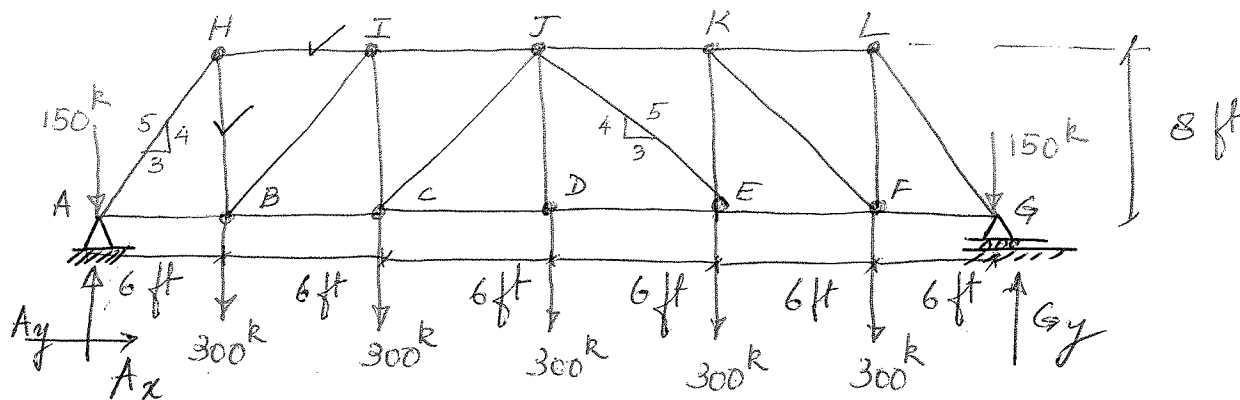
All the members are subjected to axial forces only?



lot stiffer & stronger.



Example



$$+\uparrow \sum F_y = 0$$

$$+\rightarrow \sum F_x = 0$$

$$(\curvearrowright) \sum M_A = 0$$

$$\therefore A_y - 2 \times 150 - 300 \times 5 + G_y = 0$$

$$A_x = 0$$

$$\therefore A_y = 900 \text{ k}$$

$$-300(6+12+18+24+30) + G_y \times 36 - 150 \times 36 = 0$$

$$\therefore G_y = 300 \times \frac{90}{36} + 150$$

$$= 900 \text{ k}$$

How to calculate internal forces.

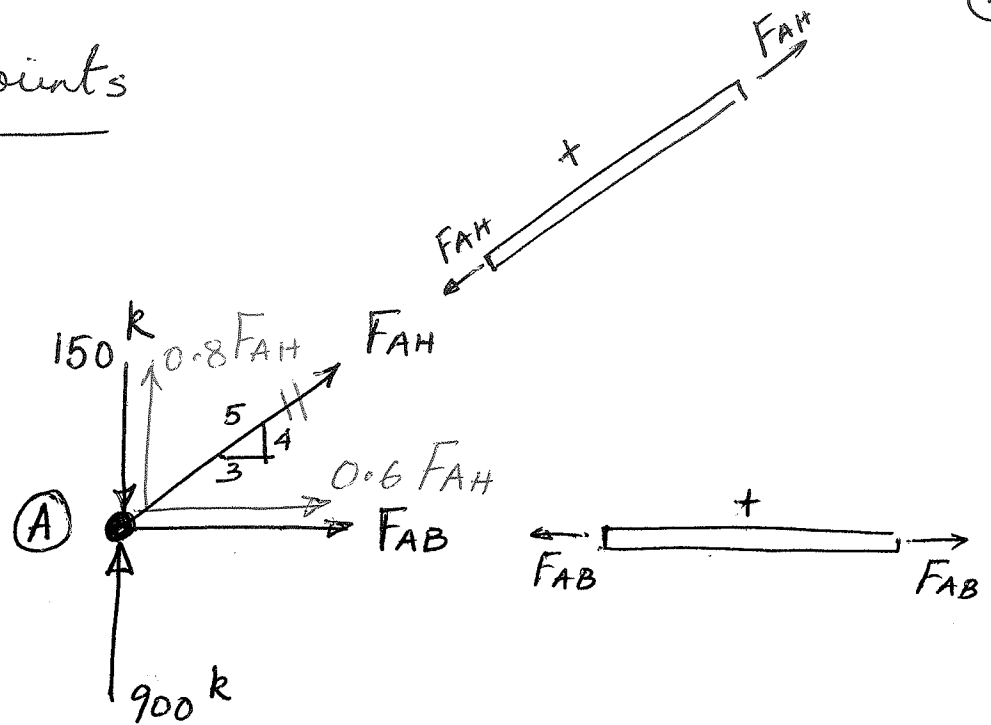
cut a section & write equations of eq.

method of joints

→ only 2 unknown forces at the joint

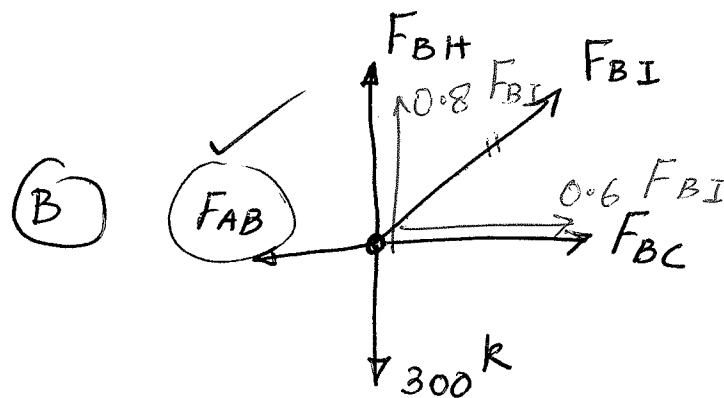
Method of Joints

(B)

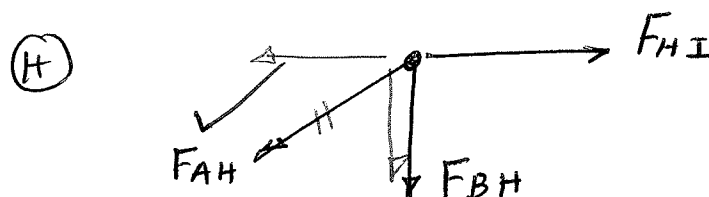


$$\rightarrow \sum F_x = 0 \quad \therefore 0.6 F_{AH} + F_{AB} = 0$$

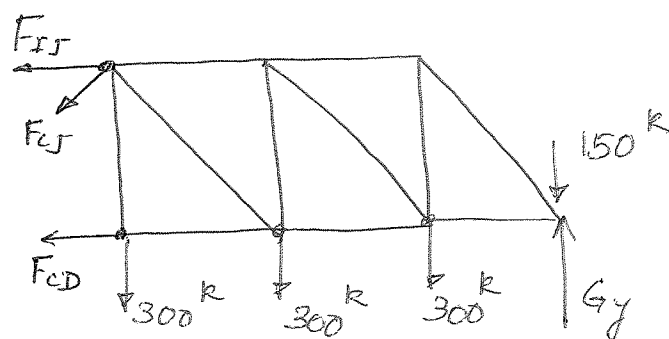
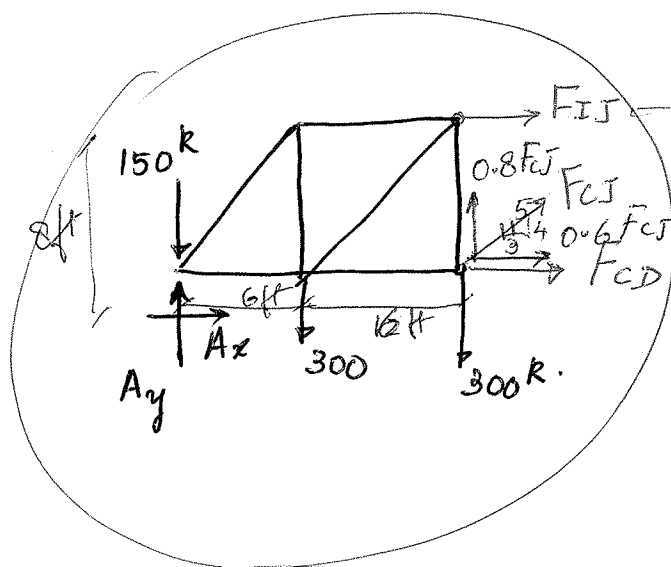
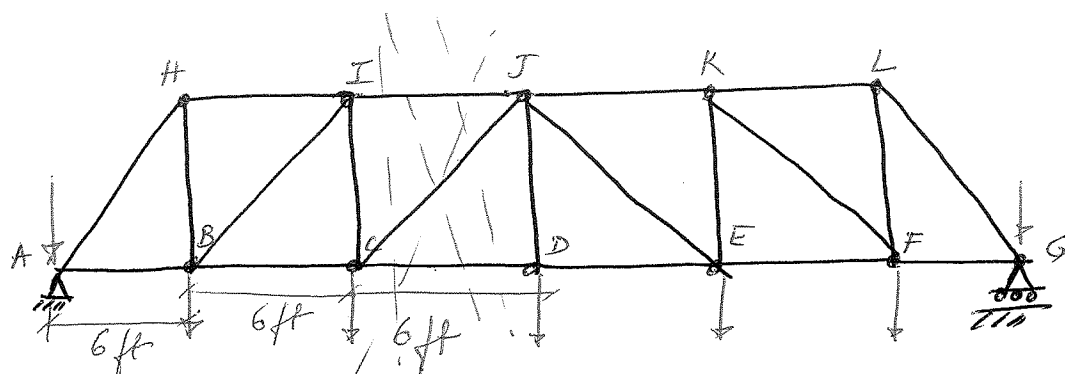
$$\uparrow \sum F_y = 0 \quad \therefore 900 - 150 + 0.8 F_{AH} = 0$$



Go to joint (H) instead



Method of Sections



$$\rightarrow \sum F_x = 0$$

$$\uparrow \sum F_y = 0$$

$$\curvearrowright \sum M_A = 0$$

$$\rightarrow Ax + 0.6 F_{CI} + F_{IJ} + F_{CD} = 0$$

$$\uparrow -150 + Ay - 300 - 300 + 0.8 F_{CI} = 0$$

$$\curvearrowright \sum M_A = 0$$

$$-300 \times 6 - 300 \times 12 + 0.8 F_{CI} \times 12$$

$$- F_{IJ} \times 8 = 0$$

Three member forces F_{IJ} , F_{CI} & F_{CD} .

AXIAL STRESS

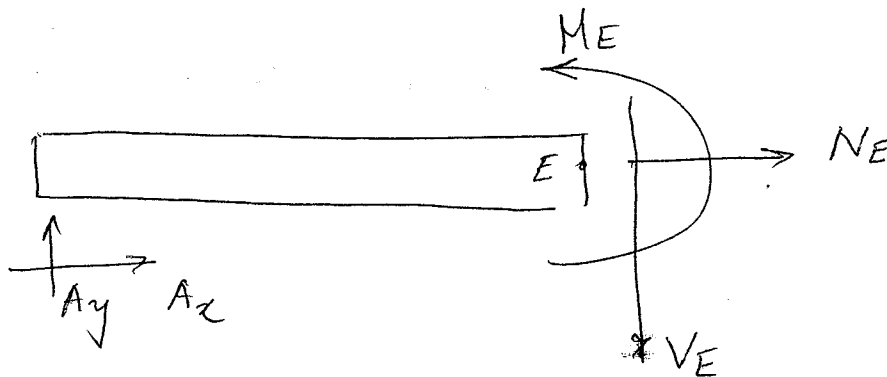
N

SHEAR STRESS

V

BENDING STRESS

M



Total Stress @ E

N_E

V_E

M_E

Is point E going to fail?

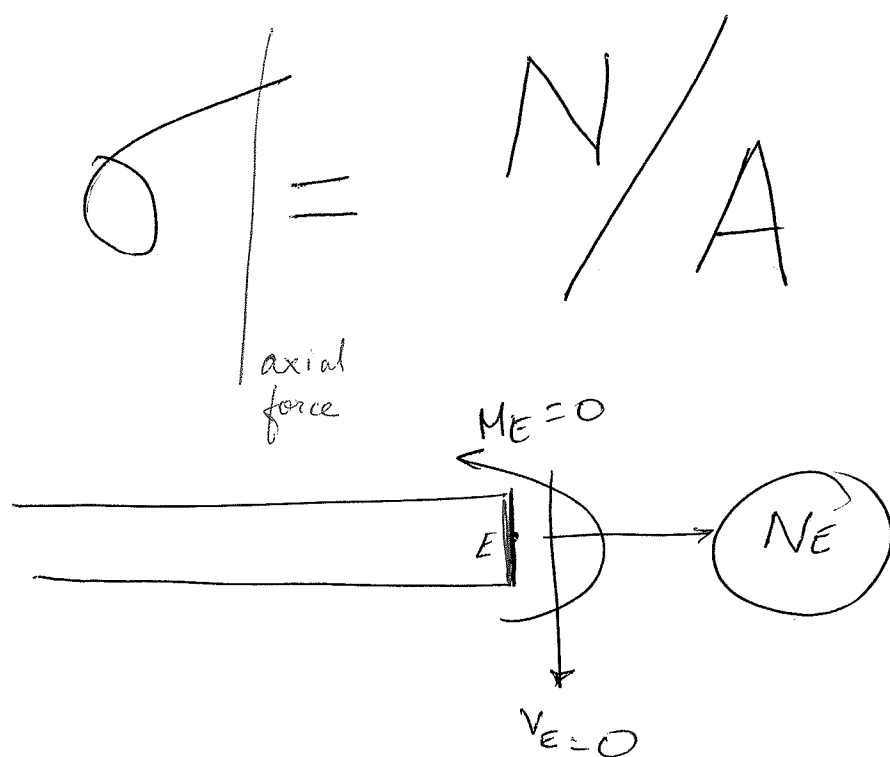
Is the total stress @ E

> material stress capacity

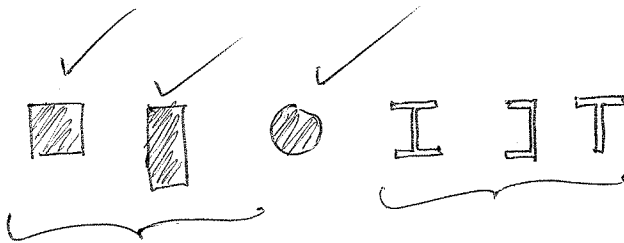
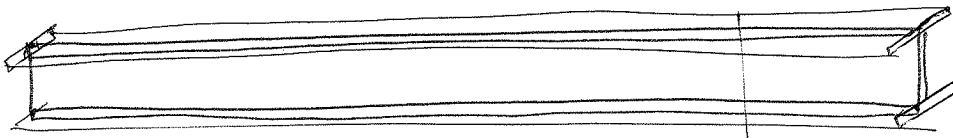
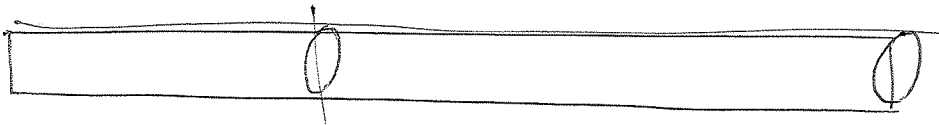
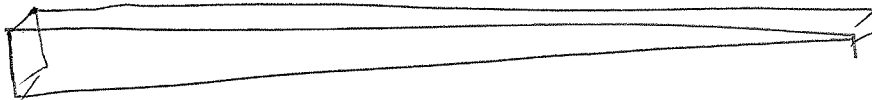
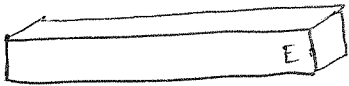
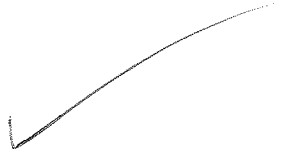
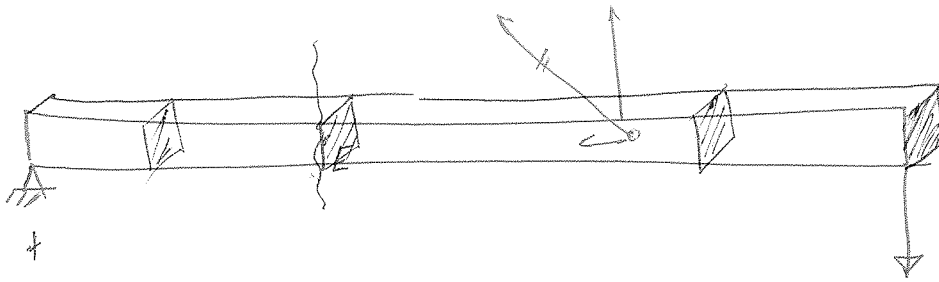
Material Stress Capacity Obtained from Tests

- AXIAL FORCE (N) $\longrightarrow$ AXIAL STRESS (σ)
- SHEAR FORCE (V) $\longrightarrow$ SHEAR STRESS (τ)

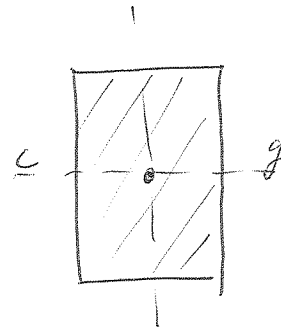
Truss members $\longrightarrow$ axial force
axial stress



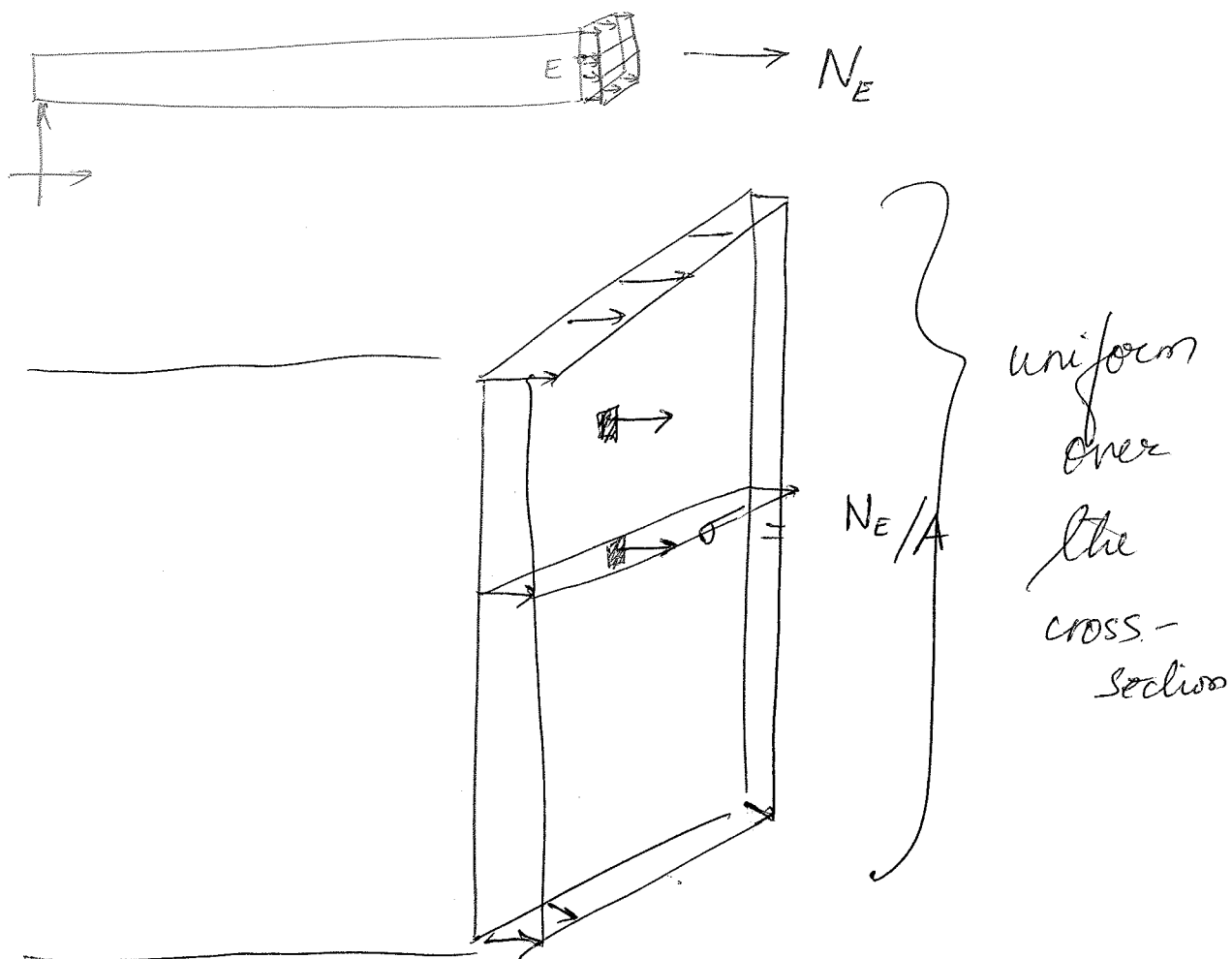
A $\longrightarrow$ cross-sectional area of member.



solid
cross-section



A
I



————— x —————

Example

$$\sigma = \frac{F_{BC}}{A_{BC}}$$

$$\frac{N}{\text{mm}^2}$$

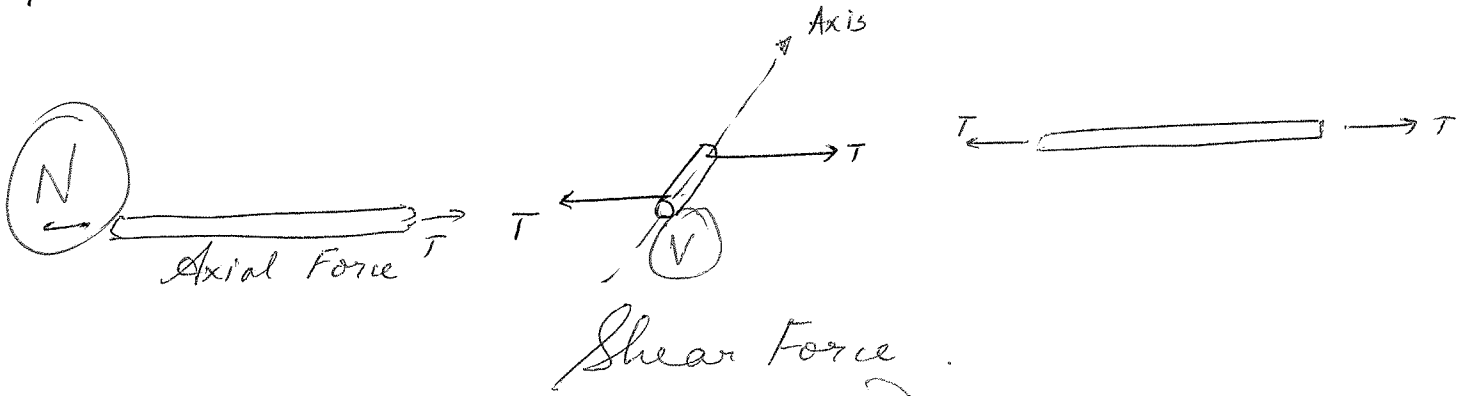
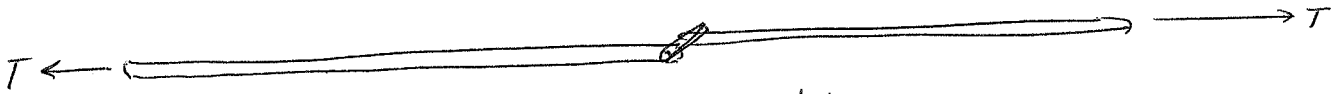
$$d_{BC} = 8 \text{ mm}$$

What are the units of σ

$$\underbrace{\frac{\text{lb}}{\text{in}^2} \quad \text{or} \quad \frac{\text{kip}}{\text{in}^2}}_{\text{in}^2} \quad \text{or} \quad \underbrace{\frac{\text{N}}{\text{mm}^2}}_{\text{MPa}}$$

(Pa)

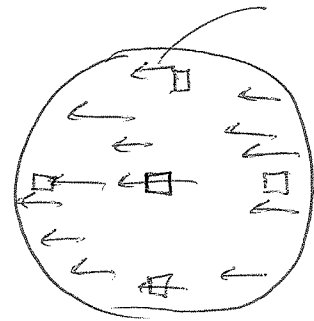
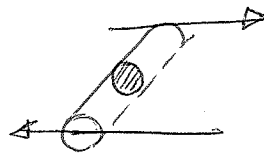
Example of Shear stress.
on Monday.

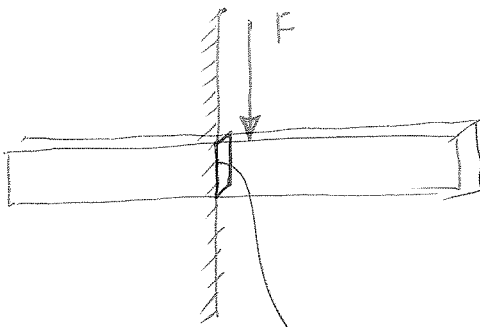


axial stress σ

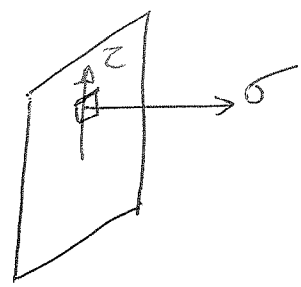
$$= \frac{N}{A}$$

fasteners / bolts. $\tau_{avg} = \frac{V}{A}$

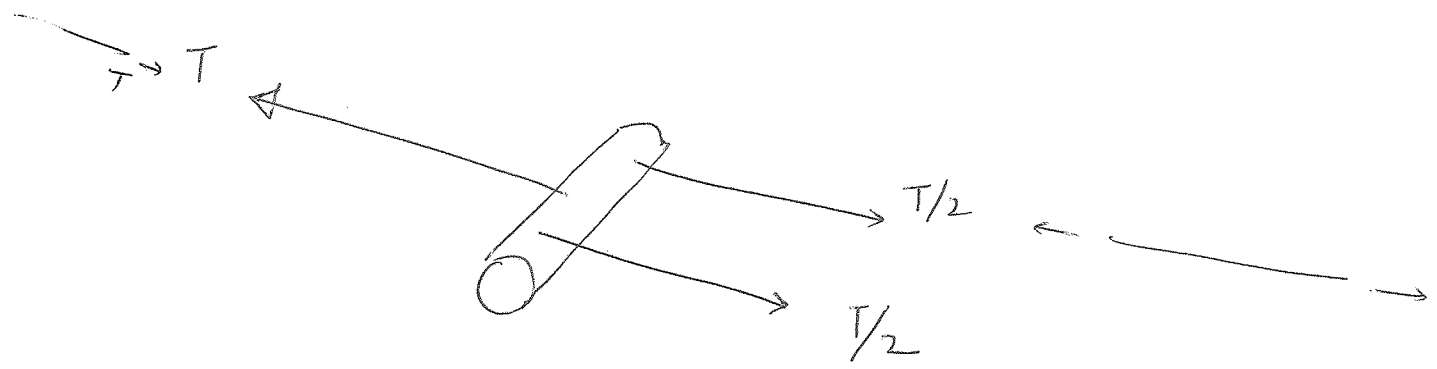




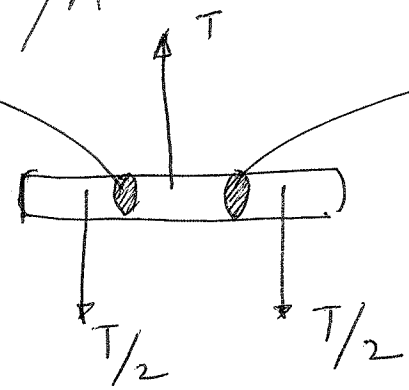
$$\tau_{avg} = \frac{F}{A}$$



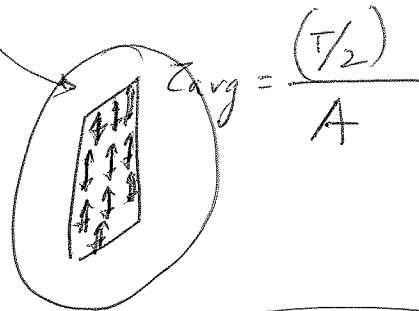
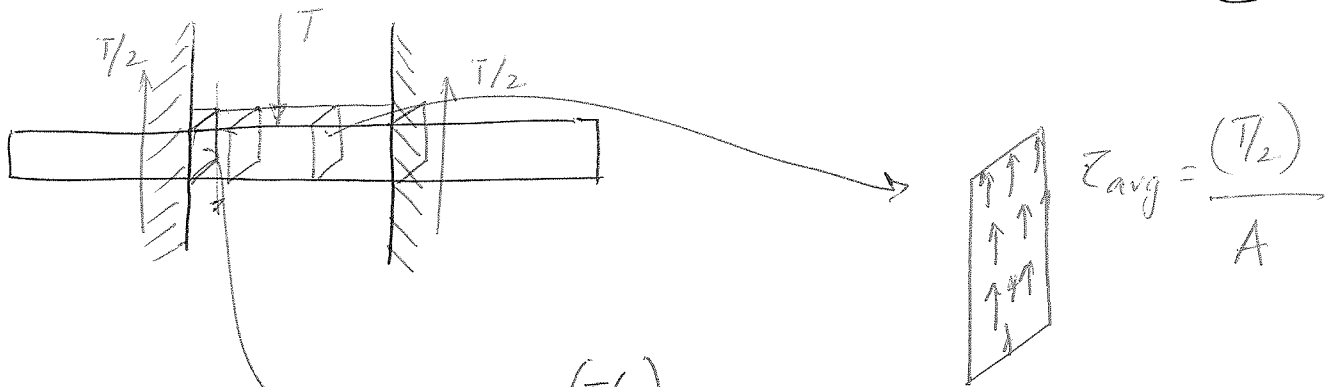
Single shear $\rightarrow$ resisted by one cross-section




$$\tau_{avg} = \frac{(T/2)}{A}$$



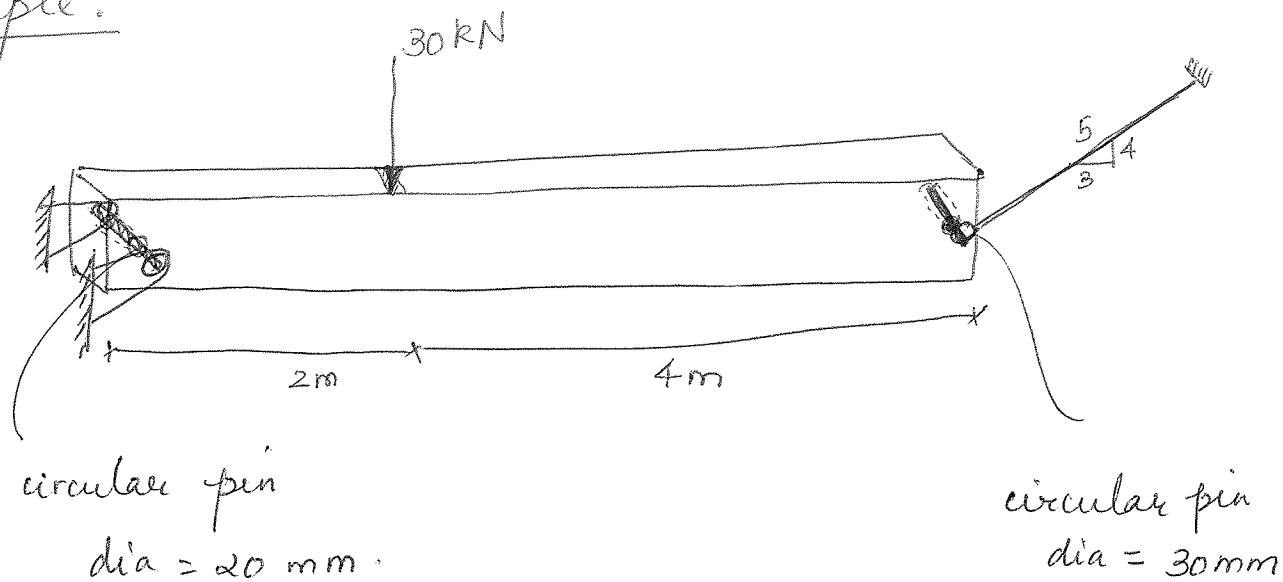

$$\tau_{avg} = \frac{T}{2A}$$



double shear

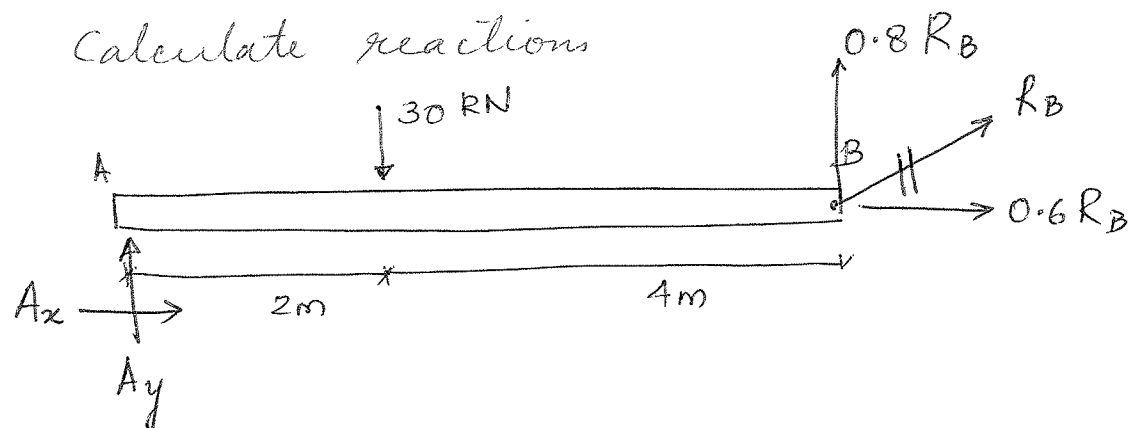
→ two cross-sections resisting
the shear force
or the shear force is
reduced to half.

design of pins, bolts, rivets or
fasteners

Example:

Obj: Calculate the average shear stress in the pins.

Step 1. Calculate reactions



$$(+\sum M_A = 0 \quad \therefore -30(2) + 0.8R_B \times 6 = 0$$

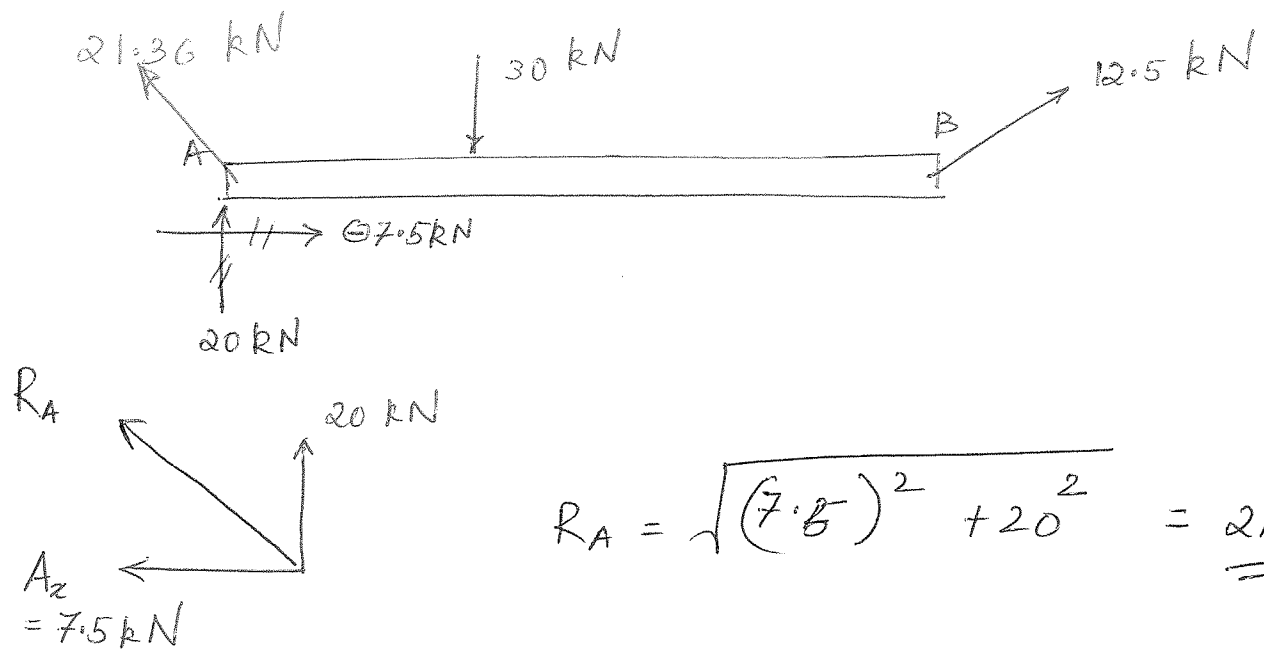
$$\therefore R_B = \frac{30 \times 2}{6 \times 0.8} = \underline{\underline{12.5 \text{ kN}}}$$

$$+\uparrow \sum F_y = 0 \quad \therefore A_y - 30 + 0.8R_B = 0$$

$$\therefore A_y = 30 - 0.8R_B = \underline{\underline{20 \text{ kN}}}$$

$$+\rightarrow \sum F_x = 0 \quad \therefore A_x + 0.6R_B = 0$$

$$\therefore A_x = -0.6 \times 12.5 = \underline{\underline{-7.5 \text{ kN}}}$$



$$R_A = \sqrt{(7.5)^2 + 20^2} = \underline{\underline{21.36 \text{ kN}}}$$

Pin @ B

diameter = 30 mm.

Force = 12.5 kN

$$\tau_{avg} \Big|_B = \frac{12.5 \text{ kN}}{A} = \frac{12.5 \text{ kN}}{\frac{\pi \times (30)^2}{4} \text{ mm}^2}$$

3 units of stress $\rightarrow$ psi, ksi, MPa
 N/mm^2

$$\tau_{avg} \Big|_B = \frac{12500 \text{ N}}{\frac{\pi \times 30^2}{4} \text{ mm}^2} = \underline{\underline{17.68 \text{ MPa}}}$$

$$A = \pi \times r^2 = \pi \times d^2/4$$

$$\tau_{avg} \Big|_A = \frac{(F/2)}{A} \rightarrow \text{double shear}$$

$$= \frac{(21.36/2) \text{ kN}}{\pi \times 20^2 / 4}$$

$$= \frac{21360 \text{ N} \times 4}{2 \times \pi \times 20^2 \text{ mm}^2}$$

$$= 34 \frac{\text{N}}{\text{mm}^2} = \underline{\underline{34 \text{ MPa}}}$$

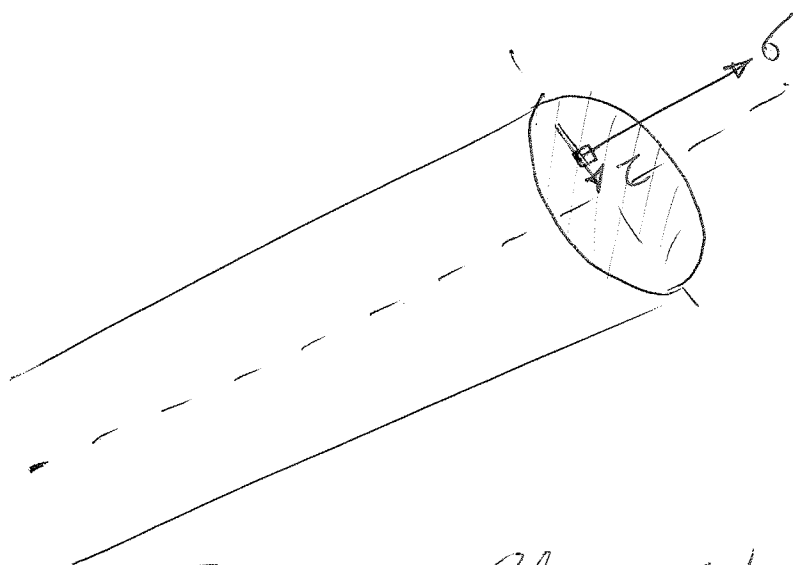
Force in Cable $R_B = 12.5 \text{ kN}$ ✓

diameter = 10 mm

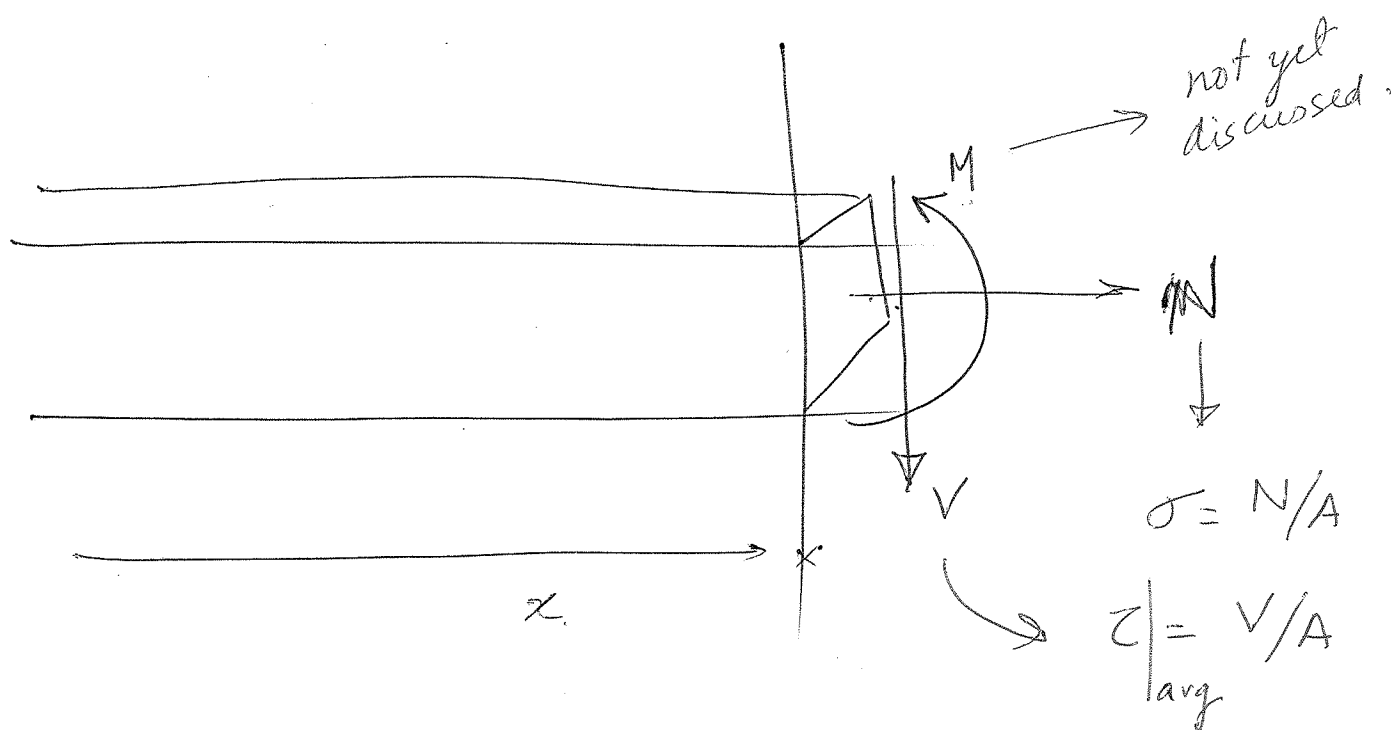
$$\therefore \sigma_{\text{normal}} = \frac{R}{A} = \frac{12500 \text{ N} \times 4}{\pi \times (10)^2 \text{ mm}} = 159.1 \frac{\text{N}}{\text{mm}^2}$$

$$\sigma = 159.1 \text{ MPa}$$

$\sigma \rightarrow$ normal stress or axial stress
or longitudinal stress



$\tau \rightarrow$ Shear stress acts in the plane of cross-sections



PURPOSE IS DESIGN

For any material

σ_{fail} } experimental
 or
 τ_{fail} } testing of material

$$\sigma_{all} = \frac{\sigma_{fail}}{\text{factor of safety}}$$

$$\tau_{all} = \frac{\tau_{fail}}{\text{factor of safety}}$$

Design

$$\sigma_{calculated} \leq$$

$$\sigma_{all} = \frac{\sigma_{fail}}{F.O.S}$$

$$\tau_{calculated} \leq$$

$$\tau_{all} = \frac{\tau_{fail}}{F.O.S}$$

$$\tau_A = 34 \text{ MPa}$$

$$\tau_B = 17.7 \text{ MPa}$$

$$\sigma_B = 159.1 \text{ MPa}$$

Calculated stresses.

Depending on the material for pin @ A

$$\tau_{\text{fail}} \Big|_{\text{bolt @ A}} \stackrel{\text{given}}{=} \underline{\underline{250 \text{ MPa}}}$$

Assume F.O.S. of 2.0

$$\tau_{\text{all}} \Big|_{\text{bolt @ A}} = 125 \text{ MPa}$$

$\therefore$ pin @ A will work

Steel

$$\tau_{fail} = 250 \text{ MPa}$$

$$\tau_{all} = 125 \text{ MPa}$$

assuming F.O.S. = 2.0

For pin @ A $\tau_{calc} = 34 \text{ MPa} < \tau_{all}$

pin @ B $\tau_{calc} = 17.7 \text{ MPa} < \tau_{all}$

Steel

$$\sigma_{fail} = 300 \text{ MPa}$$

assume F.O.S. = 2.0

$$\sigma_{all} = 150 \text{ MPa}$$

$$\sigma_{calc} \Big|_{\text{cable @ B}} = 159.1 \text{ MPa} > \sigma_{all}$$

$\therefore$ RE-DESIGN

σ_{fail} & τ_{fail}
material tests!

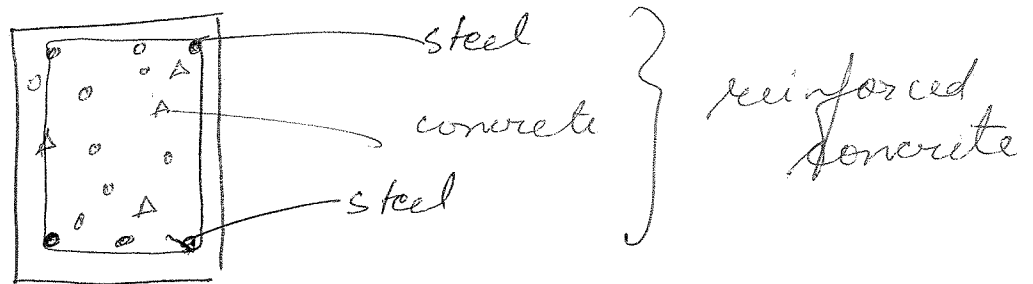
Steel, concrete, wood, masonry

↓
axial tension

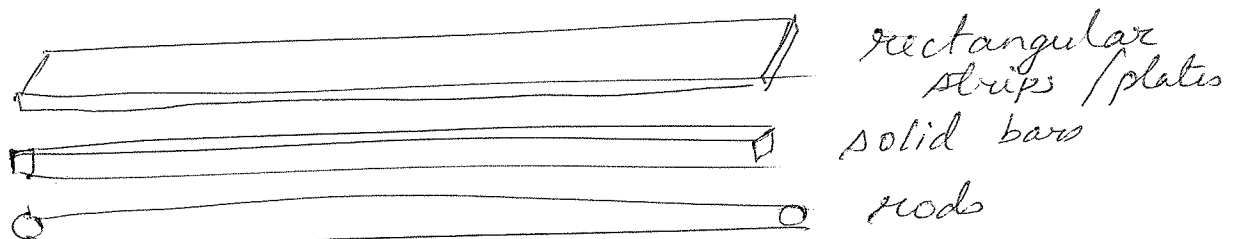
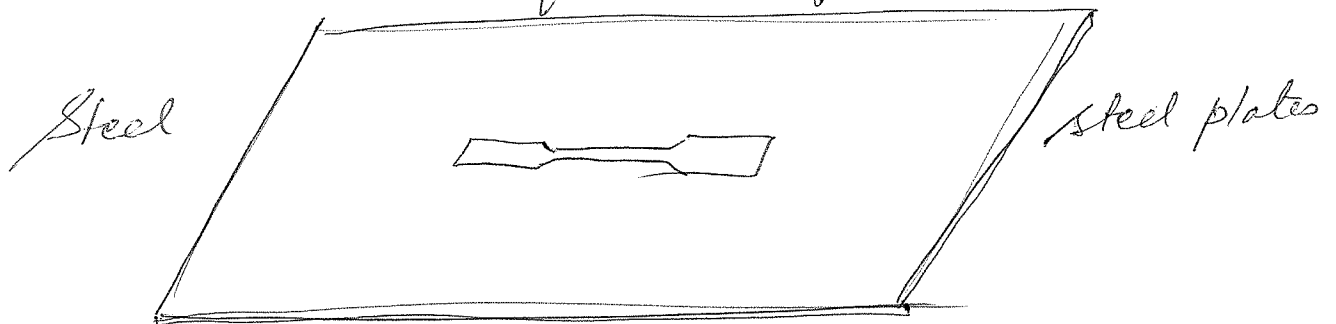
↓
axial compression

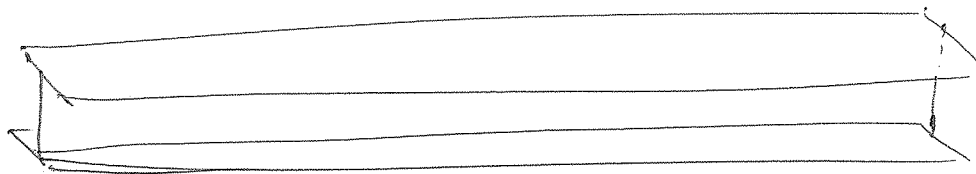
axial compression
buckling problem

axial tension almost zero strength

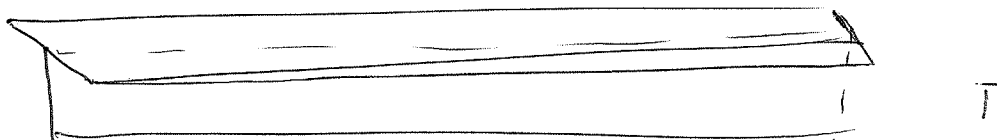
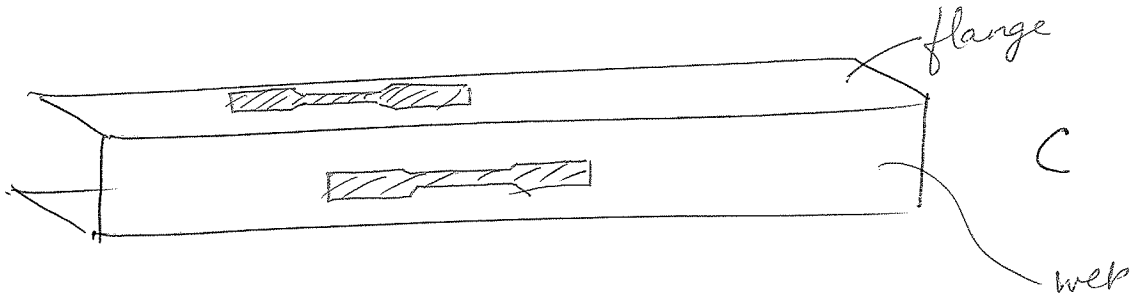


behavior & failure of these materials



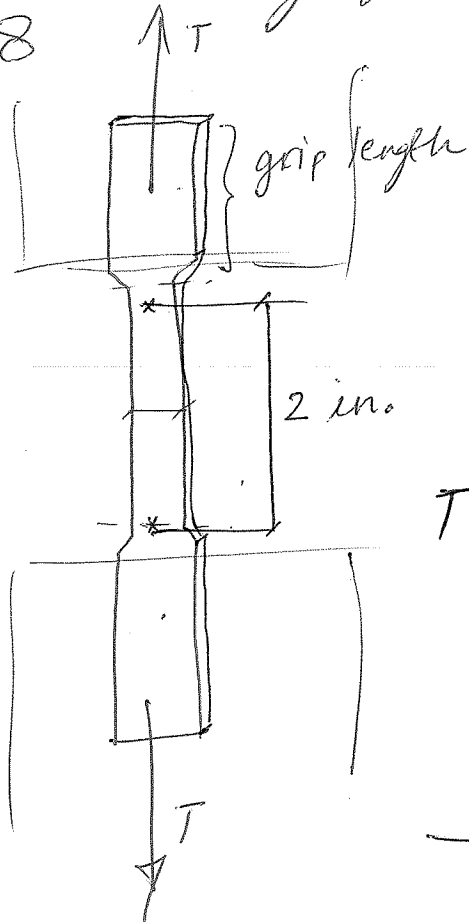


I shaped
structural
members



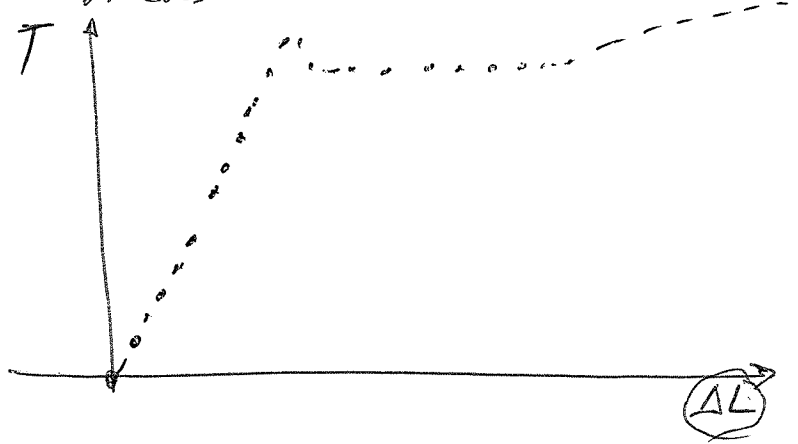
American Society for Testing & Materials (ASTM)

E8



Axial tension

apply T
measure



✓ Calculation of σ and τ

σ_{fail} & τ_{fail}

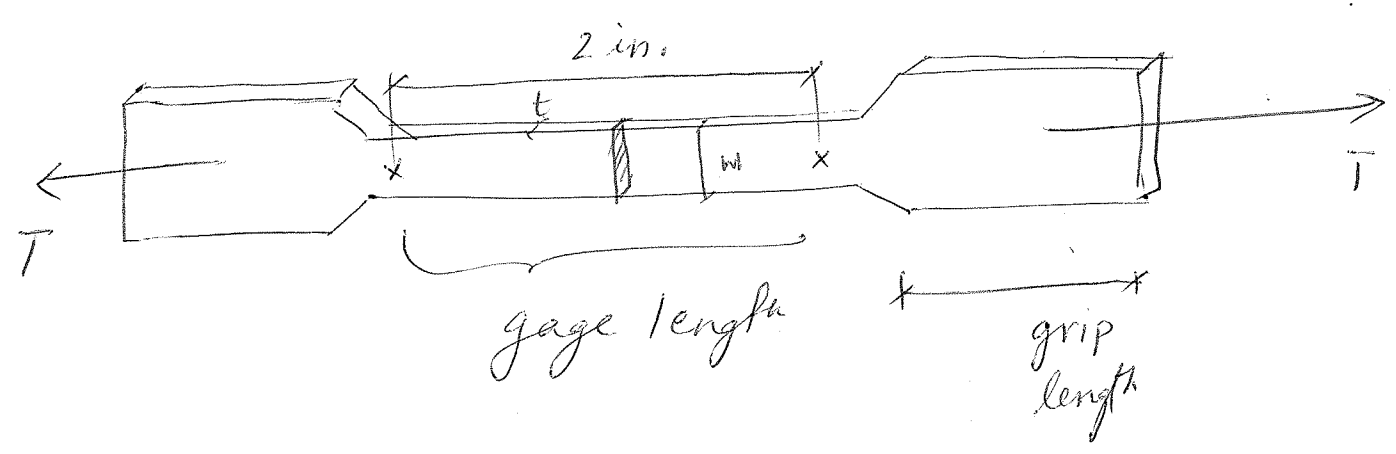
Allowable stress = $\sigma_{all} = \frac{\sigma_{fail}}{F.O.S.}$

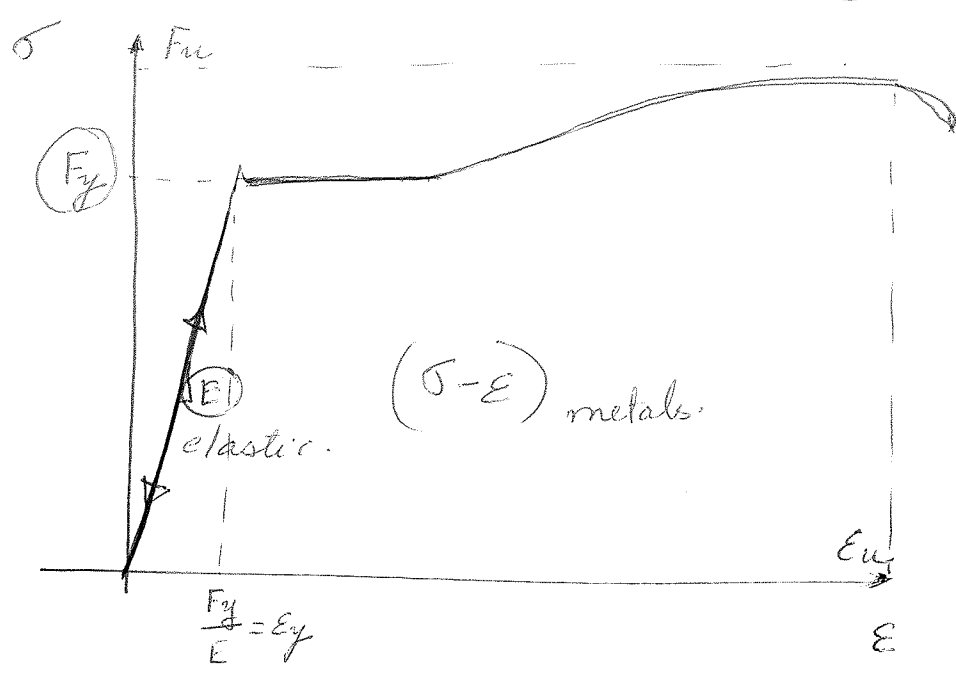
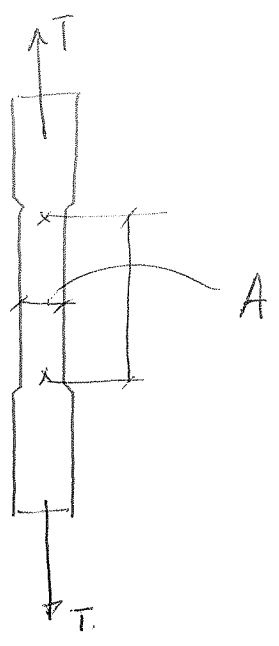
calculated $\sigma \leq \sigma_{all}$ ← Design Obj.

calculated $\tau \leq \tau_{all} = \frac{\tau_{fail}}{F.O.S.}$ ← Design Obj.

How to get σ_{fail} & τ_{fail}

ASTM standard tests of coupons from the materials.

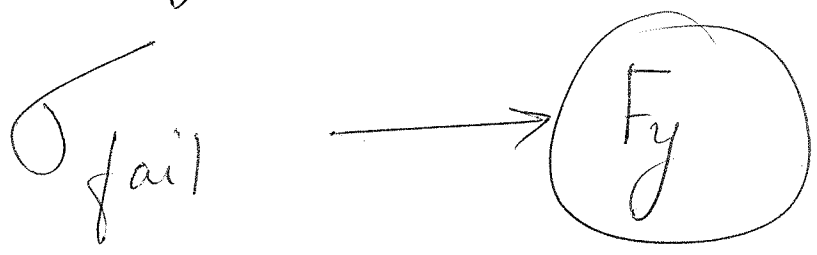




$$\left. \begin{aligned} \sigma &= T/A \\ \epsilon &= \frac{\Delta L}{L_0} \end{aligned} \right\} \text{For metals}$$

$F_y \rightarrow$ failure stress in tension

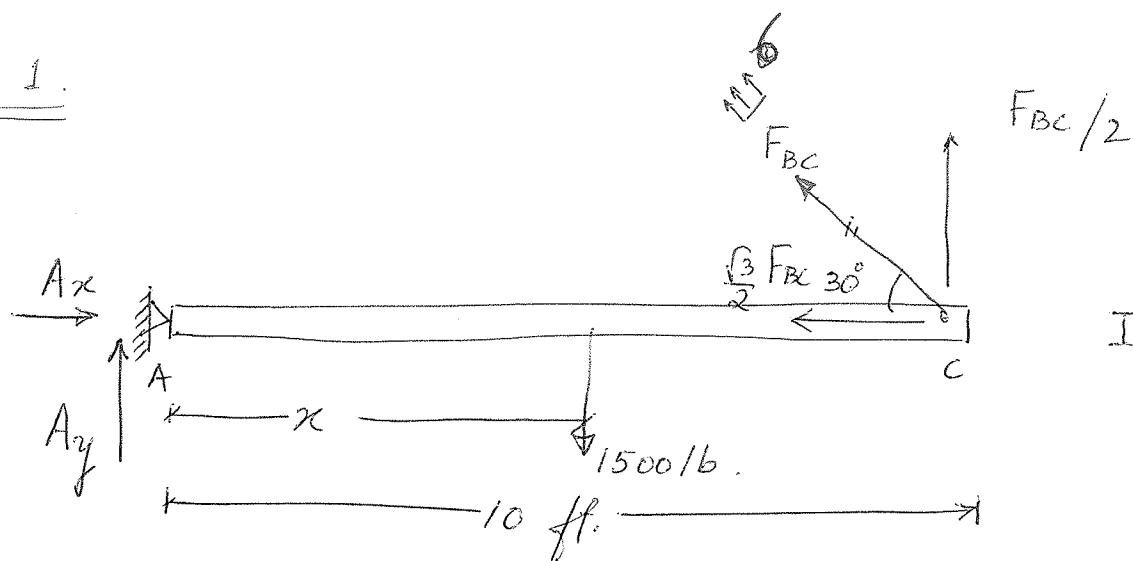
linear-elastic behavior upto
yield stress F_y



50 ksi steel or 50,000 psi steel

- ① Load is not acting @ one given location
moving along the length of the structure

Step 1.

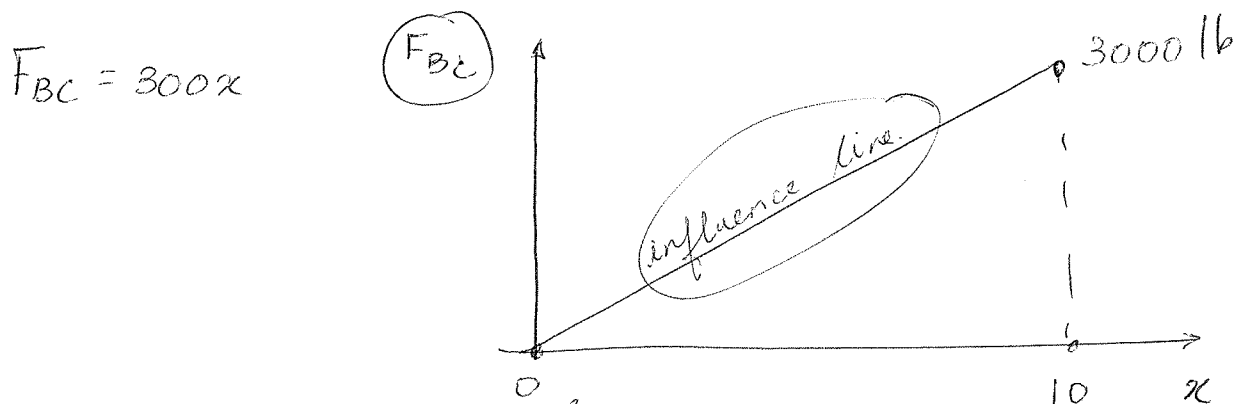


$$\rightarrow \sum F_x \quad \therefore A_x - \frac{\sqrt{3}}{2} F_{BC} = 0$$

$$\uparrow \sum F_y \quad \therefore A_y - 1500/b + \frac{F_{BC}}{2} = 0$$

$$\curvearrowright \sum M_A \quad \therefore -1500x + \frac{F_{BC}}{2} \times 10 = 0$$

$$\therefore \boxed{F_{BC}} = \frac{1500x}{5} = \boxed{300x} \text{ lb}$$



influence of moving load on F_{BC} .

(35)

$$A_x - \frac{\sqrt{3}}{2} \times F_{BC} = 0$$

$$\therefore A_x = \frac{\sqrt{3}}{2} \times 300x$$

$$(A_x) = 0.867 \times 300x = \underline{259.8x}$$

$$A_y - 1500 \text{ lb} + \frac{F_{BC}}{2} = 0$$

$$\therefore (A_y) = 1500 - \frac{300x}{2} = \underline{1500 - 150x}$$

What will be the vector reaction at A

$$R_A = \sqrt{A_x^2 + A_y^2}$$

$$= \sqrt{(259.8x)^2 + (1500 - 150x)^2}$$

$$R_A = \sqrt{67500x^2 + 2250000 + 22500x^2 - 450000x}$$

Influence of moving load on the reaction A

$$R_A|_{\max} = 2598 \underline{\underline{\text{lb}}}$$

$$F_{BC} = 300x \quad \therefore F_{BC}|_{\max} = 3000 \text{ lb}$$

$$R_A|_{\max} = 2598 \underline{\underline{\text{lb}}}$$

The rod BC diameter = $3/4$ in.

$$\text{pin @ B} = 5/8 \text{ in.}$$

$$\text{pin @ A} = 1/2 \text{ in.}$$

$$\begin{aligned} \therefore \sigma_{BC}|_{\max} &= \frac{F_{BC}|_{\max}}{A} = \frac{3000 \text{ lb}}{\frac{\pi}{4} \times \left(\frac{3}{4}\right)^2 \text{ in}^2} \\ &= 6790 \text{ lb/in}^2 \\ &= 6790 \text{ psi} \end{aligned}$$

Shear stress in the pin @ B
single or double

$$\therefore \tau_B|_{\max} = \frac{F_{BC}}{A} = \frac{3000 \text{ lb}}{\frac{\pi \times (5/8)^2}{4}}$$

Shear stress in the
pin @ B

$$\begin{aligned}\tau_B \Big|_{\max} &= \frac{F_{BC}/2}{\frac{\pi}{4} \times (5/8)^2} \\ &= \frac{1500 \text{ lb}}{\frac{\pi}{4} \times (5/8)^2} \\ &= 4889 \text{ psi}\end{aligned}$$

Shear stress in the
pin @ C

$$\begin{aligned}\tau_C \Big|_{\max} &= \frac{F_{BC}}{\frac{\pi}{4} \times (5/8)^2} \\ \tau_C \Big|_{\max} &= \frac{3000 \text{ lb}}{\frac{\pi}{4} \times (5/8)^2} \\ &= 2 \times 4889 \text{ psi}\end{aligned}$$

Shear stress in the pin
@ A

double shear

$$\begin{aligned}\tau_A \Big|_{\max} &= \frac{R_A/2}{\frac{\pi}{4} \times (1/2)^2} \\ &= \frac{(2398/2) \text{ lb}}{\frac{\pi}{4} \times 0.5^2} = 6615 \text{ psi}\end{aligned}$$

Stress - Strain Behavior of Materials

① $\sigma_{fail} \rightarrow \sigma_{all} \rightarrow \text{Design}$

② $\tau_{fail} \rightarrow \tau_{all} \rightarrow \text{Design}$

internal forces N, V, M

axial stress / calc = $\sigma = N/A$

shear stress / avg = $V/A = \tau$

① Hooke's Law

Linear - Elastic behavior

$$F \propto \Delta$$

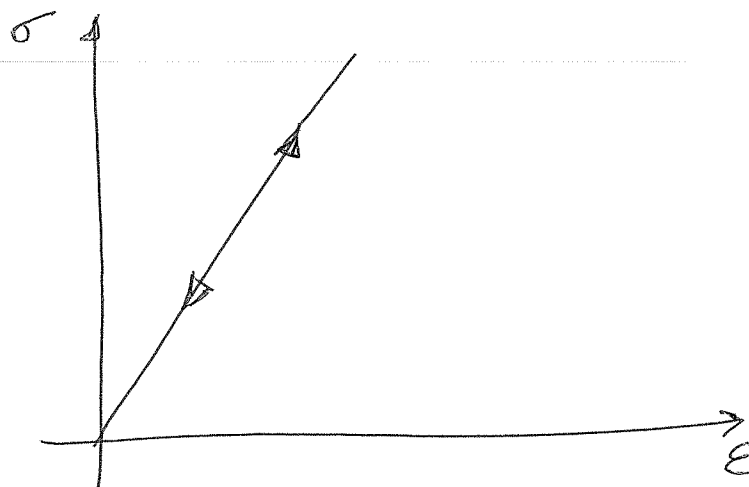
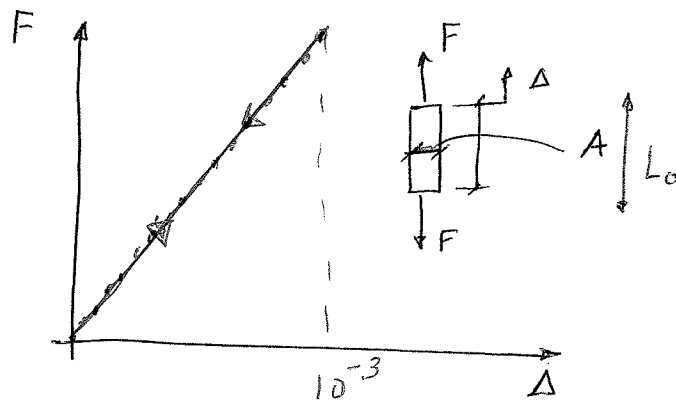
$$F = k \cdot \Delta$$

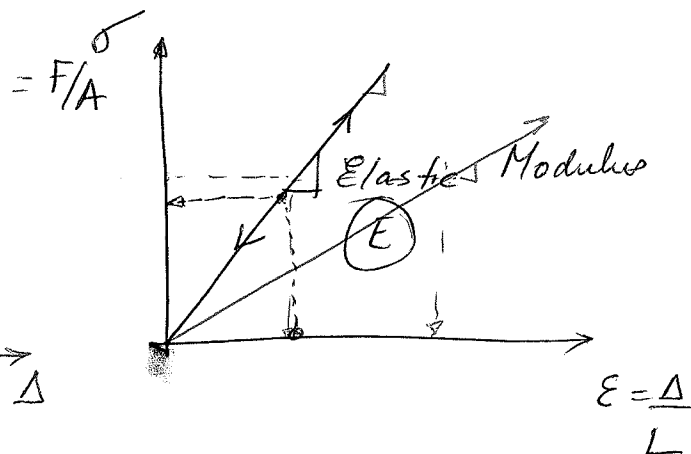
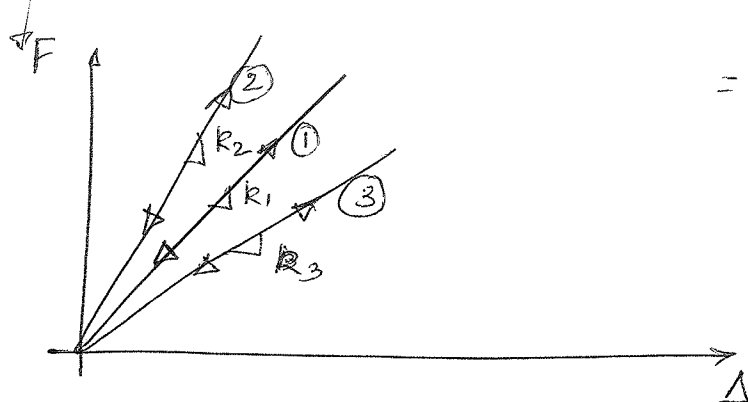
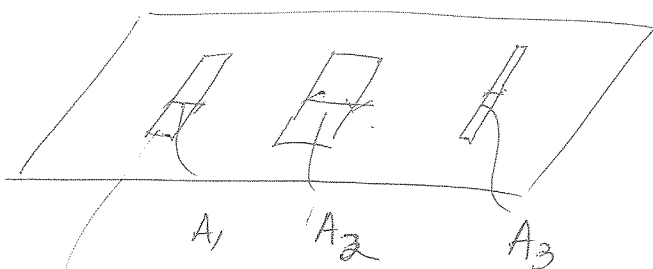
↓
spring constant

$$\sigma = \frac{F}{A}$$

$$\epsilon = \frac{\Delta}{L_0}$$

change in length





$$\sigma = E \cdot \epsilon$$

Hooke's Law.

elastic modulus.

$$E_s = 29000 \text{ ksi} \quad \text{or} \quad 200,000 \text{ MPa}$$

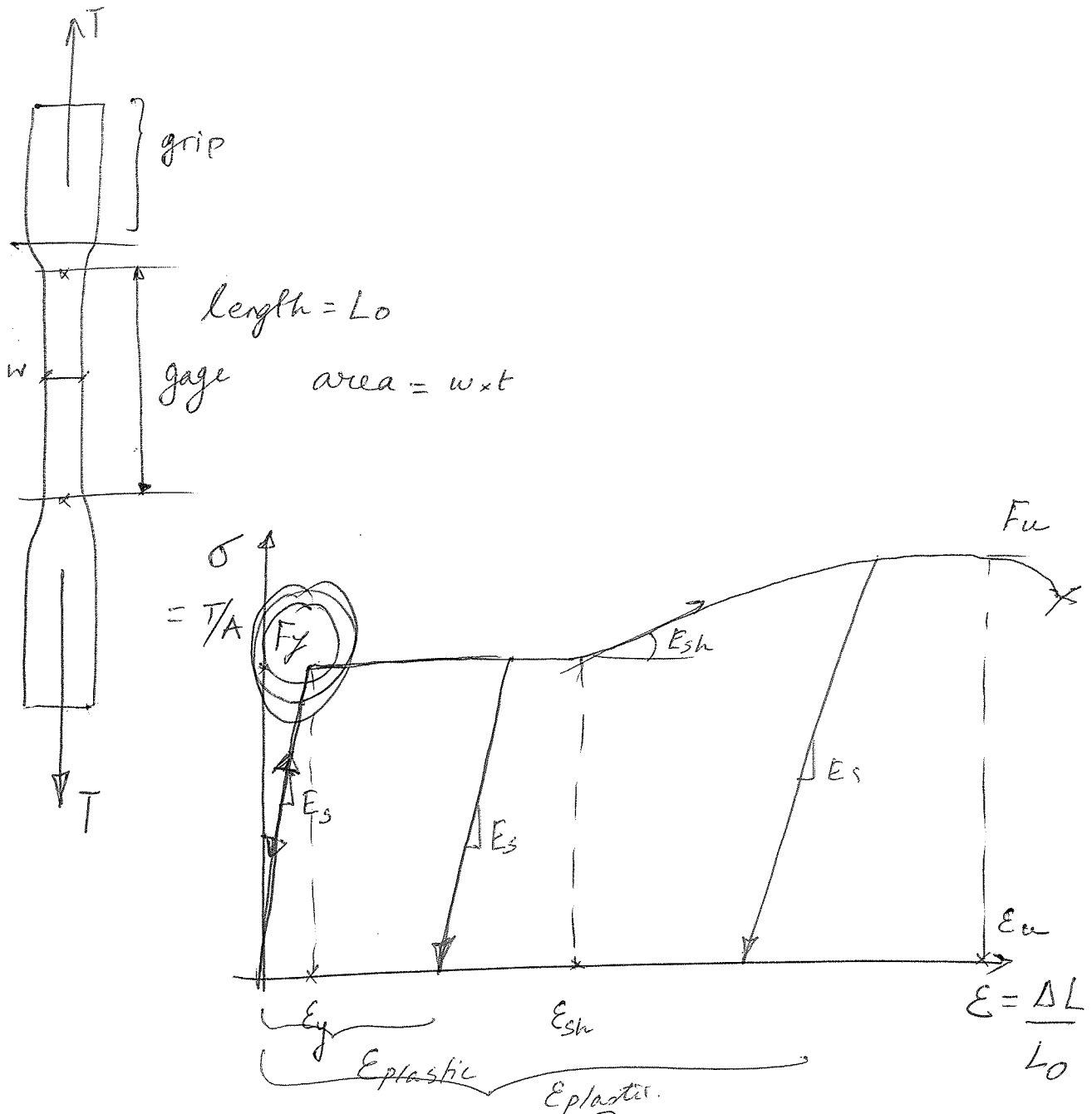
carbon or mild steel

$$\text{Stainless steel} \quad E_s \approx 27000 \text{ ksi}$$

$$\text{Aluminium} \rightarrow E_{Al} \approx 10,000 \text{ ksi}$$

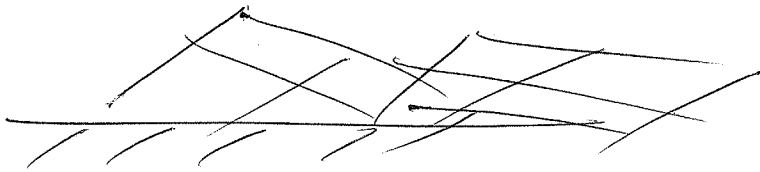
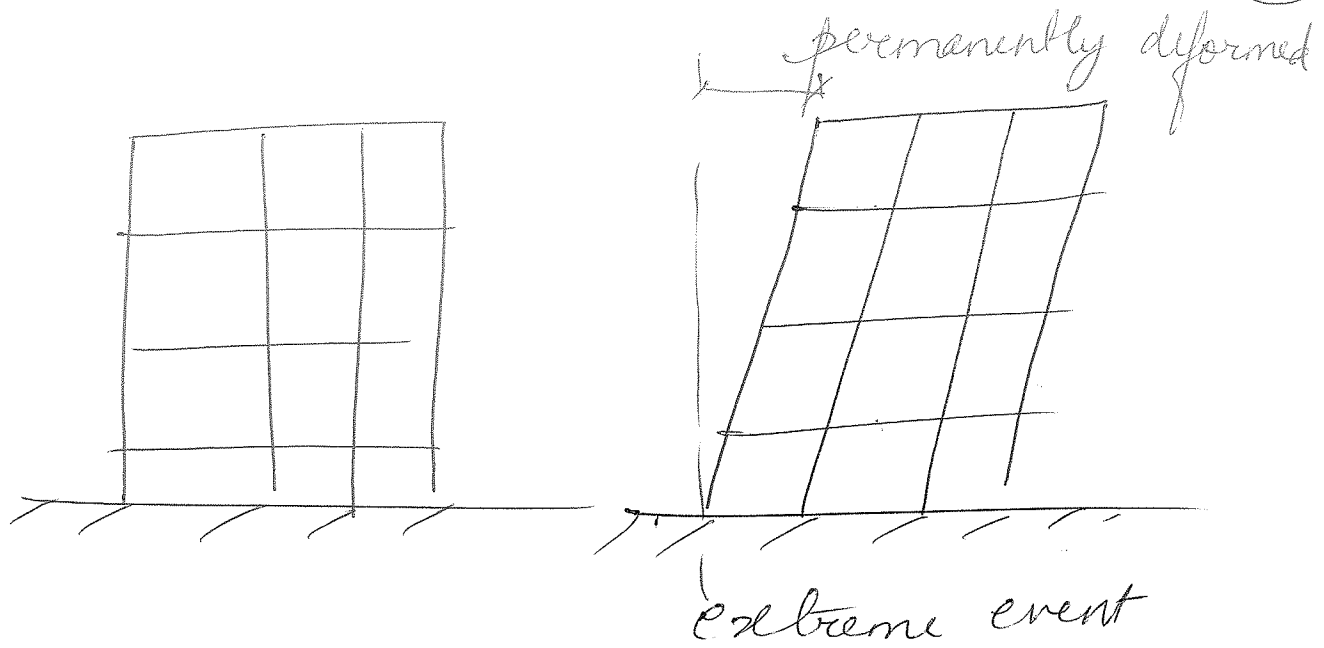
σ - ϵ behavior of steel.

→ ASTM E8 test on a standard coupon



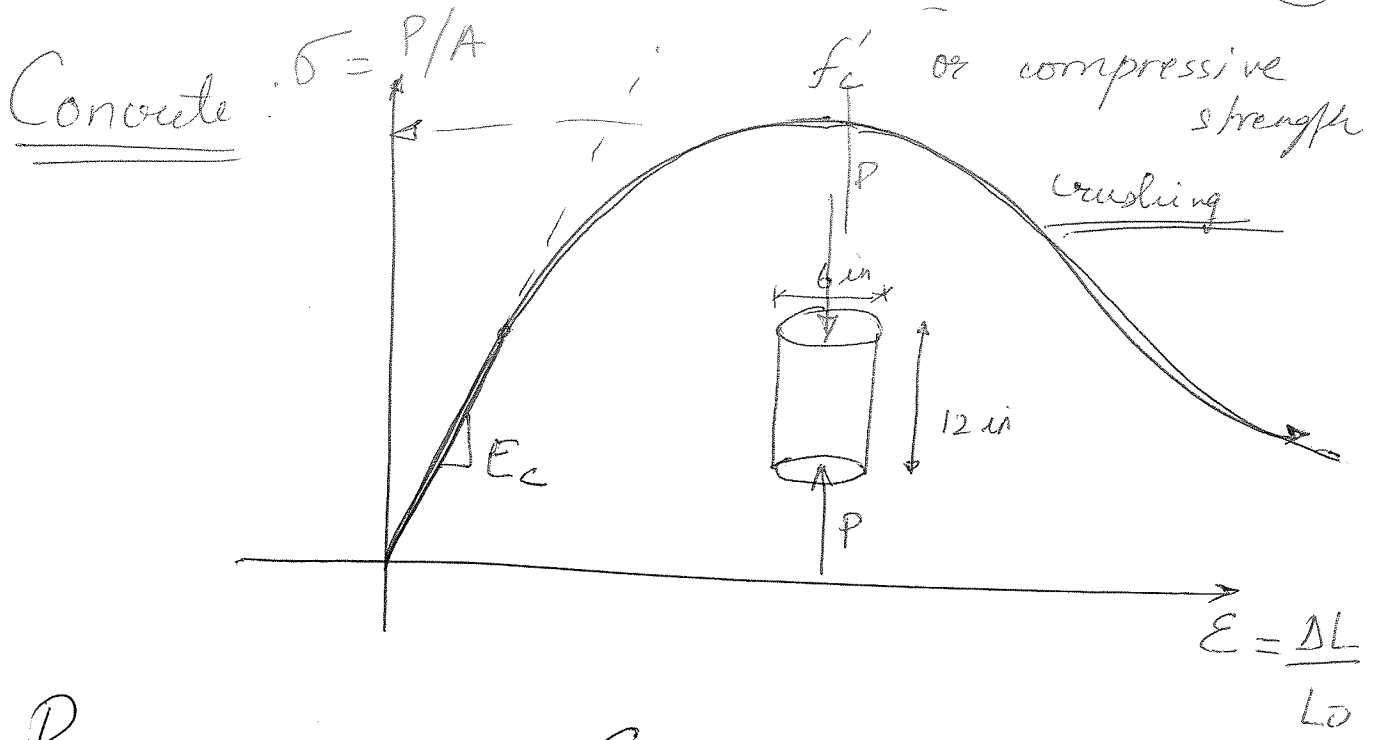
reserve strength } excellent
reserve ductility } construction material

extreme event material can deform a lot before fracture

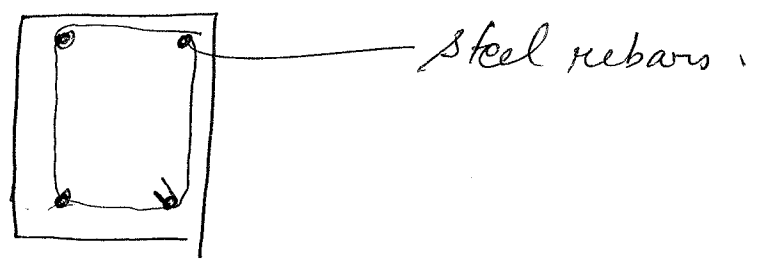


$$\sigma_{calc} \leq \sigma_{all} = \frac{F_y}{F.O.S.}$$

↓
* for service-level loads



REINFORCED CONCRETE



$f'_c \rightarrow 3000$ — 6000 psi concrete

$\swarrow$ pavements $\downarrow$ 5000 psi buildings $\searrow$ bridges

E_s

29000 ksi

F_y

36 ksi

50 ksi

70

100 ksi

E_c

depends on f'_c

$\approx 57000 \sqrt{f'_c}$
in psi

f'_c

3000 psi

4000 psi

⋮

10,000 psi

Strain

σ → axial stress

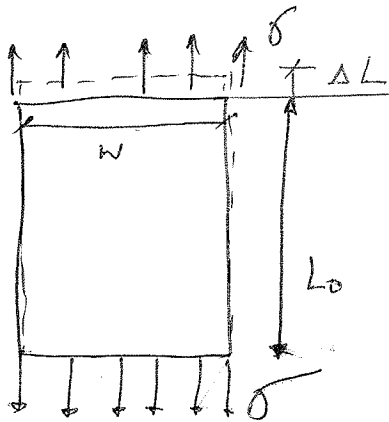
τ → shear stress

ϵ → axial strain

γ → shear strain

no such thing as a rigid body
Every thing deforms!

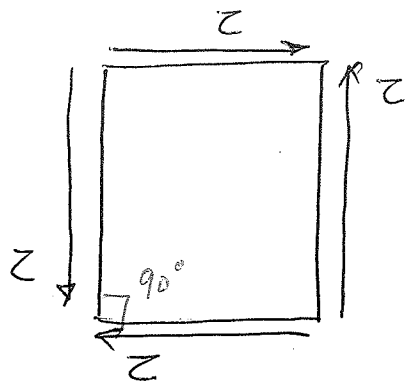
What is axial strain



$$\epsilon = \frac{\text{change in length}}{\text{original length}}$$

$$\sigma = E \cdot \epsilon$$

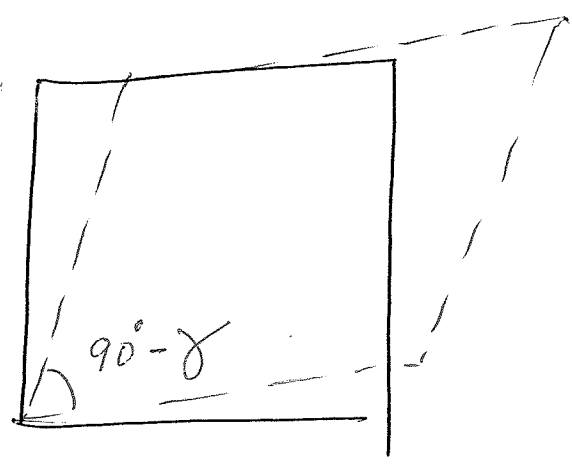
What is Shear strain



not longer
not shorter

no change in length

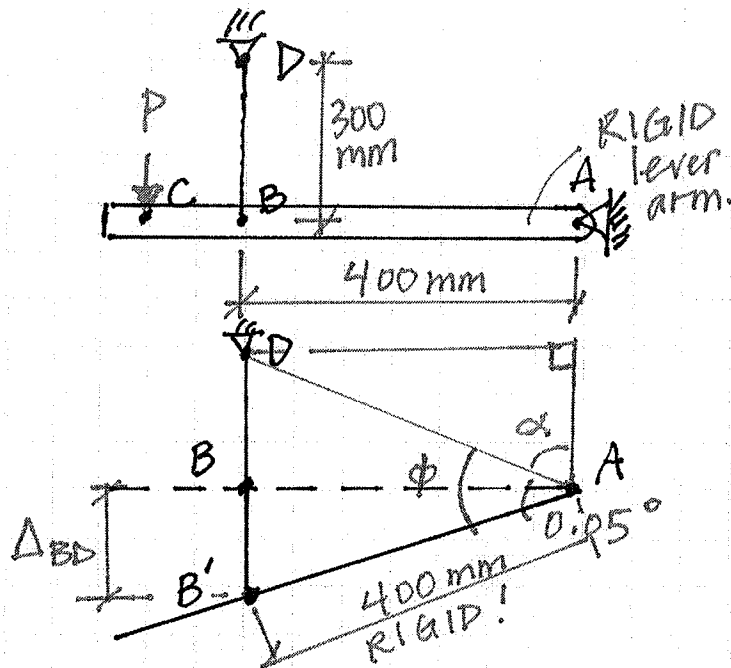
no ΔL



$\gamma \rightarrow$ shear strain

CE 270 2/4/15

p. 1/3

Example 2.2 from textbook

When P is applied, arm rotates about A through angle of 0.05°
 - Determine the normal strain in wire BD.

$$L_{AB'} = 400 \text{ mm}$$

$$L_{BD} = 300 \text{ mm}$$

Solution ① use law of cosines. triangle ADB'

$$L_{B'D} = \sqrt{L_{AD}^2 + L_{AB'}^2 - 2(L_{AD})(L_{AB'})\cos\phi}$$

$$\alpha = \tan^{-1}\left(\frac{400 \text{ mm}}{300 \text{ mm}}\right) = 53.13^\circ$$

$$\phi = (90^\circ - \alpha) + 0.05^\circ = 90^\circ - 53.13^\circ + 0.05^\circ = 36.92^\circ$$

$$L_{AD} = \sqrt{(300 \text{ mm})^2 + (400 \text{ mm})^2} = 500 \text{ mm}$$

$$L_{B'D} = \sqrt{(500 \text{ mm})^2 + (400 \text{ mm})^2 - 2(500 \text{ mm})(400 \text{ mm})\cos 36.92^\circ}$$

$$L_{B'D} = 300.35 \text{ mm.}$$

normal strain? $\epsilon_{BD} = \frac{L_{B'D} - L_{BD}}{L_{BD}} = \frac{300.35 \text{ mm} - 300 \text{ mm}}{300 \text{ mm}}$

$$= \boxed{0.001167 \text{ mm/mm.}}$$

Example 2.2, cont'd

2/3

Solution ②

small deformations.

undeformed $\approx$ deformed

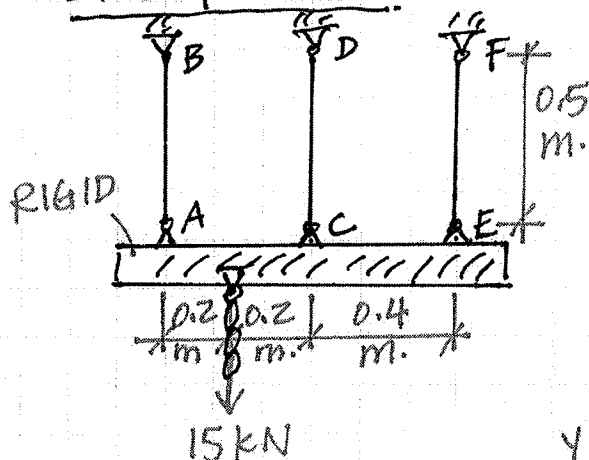
$\sin \theta \approx \theta$, $\tan \theta \approx \theta$

$$\Delta L_{BD} = \theta L_{AB} = \left[\left(\frac{0.05^\circ}{180^\circ} \right) \pi \text{ rad} \right] (400 \text{ mm}) = 0.35$$

$$\tan 0.05^\circ = \frac{\Delta L_{BD}}{400 \text{ mm}}$$

$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L_{BD}} = \frac{0.35 \text{ mm}}{300 \text{ mm}} = \boxed{0.001167 \text{ mm/mm}}$$

Example 4.1e.



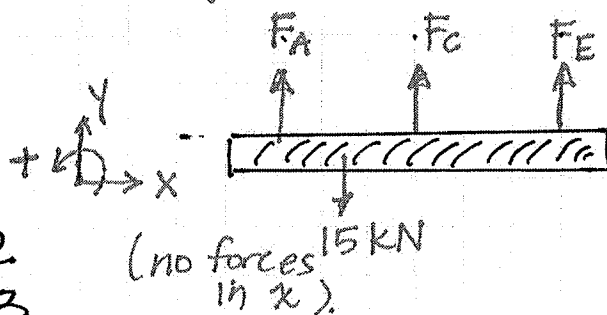
$$A_{AB} = A_{EF} = 50 \text{ mm}^2$$

$$A_{CD} = 30 \text{ mm}^2$$

all A992 steel ☺

What is the force developed in each bar?

Equilibrium.



equations? 2
unknowns? 3

(no forces in x)

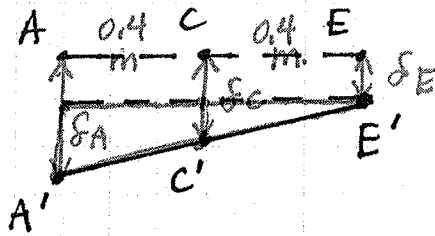
$$+\uparrow \sum F_y = 0 \quad (1) \quad F_A + F_C + F_E - 15 \text{ kN} = 0$$

$$+\curvearrowright \sum M_C = 0 \quad -F_A(0.4 \text{ m}) + 15 \text{ kN}(0.2 \text{ m}) + F_E(0.4 \text{ m}) = 0$$

$$(2) \quad -F_A + F_E + 7.5 = 0$$

→

EX. 4.6, cont'd.

Compatibility of deformations!

$$\frac{\delta_C - \delta_E}{0.4 \text{ m}} = \frac{\delta_A - \delta_E}{0.8 \text{ m}}$$

$$0.8 \delta_C - 0.8 \delta_E = 0.4 \delta_A - 0.4 \delta_E$$

$$\delta_C = \frac{\delta_A}{2} + \frac{\delta_E}{2}$$

$$\left. \begin{array}{l} \sigma = F/A \\ \epsilon = \delta/L \\ \sigma = E\epsilon \end{array} \right\} \rightarrow \delta = \frac{FL}{EA}$$

$$\frac{F_C L}{E_{st}(30 \text{ mm}^2)} = \frac{1}{2} \left[\frac{F_A L}{E_{st}(50 \text{ mm}^2)} + \frac{F_E L}{E_{st}(50 \text{ mm}^2)} \right]$$

$$(3) \quad F_C = 0.3 F_A + 0.3 F_E$$

solve simultaneously with (1) & (2)

$$(2) \quad F_A = F_E + 7.5$$

$$\text{into (3)} \quad F_C = 0.3(F_E + 7.5) + 0.3 F_E = 0.6 F_E + 2.25$$

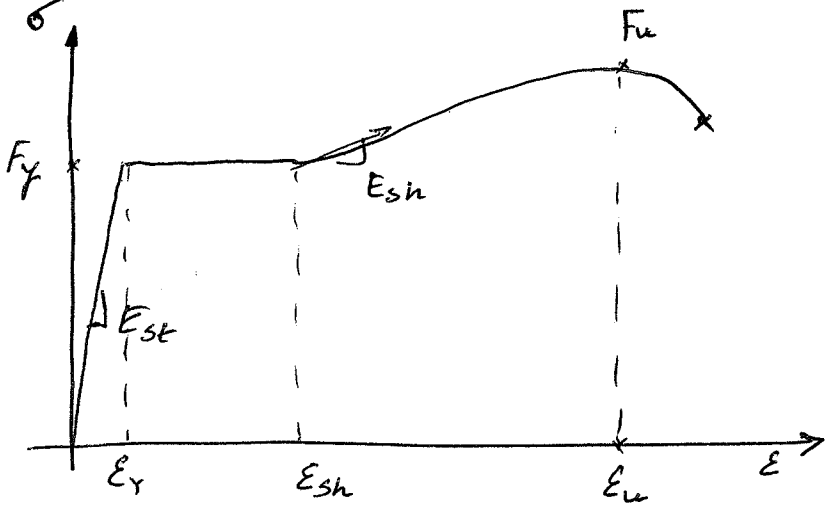
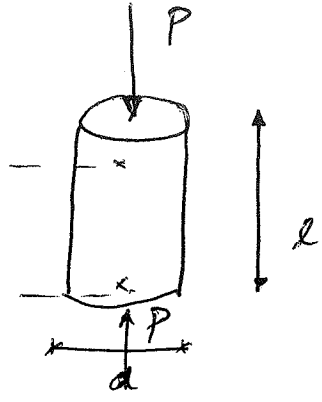
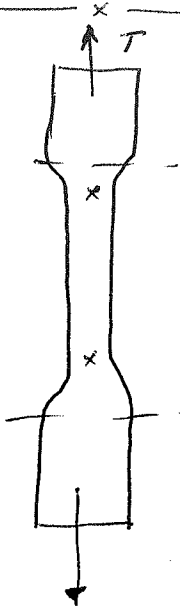
$$\text{into (1)} \quad (F_E + 7.5) + (0.6 F_E + 2.25) + F_E - 15 = 0$$

$$2.6 F_E = 5.25$$

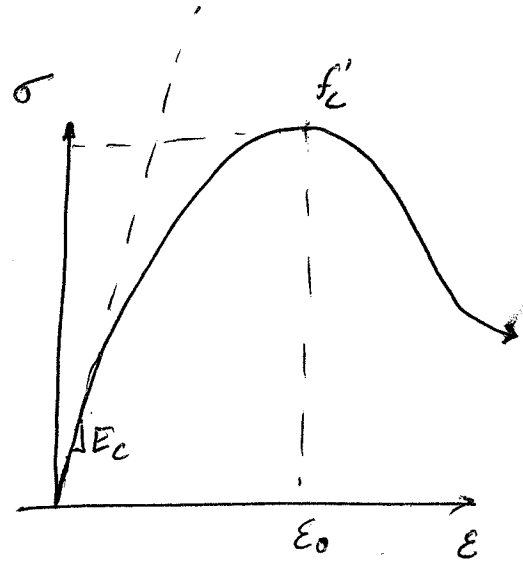
$$\begin{array}{l} F_E = 2.02 \text{ kN} \\ F_A = 9.52 \text{ kN} \\ F_C = 3.46 \text{ kN} \end{array}$$

Statically
indeterminate
problem

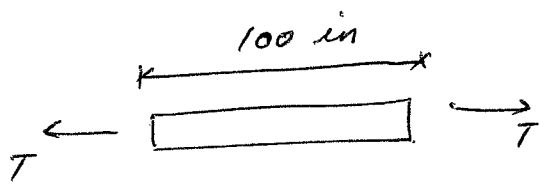
AXIAL FORCE - DEFORMATION BEHAVIOR



$$\epsilon_y \approx 0.0015 - 0.003$$



$$\epsilon_0 \approx 0.002 - 0.003$$

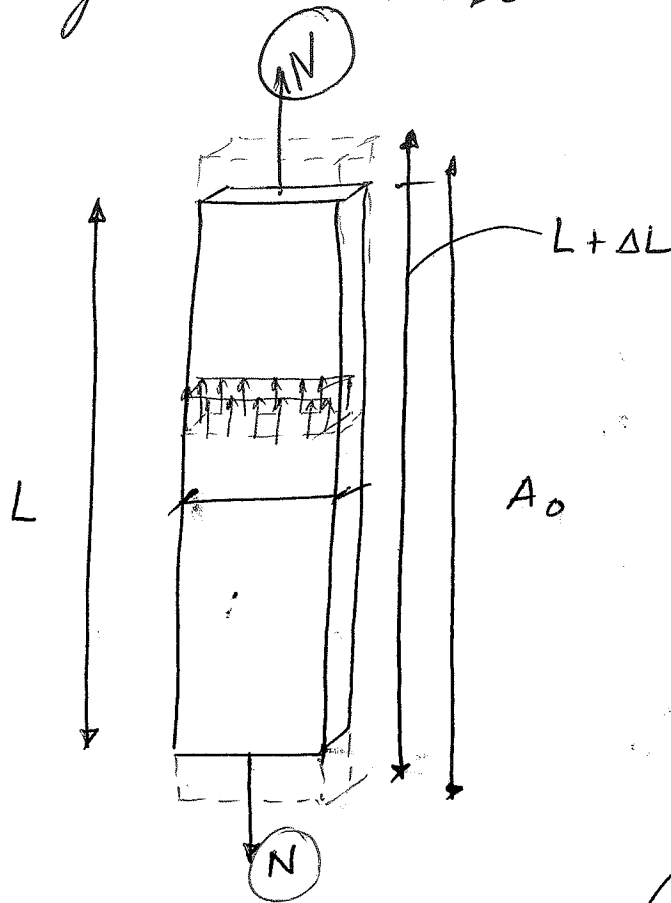


$$\epsilon \approx 0.002$$

$$\begin{aligned} \text{change in length} &= 0.002 \times 100 \text{ in} \\ &= 0.2 \text{ in} \end{aligned}$$

$$\underline{\underline{100.2 \text{ in}}}$$

Any Axial Member.

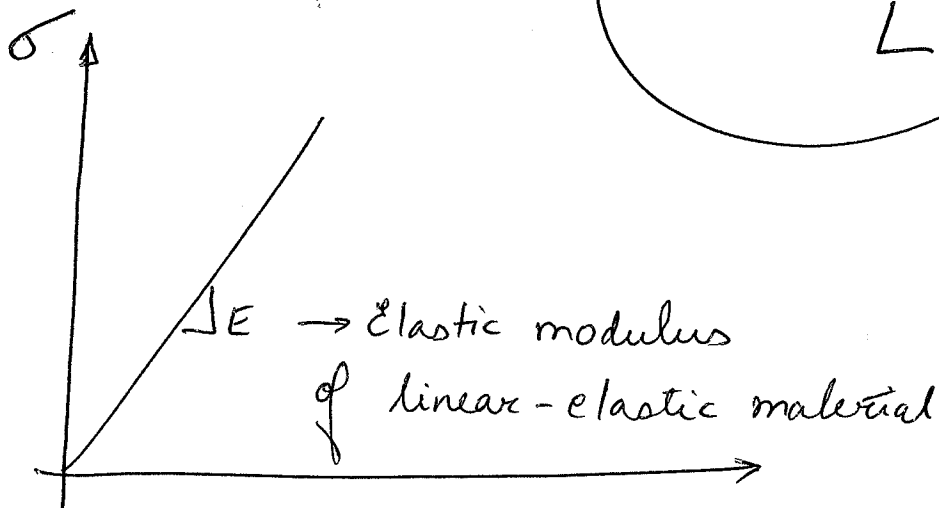


Assume

- ① N does not vary along L
- ② A_0 does not vary along L
- ③ linear elastic material

$$\sigma = \frac{N}{A_0}$$

$$\epsilon = \frac{\Delta L}{L}$$



Hooke's Law

$$\sigma = E \cdot \epsilon$$

$$\sigma = E \cdot \epsilon$$

$$\therefore \left(\frac{N}{A_0} \right) = E \cdot \left(\frac{\Delta L}{L} \right)$$

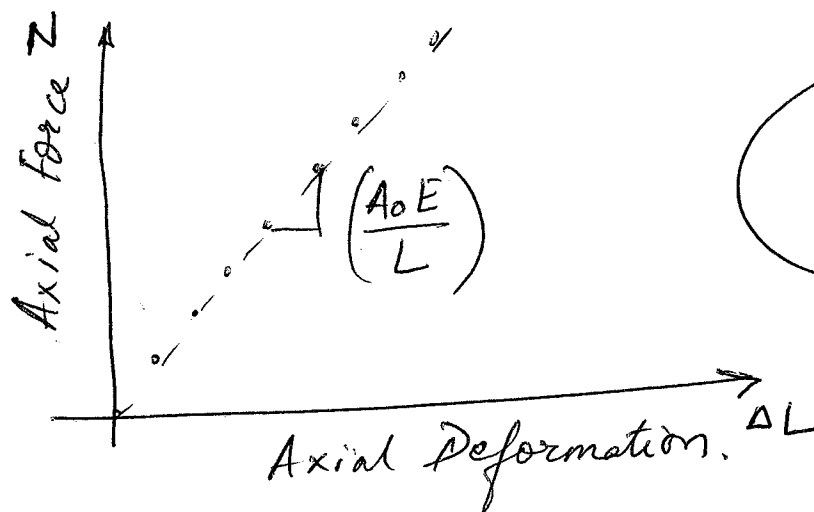
Because

$$\sigma = N/A_0$$

$$\epsilon = \frac{\Delta L}{L}$$

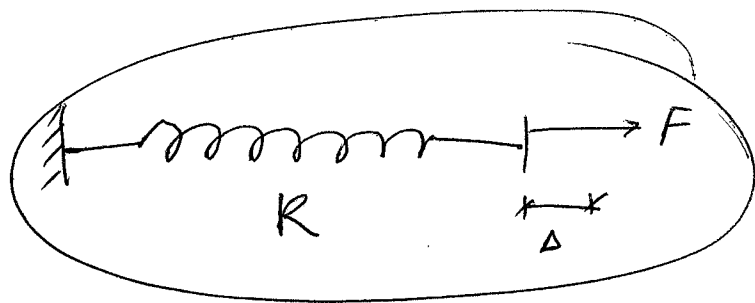
$$\therefore \Delta L = N \times \frac{L}{A_0 E}$$

$$\therefore N = \Delta L \times \left(\frac{A_0 E}{L} \right)$$



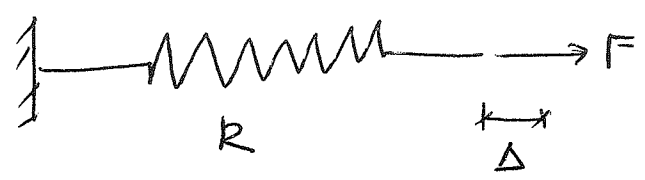
$$N \propto \Delta L$$

$$N = \left(\frac{AE}{L} \right) \Delta L$$

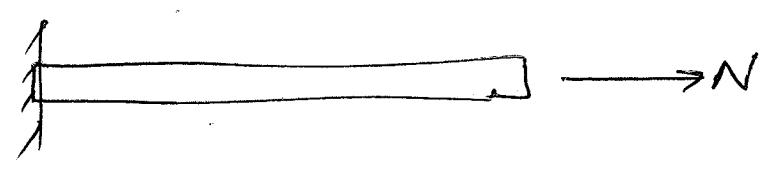


A
L
E

Consider the spring



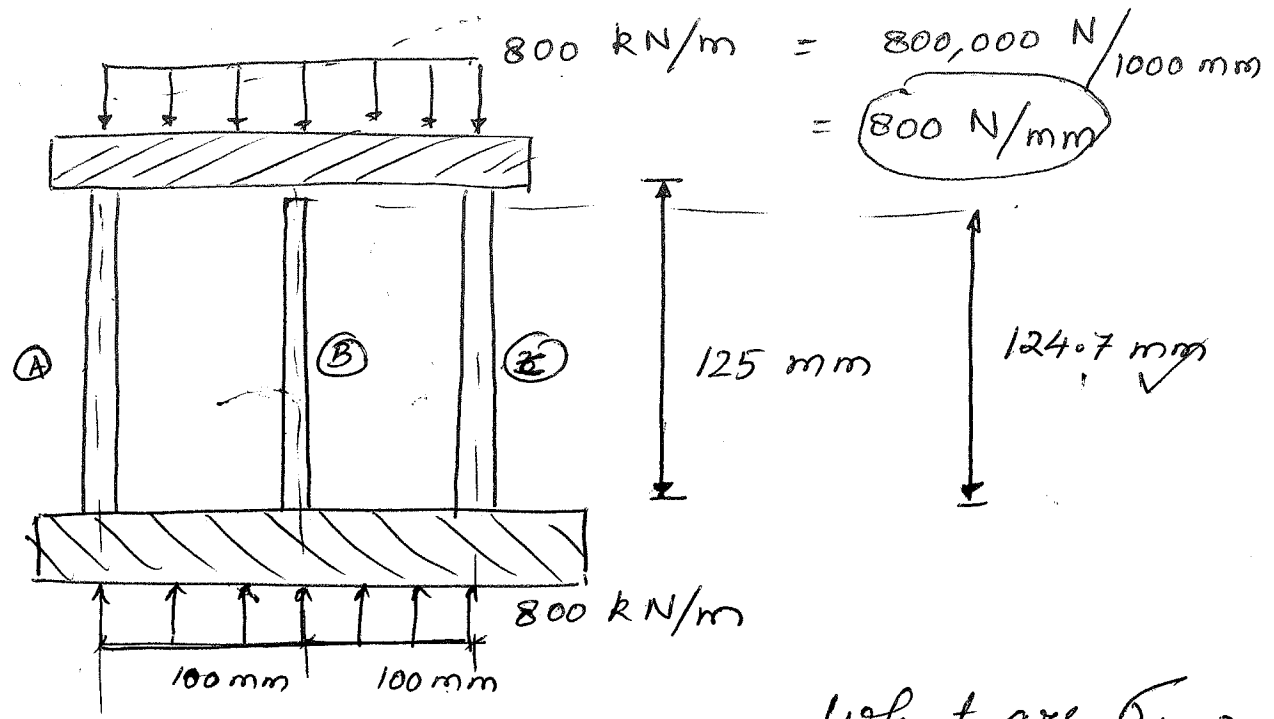
$$F = k \Delta$$



$$N = \left(\frac{AE}{L} \right) \Delta$$

→ Stiffness
member geometry
(A, L)
material (E)

Example: 4-64



What are $\sigma_A, \sigma_B, \sigma_C$

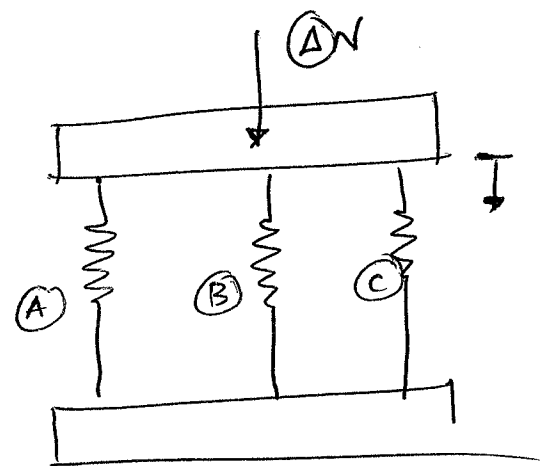
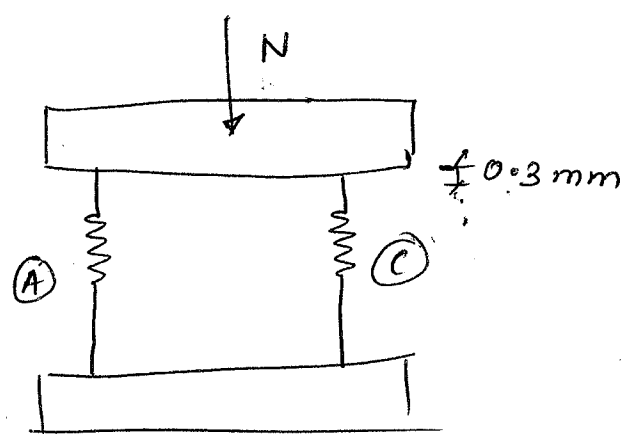
$A = 400 \text{ mm}^2$
 $E = 70 \text{ GPa}$

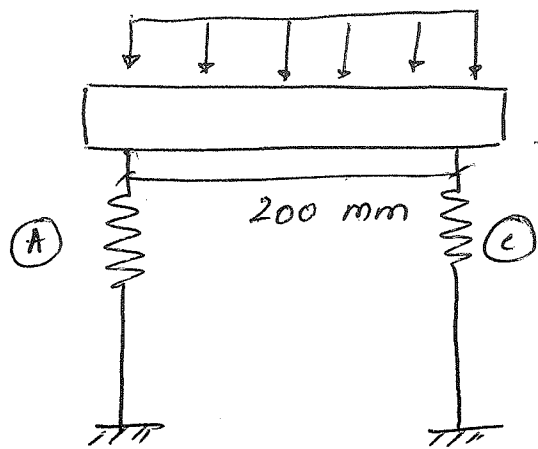
Axially loaded members A, B, C.

$$\left(\frac{AE}{L} \right)_{A, B, C} = \frac{400 \text{ mm}^2 \times 70,000 \text{ N/mm}^2}{125 \text{ mm}}$$

$E = 70,000 \text{ MPa}$
 $= 70,000 \text{ N/mm}^2$

$= 224,000 \text{ N/mm} \checkmark$





$$\Delta = 0.3 \text{ mm} \quad F_A = \left(\frac{AE}{L} \right) \times \Delta$$

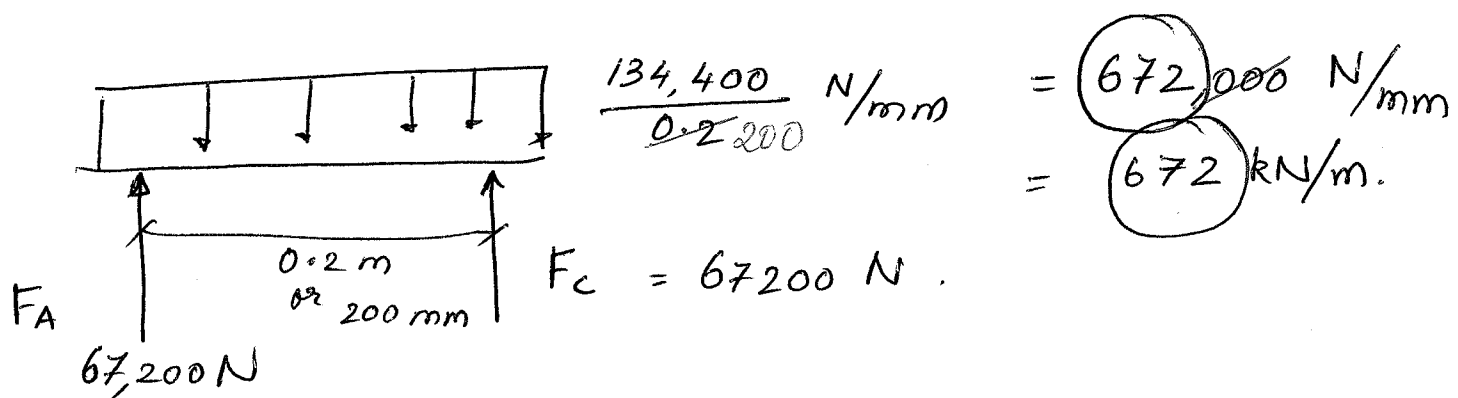
$$F_A = \frac{224000 \text{ N}}{\text{mm}} \times 0.3 \text{ mm}$$

$$F_C = \frac{224000 \text{ N}}{\text{mm}} \times 0.3 \text{ mm}$$

$$F_A = F_C = 67.2 \text{ kN}$$

$$= 67,200 \text{ N}$$

$$\therefore \text{Total force} = F_A + F_C = 134,400 \text{ N}$$



$$\frac{134,400 \text{ N}}{0.2 \text{ m}} = 672,000 \text{ N/m}$$

$$= 672,000 \text{ N/m}$$

$$= 672 \text{ kN/m}$$

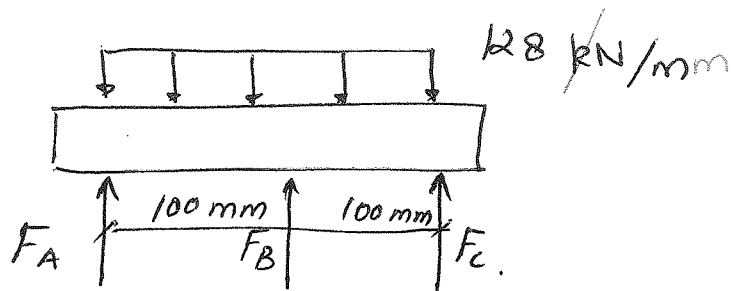
$$F_C = 67200 \text{ N}$$

$$\text{Applied force} = 800 \frac{\text{kN}}{\text{mm}}$$

of which the first 672 $\frac{\text{kN}}{\text{mm}}$ members A & C

the remaining force will be resisted by 3 springs.

$$= 128 \text{ kN/mm}$$

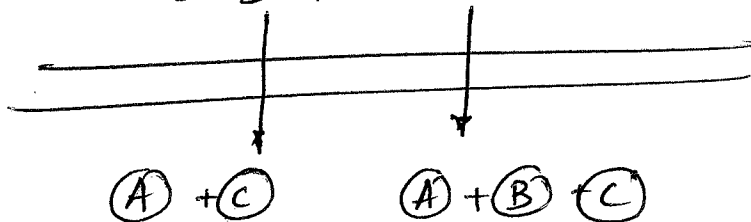


$$\left(\frac{AE}{L} \right)_{A,B,C} = 224,000 \text{ N/mm}$$

$\therefore$ $\underbrace{128 \times \cancel{1000} \frac{\text{N}}{\text{mm}} \times \cancel{0.2 \text{ m}}}_{\text{distributed } 200 \text{ mm}} = \underbrace{224,000 \frac{\text{N}}{\text{mm}} \times 3}_{\times \delta}$

$$\therefore \delta = \frac{128 \times \cancel{1000} \times 200}{224,000 \times 3} = 0.038 \text{ mm}$$

$$\text{Total deformation} = 0.3 + 0.038 \text{ mm}$$



$$F_A = \left(\frac{AE}{L} \right) \times 0.338 \text{ mm} = 75,700 \text{ N}$$

$$F_C = \left(\frac{AE}{L} \right) \times 0.338 \text{ mm} = 75,700 \text{ N}$$

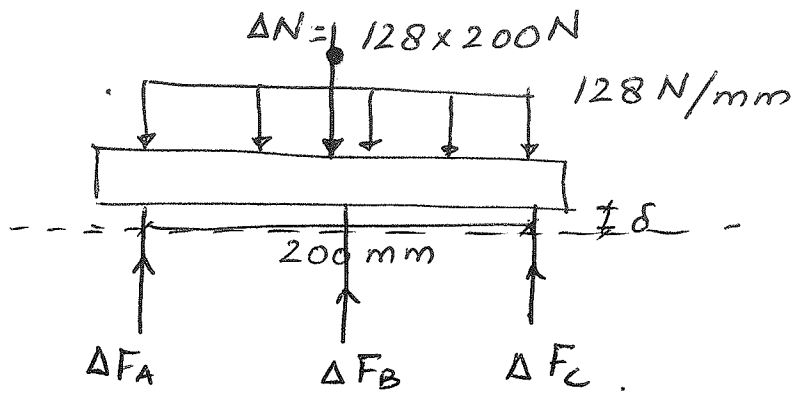
$$F_B = \left(\frac{AE}{L} \right) \times 0.038 \text{ mm} = 8,500 \text{ N}$$

$\swarrow$
 $224,000 \text{ N/mm}$

$$\therefore \sigma_A = \frac{F_A}{400 \text{ mm}^2} = \frac{75,700 \text{ N}}{400 \text{ mm}^2} = 189 \text{ MPa}$$

$$\sigma_C = 189 \text{ MPa}$$

$$\sigma_B = \frac{8,500 \text{ N}}{400 \text{ mm}^2} = \underline{\underline{21.4 \text{ MPa}}}$$



$$128 \times 200 \text{ N} = 3 \times 224,000 \frac{\text{N}}{\text{mm}} \times \delta$$

$$\Delta N = 3 \times (\Delta F)$$

$$\therefore \delta = \frac{128 \times 200 \text{ N}}{3 \times 224,000 \text{ N/mm}} = \underline{\underline{0.038 \text{ mm}}}$$

Initially

$$N = 672 \text{ N/mm}$$

$$\Delta N = 128 \text{ N/mm}$$

$$\Delta = 0.3 \text{ mm}$$

$$\delta = 0.038 \text{ mm}$$

only 2 members A & C

All 3 members A, B & C.

$$F_A = \left(\frac{AE}{L} \right) \times (\Delta + \delta) = 224,000 \frac{\text{N}}{\text{mm}} \times (0.3 + 0.038) = 75,700 \text{ N}$$

$$F_B = \left(\frac{AE}{L} \right) \times \delta = 224,000 \frac{\text{N}}{\text{mm}} \times 0.038 = 8,500 \text{ N}$$

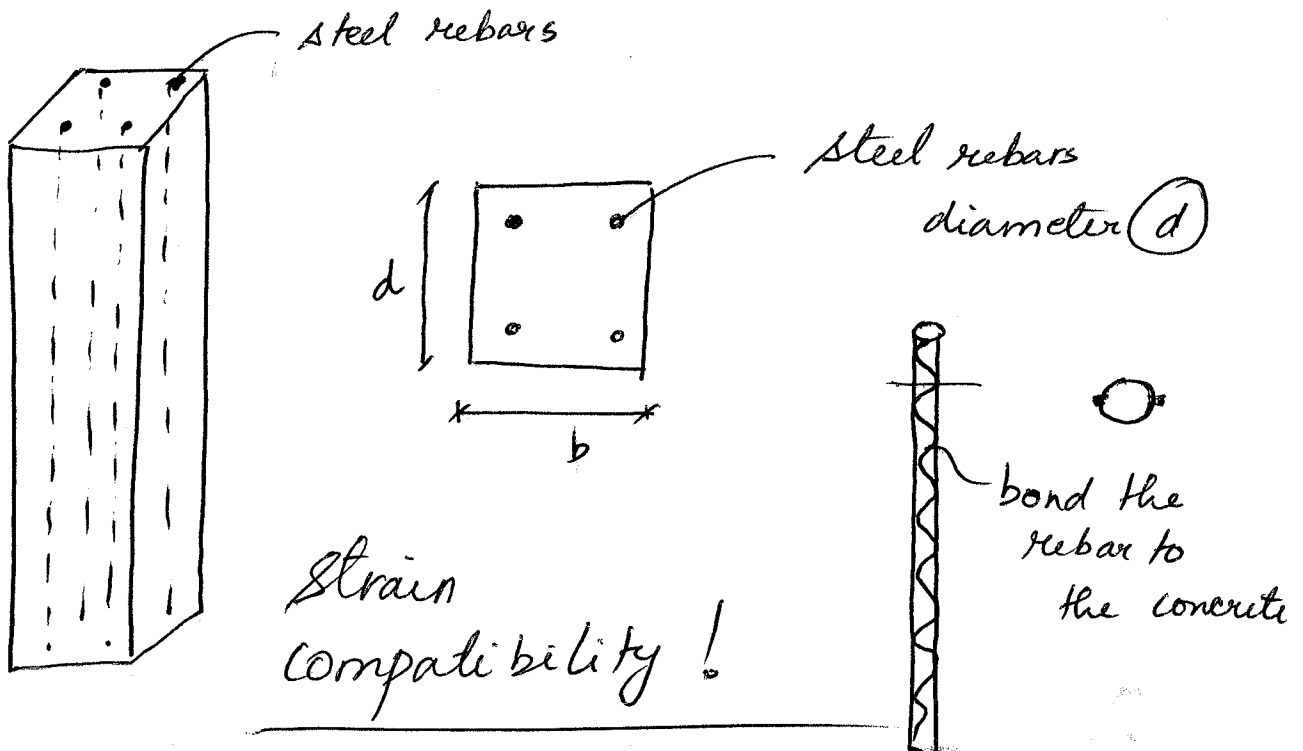
$$F_C = \left(\frac{AE}{L} \right) \times (\Delta + \delta) = 75,700 \text{ N}$$

$$\sigma_{A,C} = \frac{75,700 \text{ N}}{400 \text{ mm}^2} = 189 \text{ N/mm}^2 = 189 \text{ MPa}$$

$$\sigma_B = \frac{8,500 \text{ N}}{400 \text{ mm}^2} = 21.4 \text{ MPa}$$

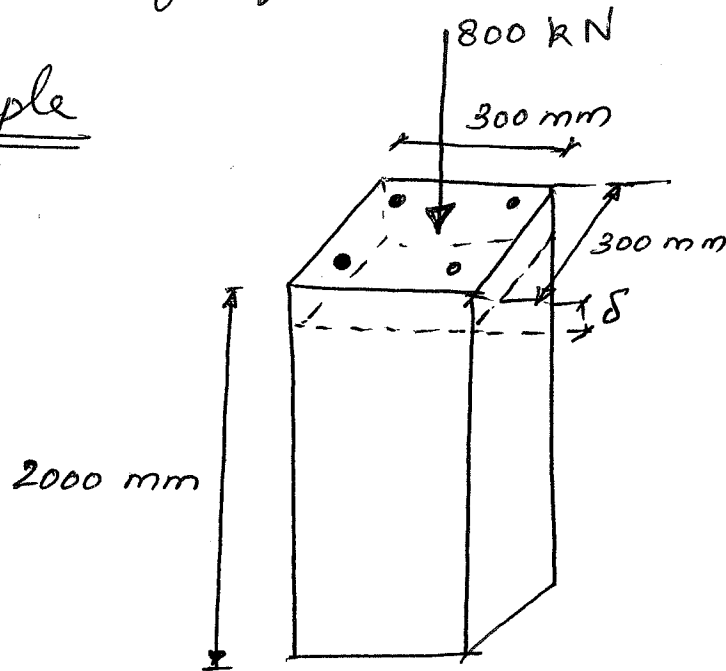
Design of Structural Members-

- Reinforced Concrete Members
- Concrete Filled Composite Members
- RC walls / columns.



- Strains in the steel and concrete are compatible
- steel is fully bonded to the concrete

Example



$$E_{st} = 200 \text{ GPa}$$

$$E_c = 25 \text{ GPa}$$

Determine the required area of each rebar so that $\frac{1}{4}$ th of the load is carried by steel and $\frac{3}{4}$ by concrete

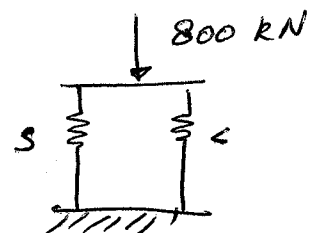
- Assume member shortening = δ

Then what is shortening of steel rebars = $\delta_s = \delta$
 shortening of concrete = $\delta_c = \delta$

What is the force carried by steel rebars.

$$P_s = \left(\frac{AE}{L} \right)_s \times \delta_s$$

$$P_c = \left(\frac{AE}{L} \right)_c \times \delta_c$$



$$P_s = \left(\frac{AE}{L} \right)_s \times \delta = \frac{1}{4} P$$

$$P_c = \left(\frac{AE}{L} \right)_c \times \delta = \frac{3}{4} P$$

$$\therefore \left(\frac{AE}{L} \right)_c \times \cancel{\delta} = 3 \left(\frac{AE}{L} \right)_s \times \cancel{\delta}$$

$$P_c = 3 P_s$$

$$A_c E_c = 3 A_s E_s$$

$$\downarrow$$

$$\sqrt{(300 \text{ mm})^2} \times 25,000 \frac{\text{N}}{\text{mm}^2} = 3 \times A_s \times 200,000 \frac{\text{N}}{\text{mm}^2}$$

$$\therefore A_s = \frac{(300)^2 \times 25000}{3 \times 200,000} = 3750 \text{ mm}^2$$

$$\downarrow$$

$$A_b \times 4 |_{\text{bars}} = \frac{\pi \times d^2}{4} \times 4 = 3750 \text{ mm}^2$$

$$\therefore d = 34.5 \text{ mm}$$

4-34. If column AB is made from high strength pre-cast concrete and reinforced with four $\frac{3}{4}$ in. diameter A-36 steel rods, determine the average normal stress developed in the concrete and in each rod. Set $P = 75$ kip.

$$E_s = 29000 \text{ ksi}$$

$$E_c = 4200 \text{ ksi}$$

Equation of Equilibrium: Referring to the free-body diagram of the cut part of the concrete column shown in Fig. a ,

$$+\uparrow \Sigma F_y = 0; \quad P_{\text{con}} + 4P_{\text{st}} - 2(75) = 0 \quad (1)$$

Compatibility Equation: Since the steel bars and the concrete are firmly bonded, their deformation must be the same. Thus,

$$\delta_{\text{con}} = \delta_{\text{st}}$$

$$\frac{P_{\text{con}}(10)(12)}{\left[(9)(9) - 4\left(\frac{\pi}{4}\right)\left(\frac{3}{4}\right)^2\right](4.20)(10^3)} = \frac{P_{\text{st}}(10)(12)}{\frac{\pi}{4}\left(\frac{3}{4}\right)^2(29)(10^3)}$$

$$P_{\text{con}} = 25.974P_{\text{st}} \quad (2)$$

Solving Eqs. (1) and (2),

$$P_{\text{st}} = 5.0043 \text{ kip} \quad P_{\text{con}} = 129.98 \text{ kip}$$

Normal Stress: Applying Eq. (1-6),

$$\sigma_{\text{con}} = \frac{P_{\text{con}}}{A_{\text{con}}} = \frac{129.98}{(9)(9) - 4\left(\frac{\pi}{4}\right)\left(\frac{3}{4}\right)^2} = 1.64 \text{ ksi}$$

$$\sigma_{\text{st}} = \frac{P_{\text{st}}}{A_{\text{st}}} = \frac{5.0043}{\frac{\pi}{4}\left(\frac{3}{4}\right)^2} = 11.3 \text{ ksi}$$

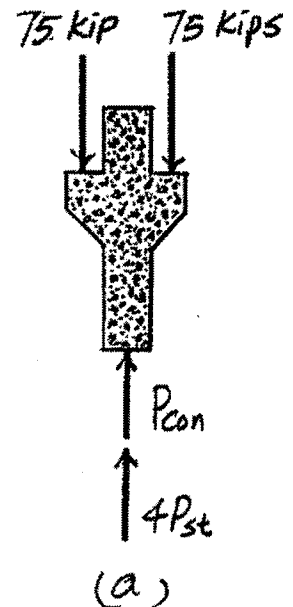
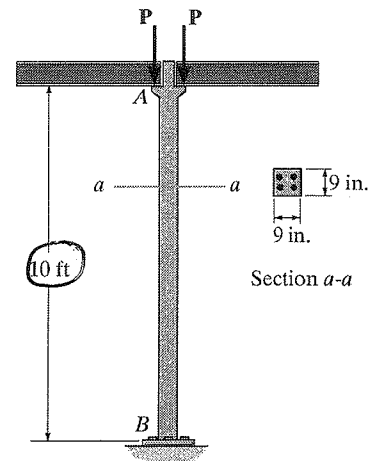
— x —

$$P = 150 \text{ k}$$

$$150 \text{ k} = \left[\left(\frac{AE}{L} \right)_s + \left(\frac{AE}{L} \right)_c \right] \times \delta$$

$$= \left[\frac{\frac{\pi}{4} \times \left(\frac{3}{4}\right)^2 \times 29000 \times 4}{10 \times 12} + \frac{9 \times 9 \times 4200}{10 \times 12} \right] \times \delta$$

$$= (106.765 \times 4 + 2835) \times \delta$$



Ans.

Ans.

$$\therefore \delta = 0.046 \text{ in.}$$

$$\therefore P_s = 5.44 \text{ k}$$

$$P_c = 130.4 \text{ k}$$

$$\therefore \sigma_c = 1.61 \text{ ksi}$$

$$\sigma_s = 11.09 \text{ ksi}$$

Ans:

$$\sigma_{\text{con}} = 1.64 \text{ ksi}, \sigma_{\text{st}} = 11.3 \text{ ksi}$$

$$P_s + P_c = 2P$$

$$P_s + P_c = 150 \text{ k}$$

$$P_s = \left(\frac{AE}{L} \right)_{\text{steel}} \times \delta$$

$$P_c = \left(\frac{AE}{L} \right)_{\text{concrete}} \times \delta$$

$$\delta_s = \delta_c = \delta$$

$$\left(\frac{AE}{L} \right)_{\text{steel}} = \frac{\frac{\pi}{4} \times \left(\frac{3}{4} \right)^2 \text{ in}^2 \times 4 \text{ bars} \times 29000 \text{ ksi}}{120 \text{ in.}}$$

$$\left(\frac{AE}{L} \right)_{\text{concrete}} = \frac{(9 \times 9 \text{ in}^2) \times 4200 \text{ ksi}}{120 \text{ in.}}$$

$$\therefore \left[\left(\frac{AE}{L} \right)_s \times \delta + \left(\frac{AE}{L} \right)_c \times \delta \right] = 150 \text{ kips.}$$

$$\therefore \delta = \frac{150 \text{ kips}}{\left(\frac{AE}{L} \right)_s + \left(\frac{AE}{L} \right)_c}$$

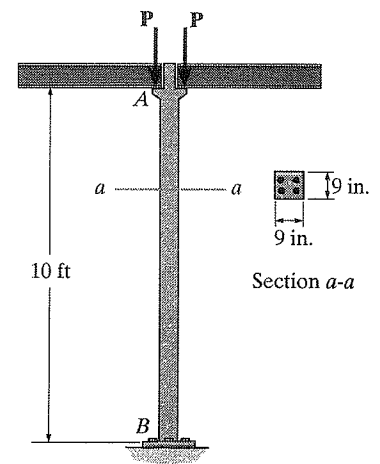
$$= \frac{150}{(4 \times 106.765 + 2835)}$$

$$\therefore \delta = 0.046 \text{ in.}$$

$$\therefore P_s = 19.6 \text{ k} \longrightarrow \sigma_s = 10.61 \text{ ksi}$$

$$P_c = 130.4 \text{ k} \longrightarrow \sigma_c = 11.09 \text{ ksi}$$

4-35. If column AB is made from high strength pre-cast concrete and reinforced with four $\frac{3}{4}$ in. diameter A-36 steel rods, determine the maximum allowable floor loadings P . The allowable normal stress for the high strength concrete and the steel are $(\sigma_{\text{allow}})_{\text{con}} = 2.5$ ksi and $(\sigma_{\text{allow}})_{\text{st}} = 24$ ksi, respectively.



Equation of Equilibrium: Referring to the free-body diagram of the cut part of the concrete column shown in Fig. a ,

$$+\uparrow \Sigma F_y = 0; \quad P_{\text{con}} + 4P_{\text{st}} - 2P = 0 \quad (1)$$

Compatibility Equation: Since the steel bars and the concrete are firmly bonded, their deformation must be the same. Thus,

$$\delta_{\text{con}} = \delta_{\text{st}}$$

$$\frac{P_{\text{con}}(10)(12)}{\left[(9)(9) - 4\left(\frac{\pi}{4}\right)\left(\frac{3}{4}\right)^2 \right] (4.20)(10^3)} = \frac{P_{\text{st}}(10)(12)}{\frac{\pi}{4}\left(\frac{3}{4}\right)^2 (29.0)(10^3)}$$

$$P_{\text{con}} = 25.974P_{\text{st}} \quad (2)$$

Solving Eqs. (1) and (2),

$$P_{\text{st}} = 0.06672P \quad P_{\text{con}} = 1.7331P$$

Allowable Normal Stress:

$$(\sigma_{\text{con}})_{\text{allow}} = \frac{P_{\text{con}}}{A_{\text{con}}}; \quad 2.5 = \frac{1.7331P}{(9)(9) - 4\left(\frac{\pi}{4}\right)\left(\frac{3}{4}\right)^2}$$

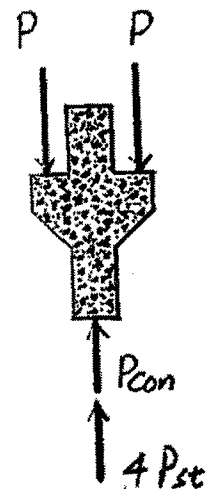
$$P = 114.29 \text{ kip} = 114 \text{ kip (controls)}$$

$$(\sigma_{\text{st}})_{\text{allow}} = \frac{P_{\text{st}}}{A_{\text{st}}}; \quad 24 = \frac{0.06672P}{\frac{\pi}{4}\left(\frac{3}{4}\right)^2}$$

$$P = 158.91 \text{ kip}$$

$$\begin{aligned} 2P &= P_{\text{st}} + P_{\text{c}} \\ &= \left[\left(\frac{AE}{L} \right)_{\text{s}} + \left(\frac{AE}{L} \right)_{\text{c}} \right] \times \delta \end{aligned}$$

$$\therefore \delta = \frac{2P}{\left(\frac{AE}{L} \right)_{\text{s}} + \left(\frac{AE}{L} \right)_{\text{c}}} = \frac{2P}{427.06 + 2835} = \frac{2P}{3262.06} = P/1631.03$$



Ans.
If $\sigma_{\text{c}} = 2.5 \rightarrow P = 116.5 \text{ k}$
 $\sigma_{\text{s}} = 24 \rightarrow P = 162 \text{ k}$ ✓

$$\therefore P_{\text{st}} = \left(\frac{AE}{L} \right)_{\text{s}} \times \frac{2P}{\left(\frac{AE}{L} \right)_{\text{s}} + \left(\frac{AE}{L} \right)_{\text{c}}}$$

$$\sigma_{\text{st}} = \left(\frac{AE}{L} \right)_{\text{s}} \times \frac{2P}{\left(\frac{AE}{L} \right)_{\text{s}} + \left(\frac{AE}{L} \right)_{\text{c}}} \times \frac{1}{A_{\text{s}}}$$

$$\sigma_{\text{con}} = \left(\frac{AE}{L} \right)_{\text{c}} \times \frac{2P}{\left(\frac{AE}{L} \right)_{\text{s}} + \left(\frac{AE}{L} \right)_{\text{c}}} \times \frac{1}{A_{\text{c}}}$$

Ans:
 $P = 114 \text{ kip}$
 $\sigma_{\text{st}} = 0.1481P$
 $\sigma_{\text{c}} = 0.0214P$

Remember,

- You want δ
- Once you have $\delta \rightarrow P_s$
- $P_s \rightarrow \sigma_s \leq \sigma_{all} \rightarrow P_{all}$
- $P_c \rightarrow \sigma_c \leq \sigma_{all} \rightarrow P_{all}$

Thermal Effects

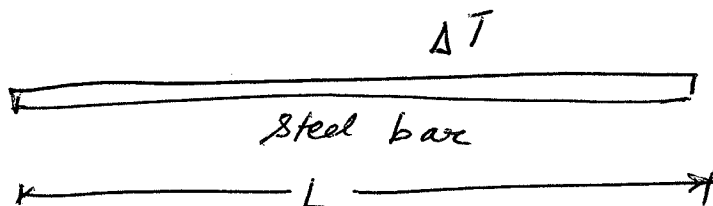
$\alpha \rightarrow$ Expansion coefficient

$$\alpha_{\text{steel}} = 12 \times 10^{-6} / ^\circ\text{C}$$

$$\alpha_{\text{Al}} = 23 \times 10^{-6} / ^\circ\text{C}$$

$$\alpha_c = 6 \times 10^{-6} / ^\circ\text{C}$$

$$T_1 \xrightarrow{\textcircled{\Delta T}} T_2$$



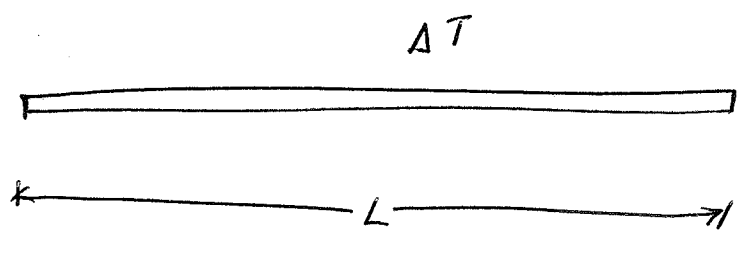
expansion $\rightarrow$ longer $\rightarrow$

$$\text{change in length} = \underbrace{\alpha_{st} \Delta T L}$$

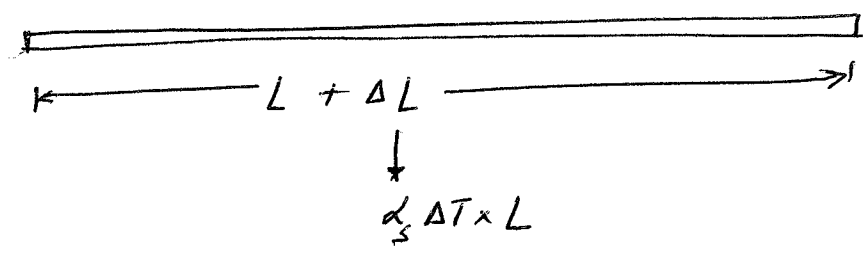
Change in our stress-strain equation

$$\epsilon_{th} = \alpha \Delta T$$

$$\sigma = E \cdot (\epsilon - \epsilon_{th})$$



α_s
 E_s
 A_s



expansion.

$$\epsilon = \frac{\alpha_s \Delta T L}{L} = \alpha_s \Delta T$$

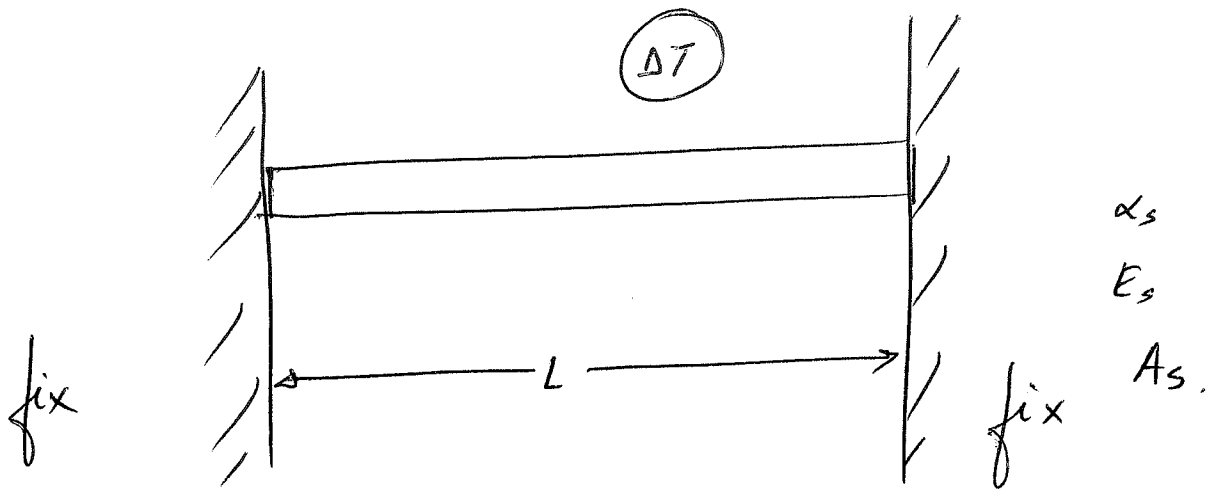
change in length

$$\epsilon_{th} = \alpha_s \Delta T$$

$$\therefore \sigma = E (\alpha_s \Delta T - \alpha_s \Delta T)$$

Thermal strain

$$= \cancel{0}$$



$$\sigma = E \cdot (\epsilon - \epsilon_{th})$$

What is $\epsilon = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L}$

$$\sigma = E (0 - \alpha_s \Delta T)$$

$$\sigma = -E \alpha_s \Delta T$$

$$\sigma = E \cdot (\epsilon - \epsilon_{th})$$

↓

$$F = A \cdot \sigma$$

$$= A \cdot E \cdot (\epsilon - \epsilon_{th})$$

↓ ↘
 $\frac{\delta}{L}$ $\propto \Delta T$

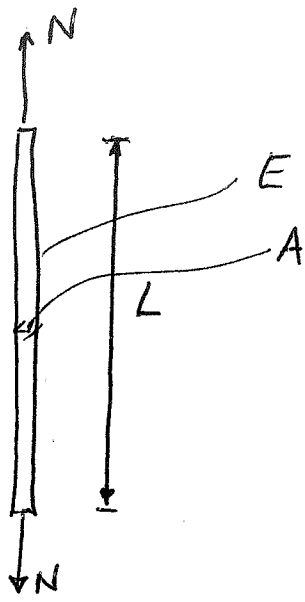
$$= A \cdot E \cdot \left(\frac{\delta}{L} - \alpha \Delta T \right)$$

$$= \frac{A \cdot E}{L} (\delta - \alpha \Delta T L)$$

$$= \frac{AE}{L} (\delta - \delta_{th})$$

↗
 change in
 length
 due to
 thermal

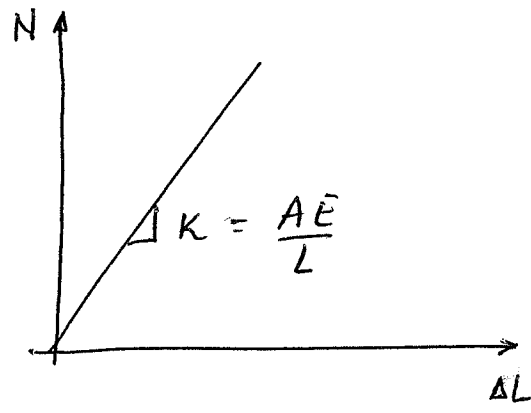
Thermal Stresses



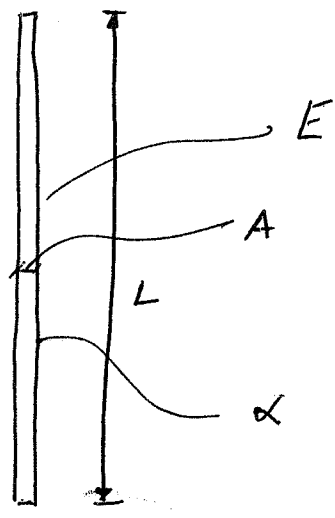
$$\frac{AE}{L} (\Delta L) = N \quad \text{or}$$

$$N = \left(\frac{AE}{L} \right) (\Delta L)$$

↑
k



$$\Delta L = \frac{N \times L}{AE}$$



Change in temperature (ΔT) $\rightarrow$ $^{\circ}\text{C}$
 $\rightarrow$ $^{\circ}\text{F}$
 when in $^{\circ}\text{C}$, mm, N, N/mm², MPa

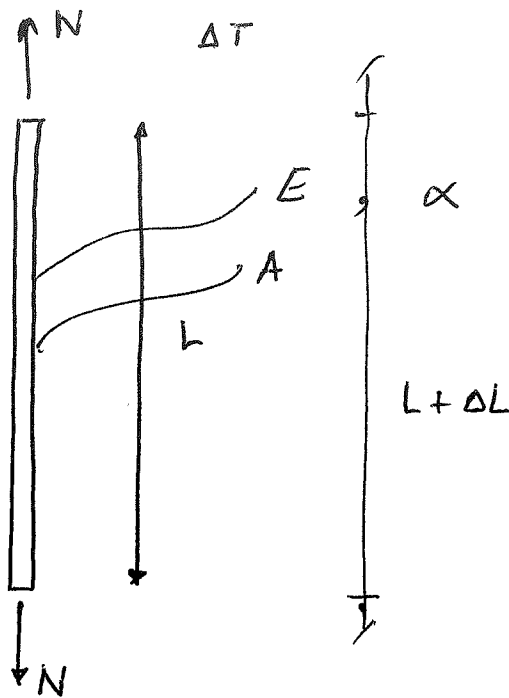
when in $^{\circ}\text{F}$, in., lb, lb/in², psi
 , ksi

$\alpha \rightarrow$ Thermal expansion coefficient

$$\alpha \approx 10 \times 10^{-6} / ^{\circ}\text{C}$$

changes based on material

thermal ^{deformation} expansion = $\alpha \Delta T L = \delta_{th}$



$$N = \frac{AE}{L} (\delta - \delta_{th})$$

$$N = \frac{AE}{L} (\Delta L - \delta_{th})$$

$\delta_{th} = \alpha \cdot \Delta T \cdot L$

Special Case ① When the member is unrestrained/free

$$\Delta L = \alpha \Delta T L$$

$$\therefore N = \frac{AE}{L} (\alpha \Delta T L - \delta_{th})$$

$\delta_{th} = \alpha \Delta T L$

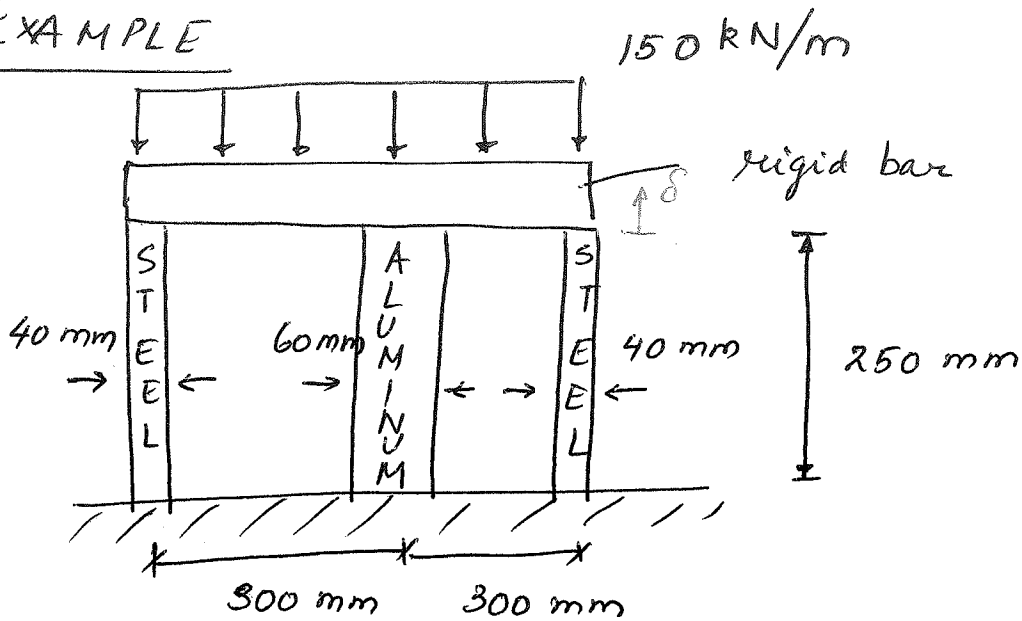
$$= 0$$

Special Case ② When the member is fully restrained/fix

$$\Delta L = 0$$

$$\therefore N = \frac{AE}{L} (0 - \delta_{th}) = -AE(\alpha \Delta T)$$

$\delta_{th} = \alpha \Delta T L$

EXAMPLE

$$T_1 = 20^\circ\text{C}$$

$$T_2 = 80^\circ\text{C}$$

$$\alpha_s = 12 \times 10^{-6}/^\circ\text{C}$$

$$\alpha_{AL} = 23 \times 10^{-6}/^\circ\text{C}$$

$$\Delta T = 60^\circ\text{C} \rightarrow \sigma_{st} \text{ \& \& } \sigma_{AL}$$

$$E_s = 200 \text{ GPa} = 200,000 \text{ MPa} = 200,000 \text{ N/mm}^2$$

$$E_{AL} = 73.1 \text{ GPa} = 73,100 \text{ MPa} = 73,100 \text{ N/mm}^2$$



$$F_s = \left(\frac{AE}{L} \right)_s \times (\delta - \delta_{th}^s)$$

$$F_{AL} = \left(\frac{AE}{L} \right)_{AL} \times (\delta - \delta_{th}^{AL})$$

where,

$$\delta_{th}^s = \alpha_s \times \Delta T \times L$$

$$= 12 \times 10^{-6} \times 60 \times 250 \text{ mm}$$

$$= 0.18 \text{ mm}$$

$$\delta_{th}^{AL} = \alpha_{AL} \times \Delta T \times L$$

$$= 23 \times 10^{-6} \times 60 \times 250$$

$$= 0.345 \text{ mm}$$

(7)

where, $\left(\frac{AE}{L}\right)_s = \frac{\pi \times 40^2}{4} \times \frac{200,000 \text{ N/mm}^2}{250 \text{ mm}}$

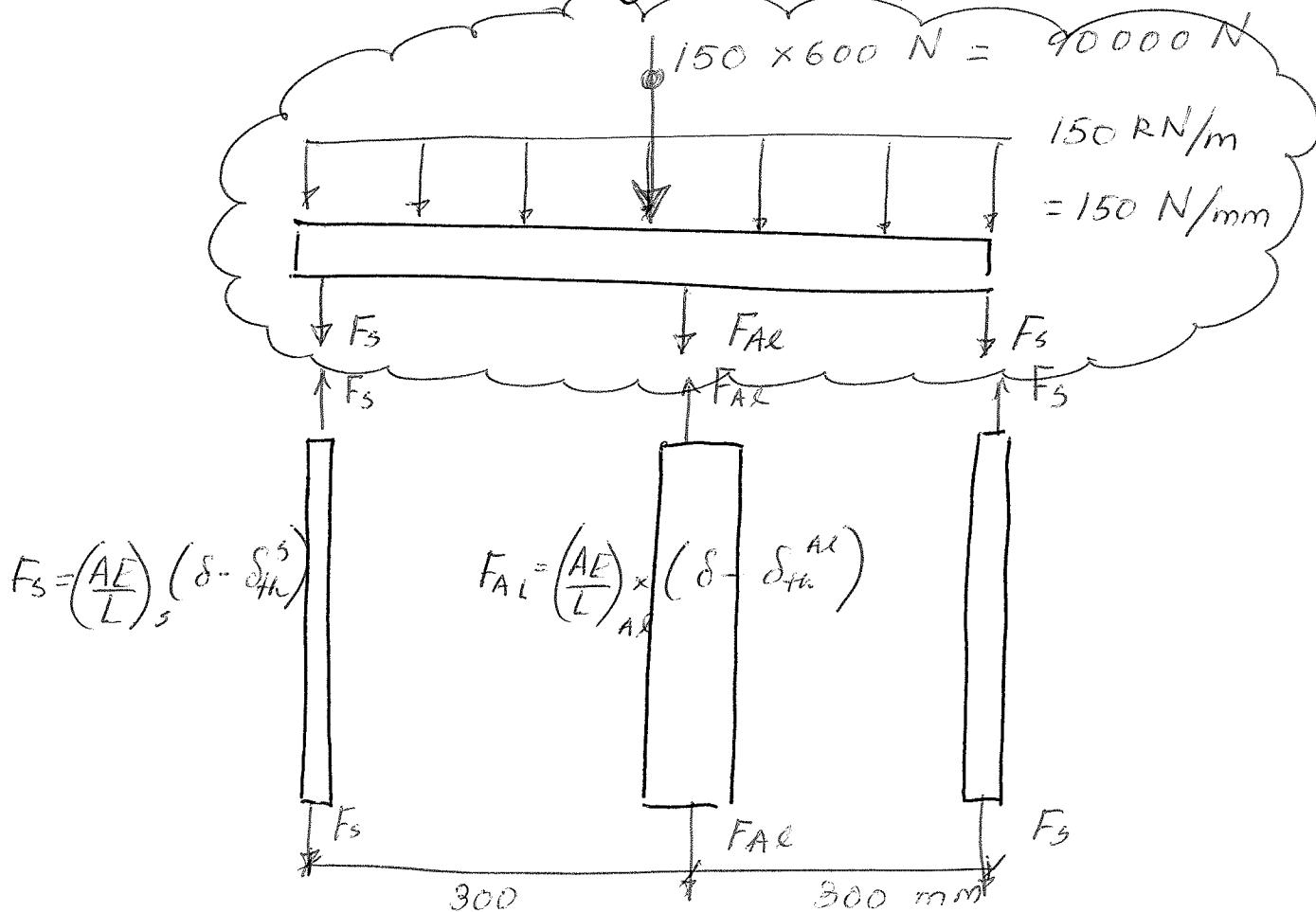
$$= 1005309.65 \text{ N/mm}$$

$$\left(\frac{AE}{L}\right)_{AL} = \frac{\pi \times 60^2}{4} \times \frac{73,100 \text{ N/mm}^2}{250 \text{ mm}}$$

$$= 826741.52 \text{ N/mm}$$

$$F_s = 1005309.65 (\delta - 0.18)$$

$$F_{AL} = 826741.52 (\delta - 0.345)$$



$$+\uparrow \sum F_y = 0 \quad -90000 - F_s - F_s - F_{Ax} = 0 \quad (72)$$

$$2F_s + F_{Ax} + 90,000 = 0$$

$$\therefore 2 \left\{ 1005309.65 (\delta - 0.18) \right\} + 826741.52 (\delta - 0.345) = -90,000$$

$$\therefore \delta = \frac{557137.3}{2837360.82} = \frac{0.196}{\underline{\underline{0.2597 \text{ mm}}}}$$

$$\begin{aligned} F_{st} &= \left(\frac{AE}{L} \right)_s (\delta - \delta_{th}^s) \\ &= 1005309.65 (0.196 - 0.18) = 16444.4 \text{ N} \\ &= \underline{\underline{16.4 \text{ kN}}} \end{aligned}$$

$$\begin{aligned} F_{Ax} &= \left(\frac{AE}{L} \right)_{Al} (\delta - \delta_{th}^{Al}) \\ &= 826741.52 (0.196 - 0.345) \\ &= -123,184 \text{ N} = \underline{\underline{-123.2 \text{ kN}}} \end{aligned}$$

most structural materials

$$\epsilon \approx 0.002$$

$$\epsilon_{th} = \alpha \Delta T$$

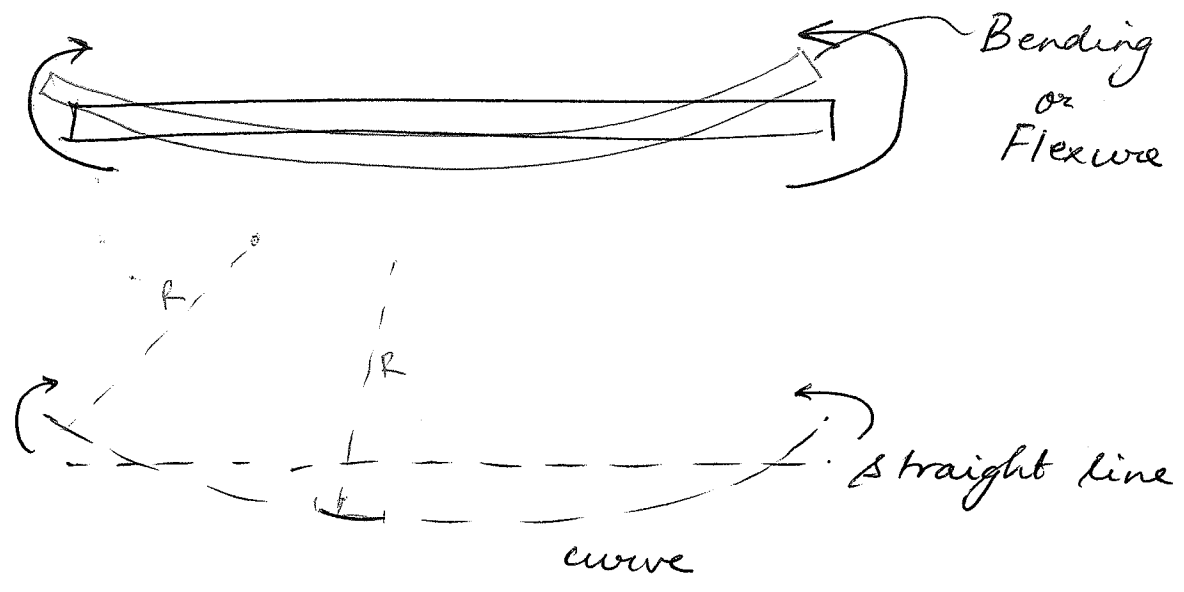
$$= 10 \times 10^{-6} \times 100$$

$$= 1000 \times 10^{-6}$$

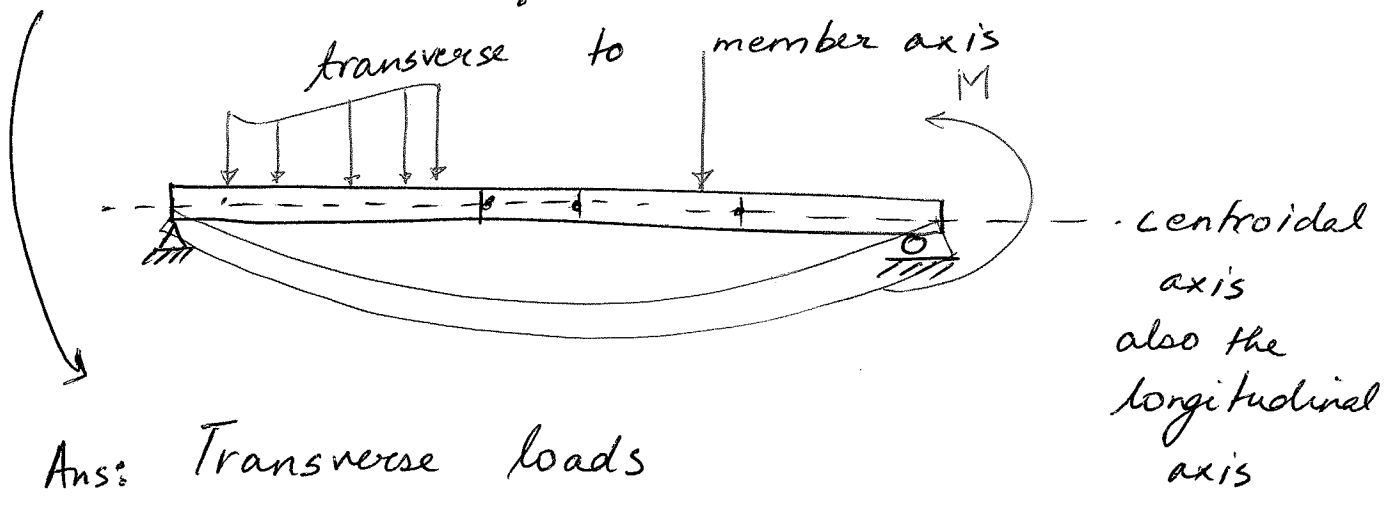
$$= \underline{0.001}$$

CE 270: Bending / Flexure

most important topics to remember

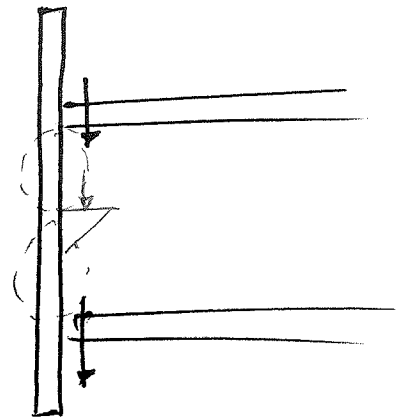


What causes it?



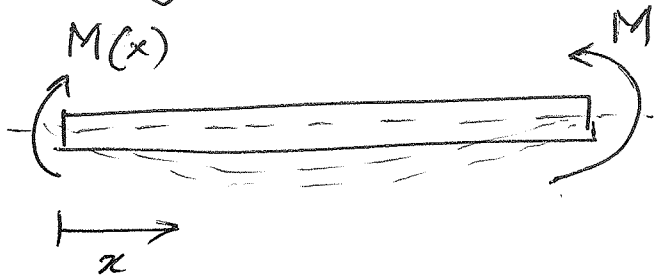
What is the difference between flexure & axial force?

Axial Force:



- load was parallel to the longitudinal axis
- Axial Force, N
- Axial deformation, Δ , shortening or lengthening
- did not become a curve
- $\sigma = N/A$
- $\epsilon = \frac{\Delta}{L}$
- $\sigma = E \cdot \epsilon$
- $N = \frac{AE}{L} (\Delta)$
- N was constant along length
- $\therefore \sigma$ was constant along length.

Bending or Flexure:



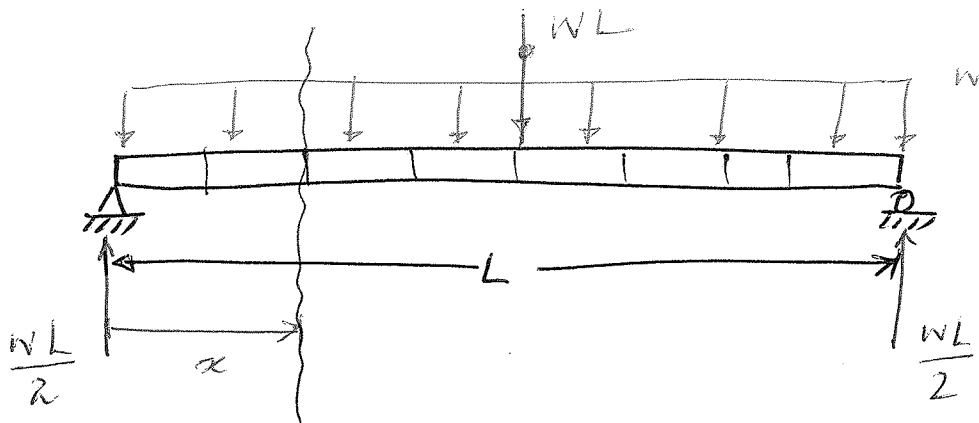
- transverse loading
even if uniform changes $M(x)$ along length
- Bending Moment, $M(x)$
- Curvature, ϕ , bending

- member has curvature after bending
- produce stresses
normal stress due to bending

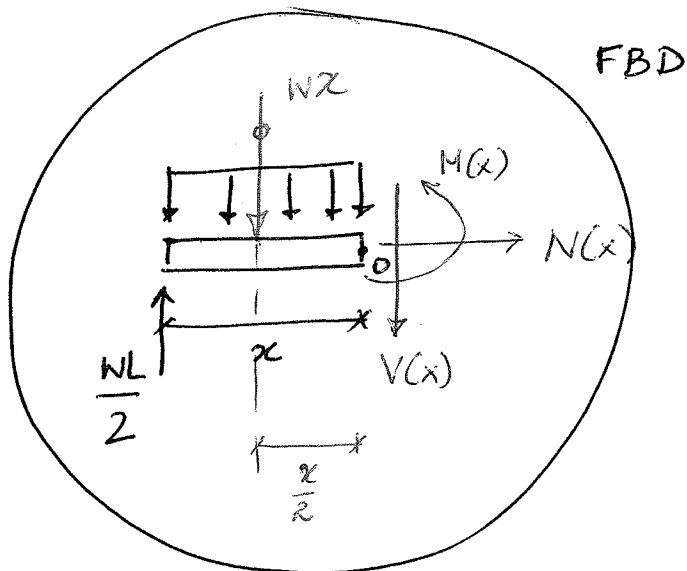
$$\sigma = \frac{M(x) y}{I}$$

$$\phi = \frac{M(x)}{EI}$$
- σ is not constant along length
- not uniform through the section

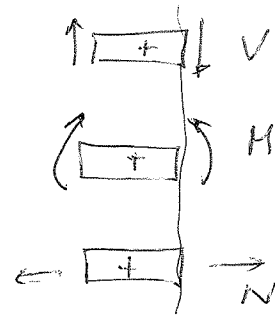
Obj: Determine $M(x)$ along member length



Q: Determine $M(x)$



Sign Convention



Equations of Equilibrium

$$\rightarrow \sum F_x = 0 \quad \therefore N(x) = 0$$

$$\uparrow \sum F_y = 0 \quad \therefore \frac{WL}{2} - wx - V(x) = 0$$

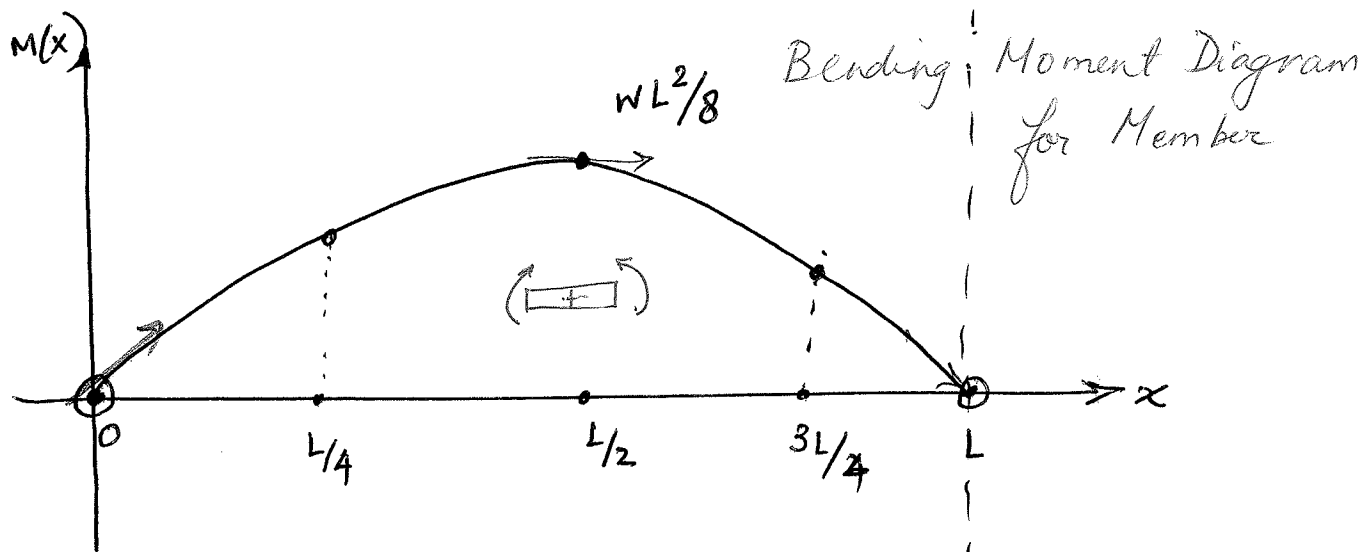
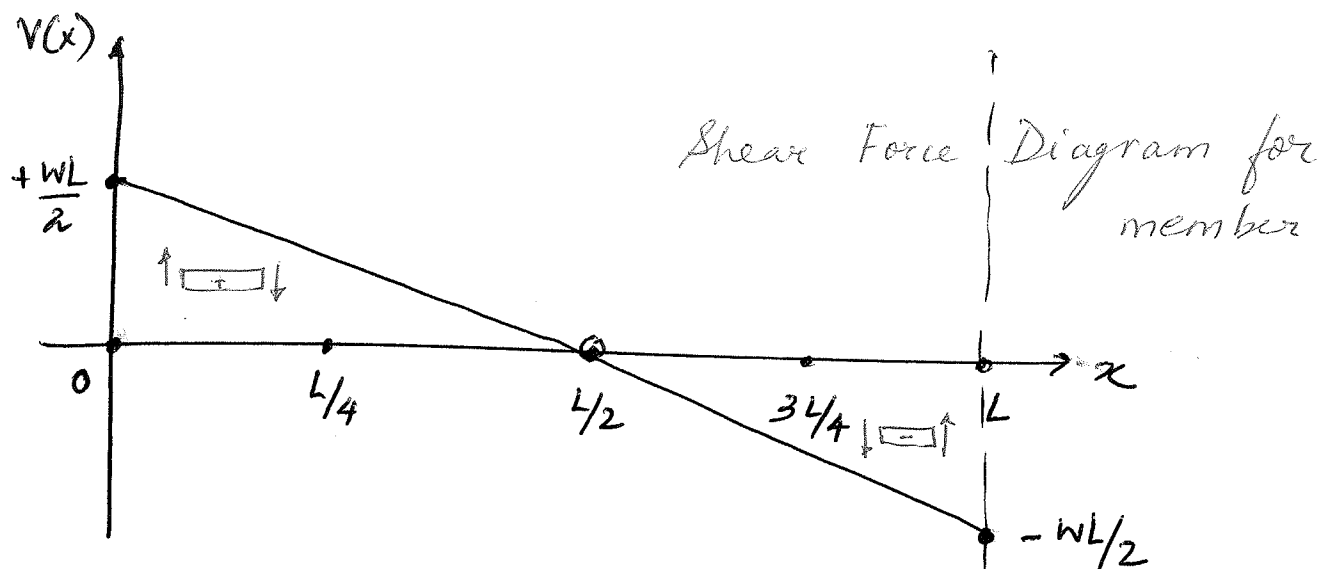
$$\therefore V(x) = w \left(\frac{L}{2} - x \right) \quad \text{Shear Force}$$

$$\curvearrowright \sum M_o = 0 \quad \therefore -\frac{WL}{2} \times x + wx \times \frac{x}{2} + M(x) = 0$$

$$\therefore M(x) = \frac{WLx}{2} - \frac{wx^2}{2}$$

Plot $V(x)$ & $M(x)$ along the member length

$$V(x) = w\left(\frac{L}{2} - x\right)$$

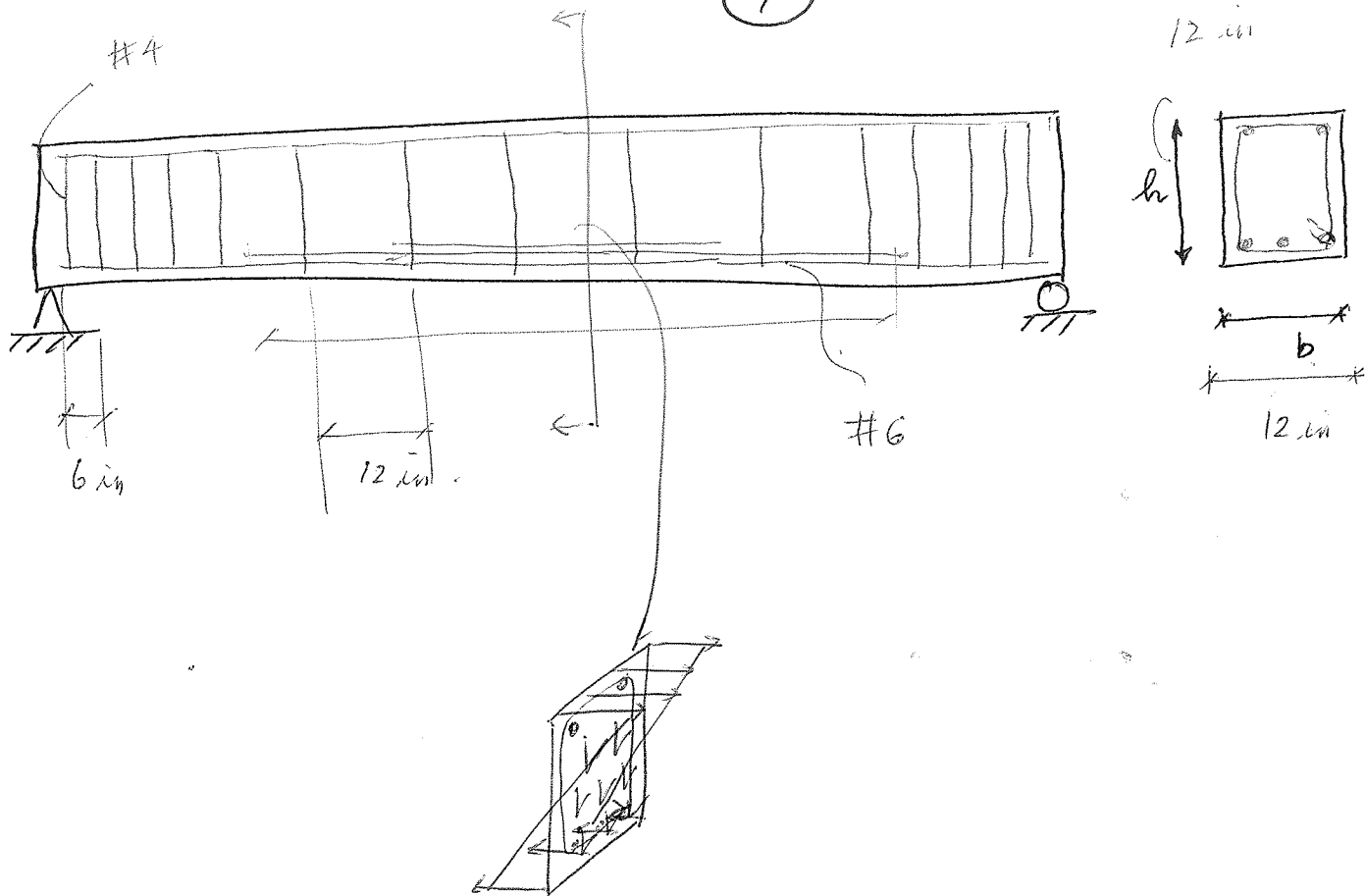


$$M(x) = w \frac{Lx}{2} - \frac{wx^2}{2}$$

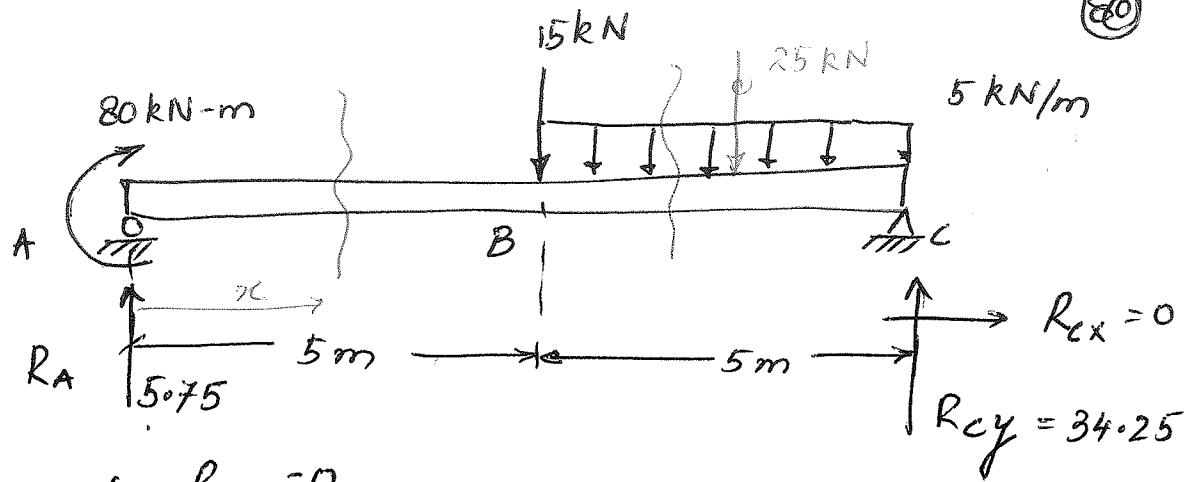
@ $L/2$

$$\begin{aligned} &= w \frac{L}{2} \times \frac{L}{2} - w \frac{L^2}{4} \times \frac{1}{2} \\ &= \frac{wL^2}{8} \end{aligned}$$

Fig



EXAMPLE



$$\rightarrow \sum F_x = 0 \quad \therefore R_{Cx} = 0$$

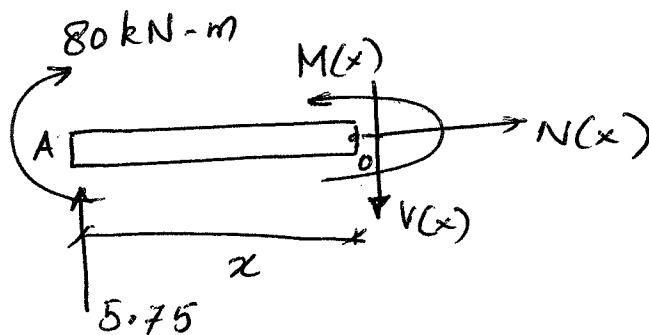
$$\uparrow \sum F_y = 0 \quad R_A - 15 - 25 + R_{Cy} = 0 \quad (\text{kN})$$

$$\curvearrowleft \sum M_A = 0 \quad \therefore -80 - 15 \times 5 - 25(7.5) + R_{Cy} \times 10 = 0 \quad (\text{kN-m})$$

$$\therefore R_{Cy} = 34.25 \text{ kN} \quad \rightarrow \quad R_A = 5.75 \text{ kN}$$

Use method of sections.

$$\forall x \quad 0 \leq x \leq 5 \text{ m}$$



$$\rightarrow \sum F_x = 0 \quad \therefore N(x) = 0$$

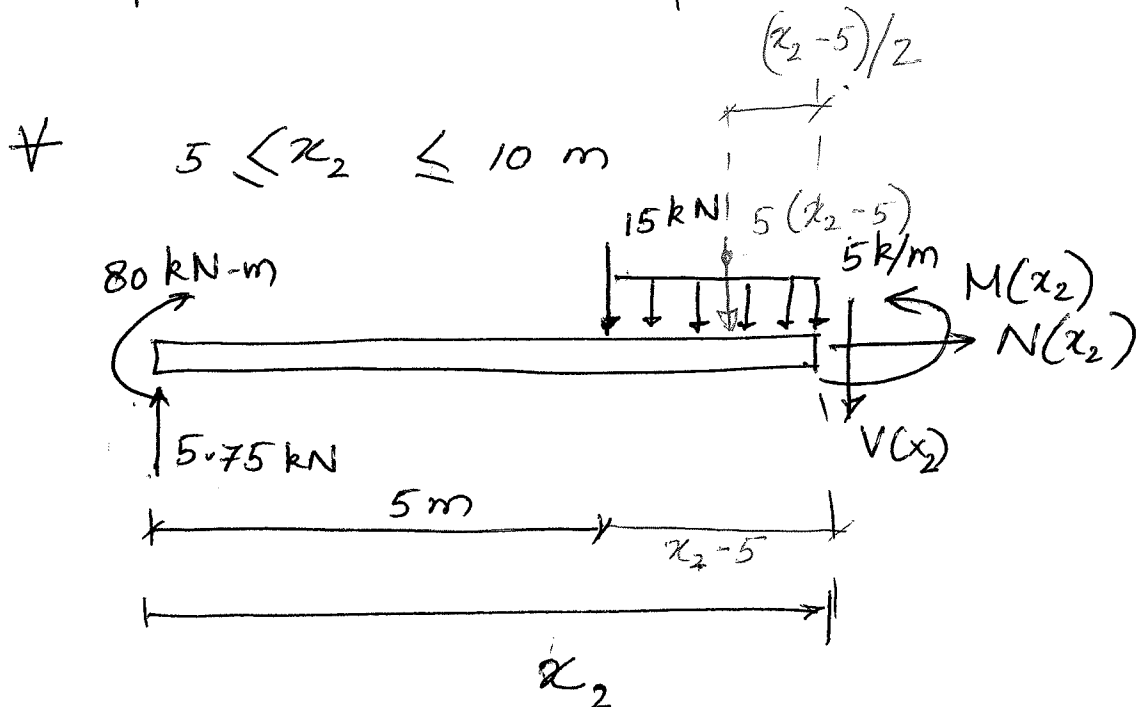
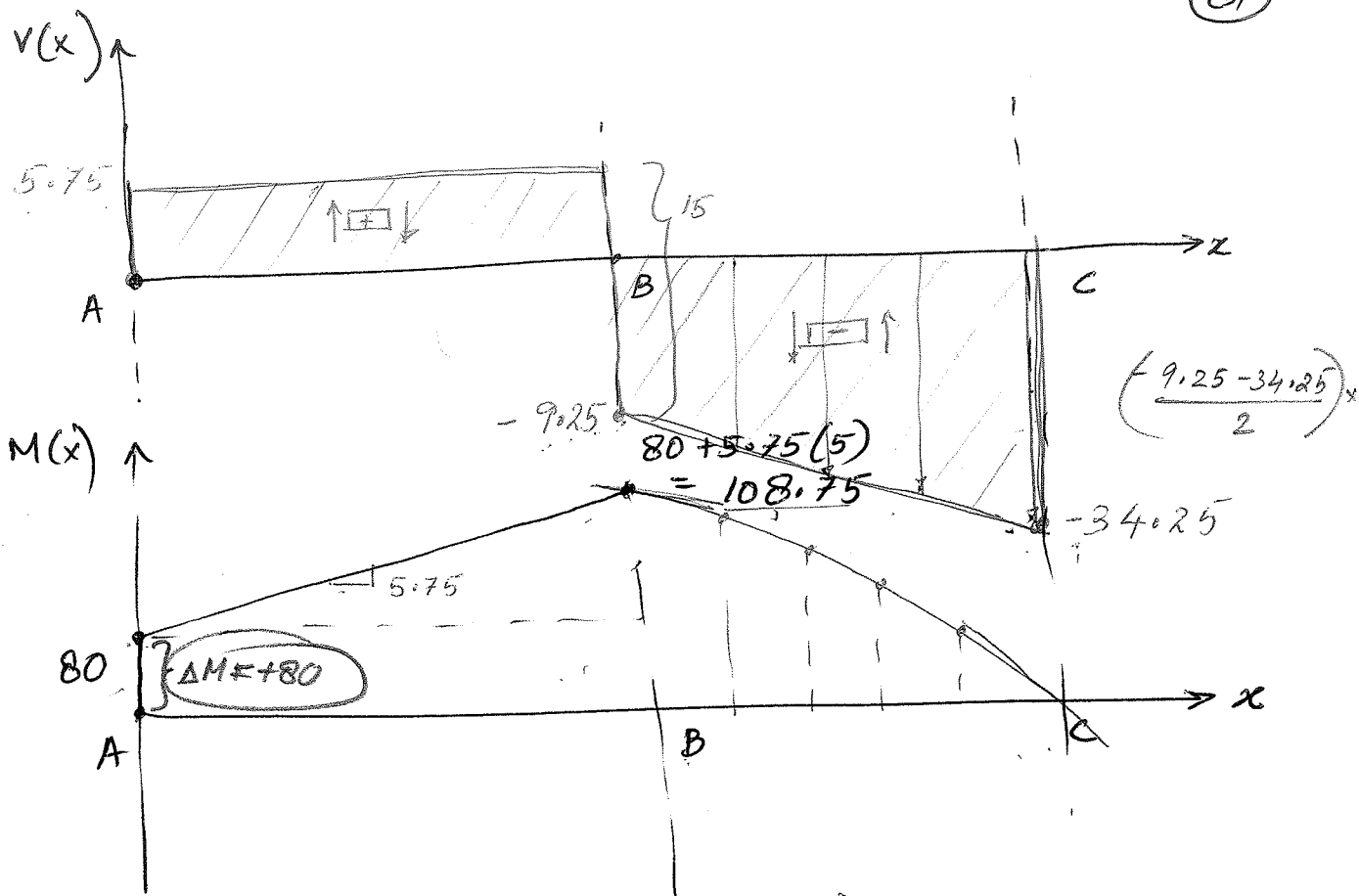
$$\uparrow \sum F_y = 0 \quad \therefore 5.75 - V(x) = 0$$

$$\therefore V(x) = 5.75 \text{ kN}$$

$$\curvearrowleft \sum M_o = 0 \quad \therefore M(x) - 80 - 5.75(x) = 0$$

$$\therefore M(x) = 80 + 5.75(x)$$

(8/)



$$\rightarrow \sum F_x = 0 \quad \therefore N(x_2) = 0$$

$$\uparrow \sum F_y = 0 \quad \therefore 5.75 - 15 - 5(x_2 - 5) - V(x_2) = 0$$

$$\therefore 5.75 - 15 - 5x_2 + 25 - V(x_2) = 0$$

$$\therefore 15.75 - 5x_2 = V(x_2)$$

$$\hookrightarrow \sum M_o = 0$$

$$\therefore M(x_2) + \underline{5(x_2 - 5)} \times \frac{(x_2 - 5)}{2} + 15 \times (x_2 - 5) - 5.75 \times x_2 - 80 = 0$$

$$\therefore M(x_2) = \left(-2.5x_2^2 + 15.75x_2 + 92.5 \right) \text{ kN-m}$$

$M(x_2)$

$$x_2 = 5$$

6

7

8

9

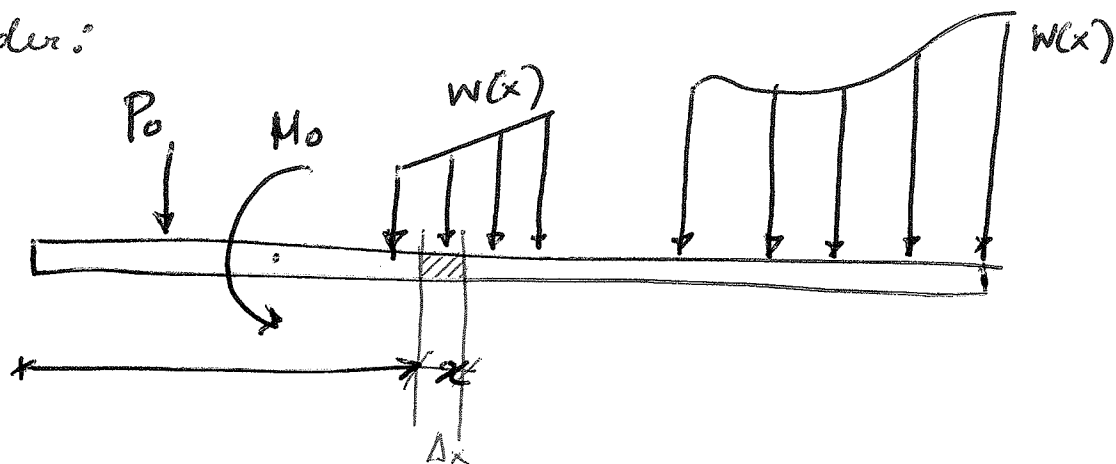
10

Slope of $M(x_2)$

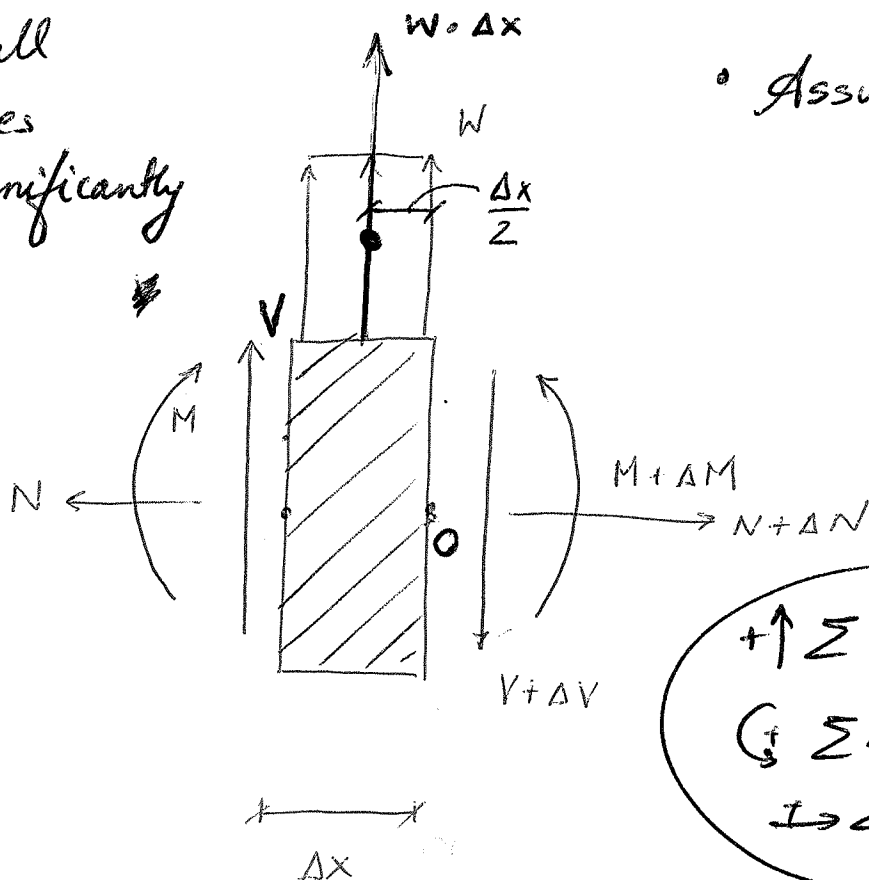
$$= \frac{dM}{dx} = 15.75 - 5x_2$$

Part 2 : Shear Force & Bending Moment Diagrams (Graphical)

Consider:



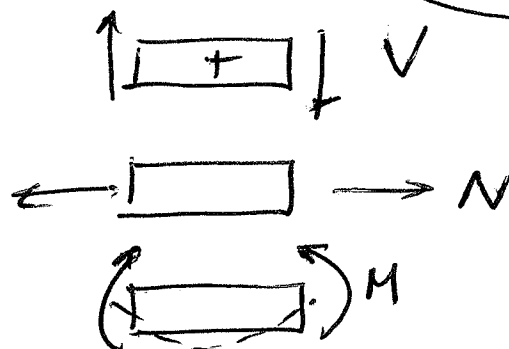
• Δx is so small
 that $w(x)$ does
 not vary significantly



• Assume sign convention
 of $w(x)$
 ↑ +ve

$$\begin{aligned} + \uparrow \sum F_y &= 0 \\ \curvearrowright \sum M &= 0 \\ \rightarrow \sum F_x &= 0 \end{aligned}$$

Sign conventions:



$$+\uparrow \sum F_y = 0 \quad \therefore \cancel{V} + W \cdot \Delta x - \cancel{V} - \Delta V = 0$$

$$\therefore \Delta V = W \cdot \Delta x$$

1

$$\text{or} \quad \frac{\Delta V}{\Delta x} = W$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta V}{\Delta x} = \frac{dV}{dx} = W$$

Slope of the shear force diagram = $W(x)$
remember sign convention

$$\curvearrowright \sum M_o = 0 \quad \therefore \cancel{M} + \Delta M - \left(W \cdot \Delta x \cdot \frac{\Delta x}{2} \right) - \cancel{M} - V \cdot \Delta x = 0$$

$$\therefore \Delta M = V \Delta x + \frac{W \cdot \Delta x^2}{2}$$

negligible

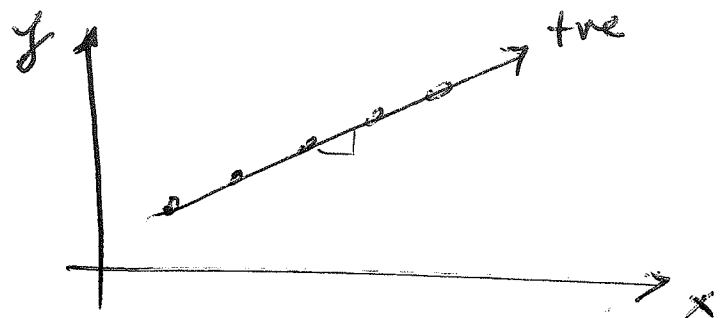
2

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta M}{\Delta x} = V$$

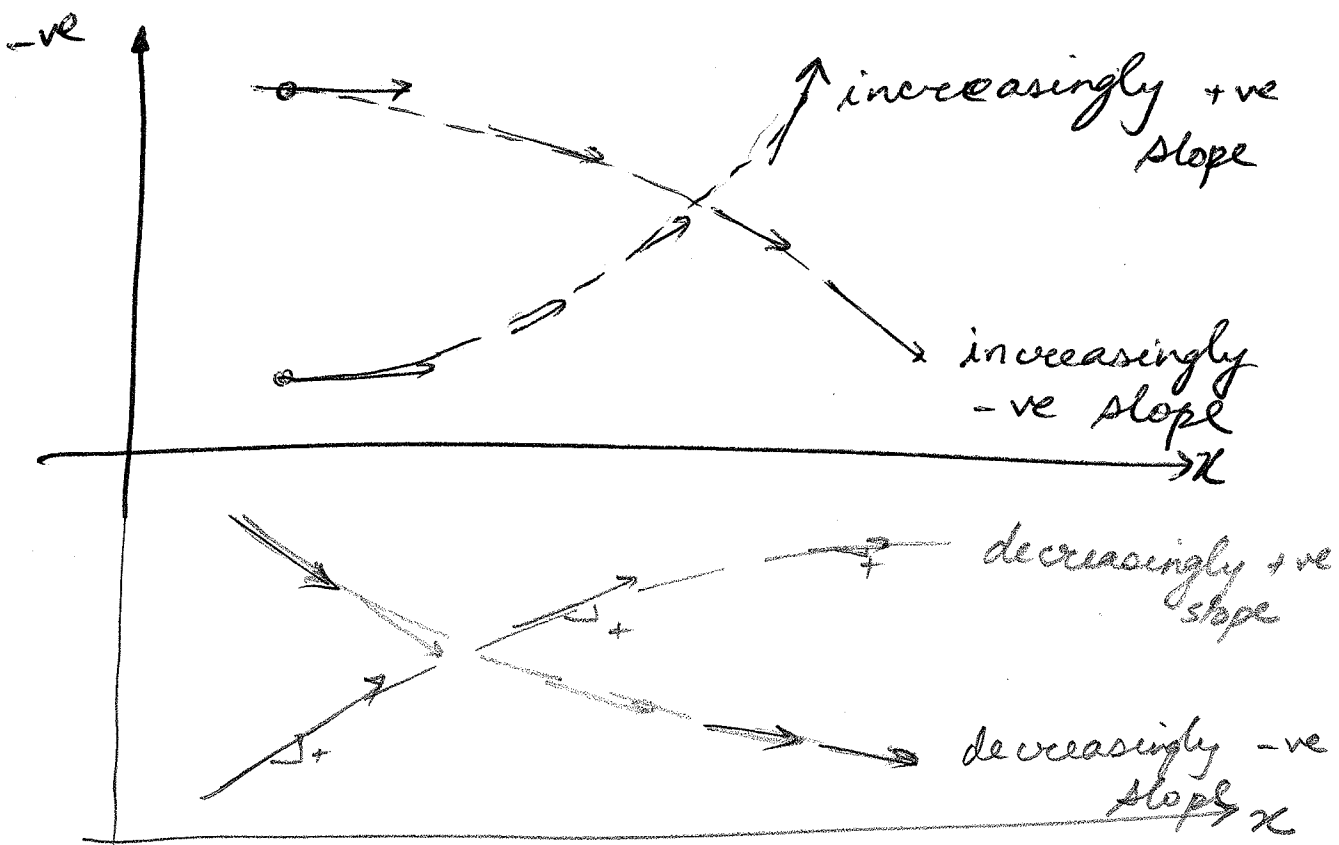
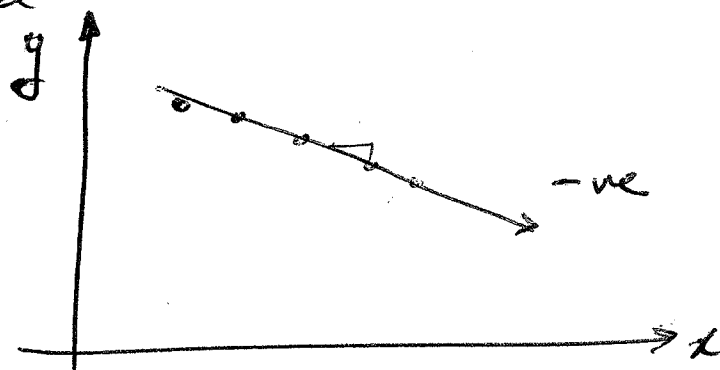
$$\frac{dM}{dx} = V$$

Slope of the moment diagram = $V(x)$
according to our sign convention

What is +ve slope?



-ve slope



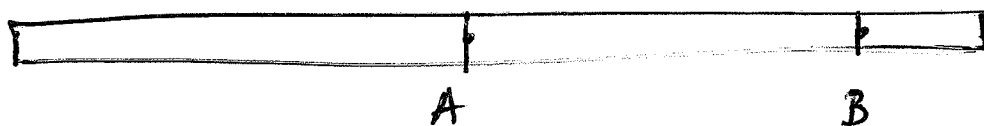
One thing about derivatives.
 $\rightarrow$ into integrals.

$$\frac{dV}{dx} = w(x)$$

$$\int_A^B dV = \int_A^B w(x) \cdot dx$$

$$\Delta V \Big|_A^B = \int_A^B w(x) \cdot dx$$

(3)



$\Delta V \Big|_A^B =$ Area under the
 w diagram
 between A & B



$$\frac{dM}{dx} = V$$

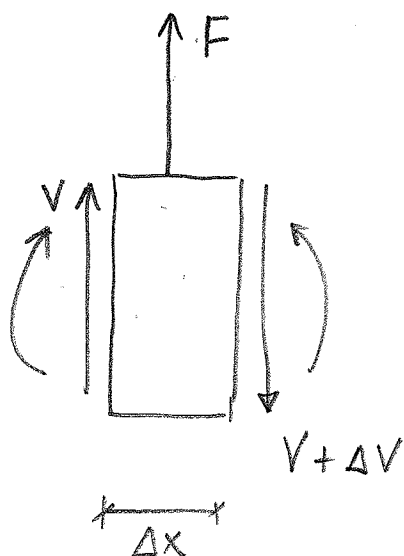
$$\therefore \int_A^B dM = \int_A^B V(x) dx$$

(4)

change in
the moment from
A $\rightarrow$ B along
member length

= Area under the
shear force diagram
between A & B

Effect of Concentrated Force / Reaction



$$+\uparrow \sum F_y = 0$$

$$\therefore V + F - \cancel{V} - \Delta V = 0$$

$$\Delta V = F$$

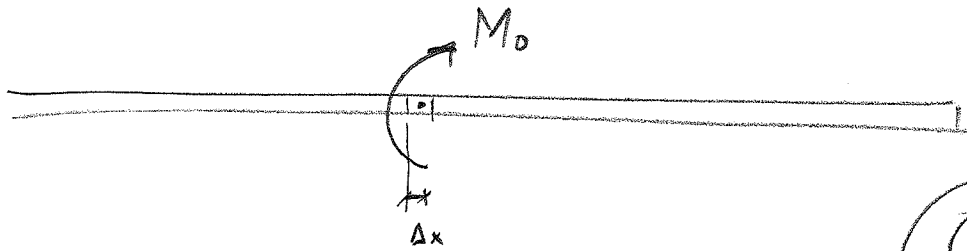
(5)

change in shear = Concentrated force F

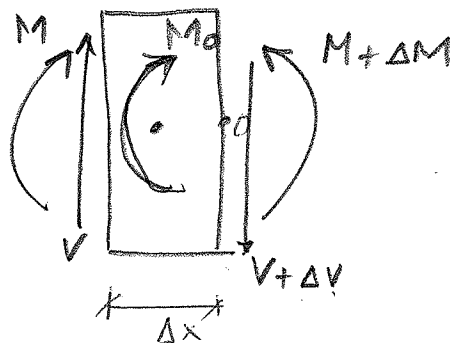
Wherever there is a concentrated force/reaction there will be a jump/discontinuity in the shear force diagram (5)

- Magnitude of Jump = F
- Direction of Jump = direction of F

Effect of Concentrated Moment M_0



(6)



$$\sum M_0 = 0$$

$$\therefore M + \Delta M - M - M_0 = 0$$

$$\Delta M = M_0$$

Wherever there is a concentrated moment there will be a jump / discontinuity in the bending moment diagram

- Clockwise conc. moment is +ve

(6)

- Magnitude of jump = M_0

- Direction of jump = sign convention for M_0

+ve for clockwise

-ve for counter clockwise

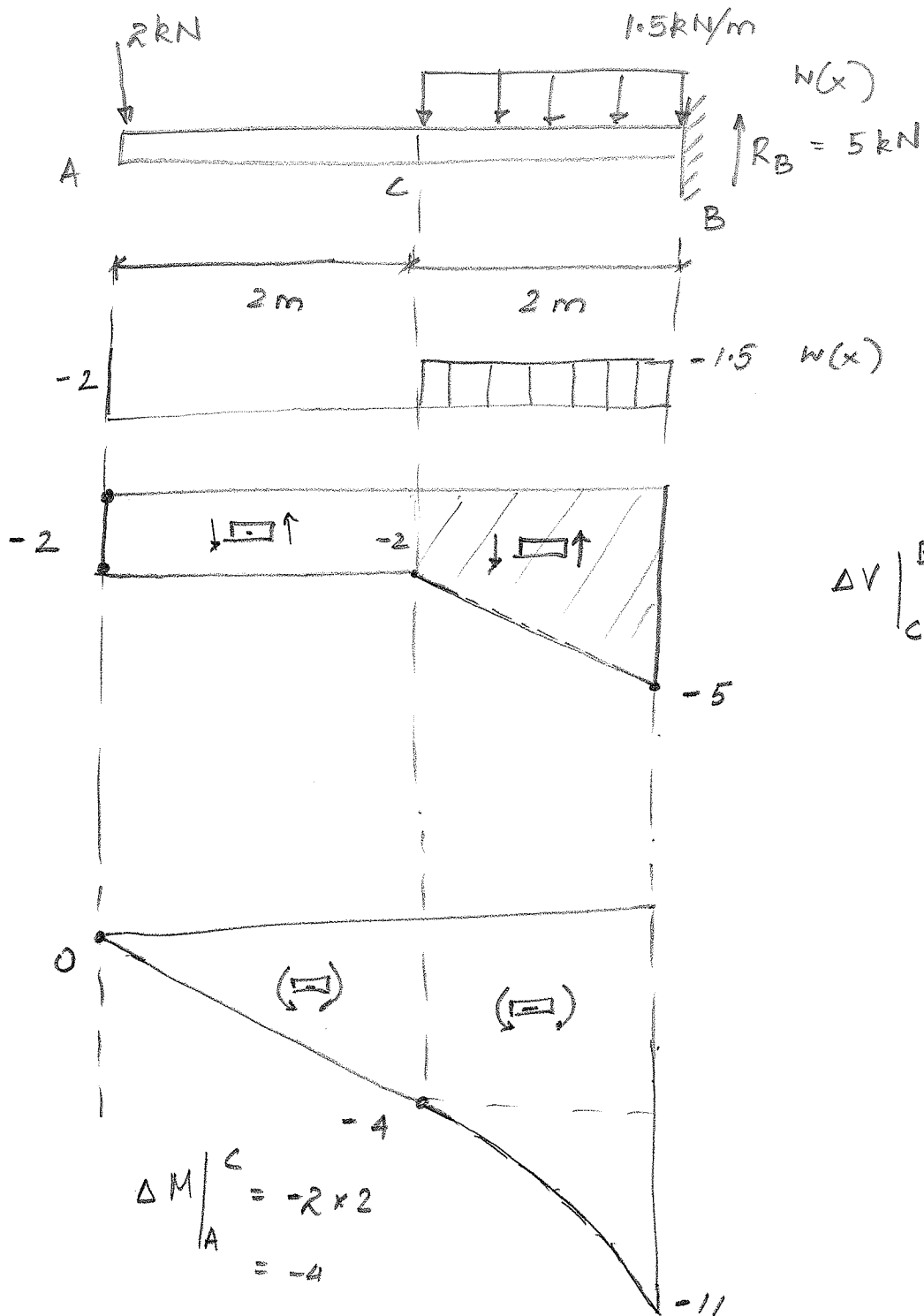


Slope of Moment diagram $\frac{dM}{dx} = V$

(7)

∴ Max. moment
or
Min. moment } occur when $V = 0$

Shear Force & Bending Moment Diagram



$$(1) \frac{dV}{dx} = w$$

$$(2) \frac{dM}{dx} = V$$

$$(3) \Delta V /_A^B = \int_A^B w(x) dx$$

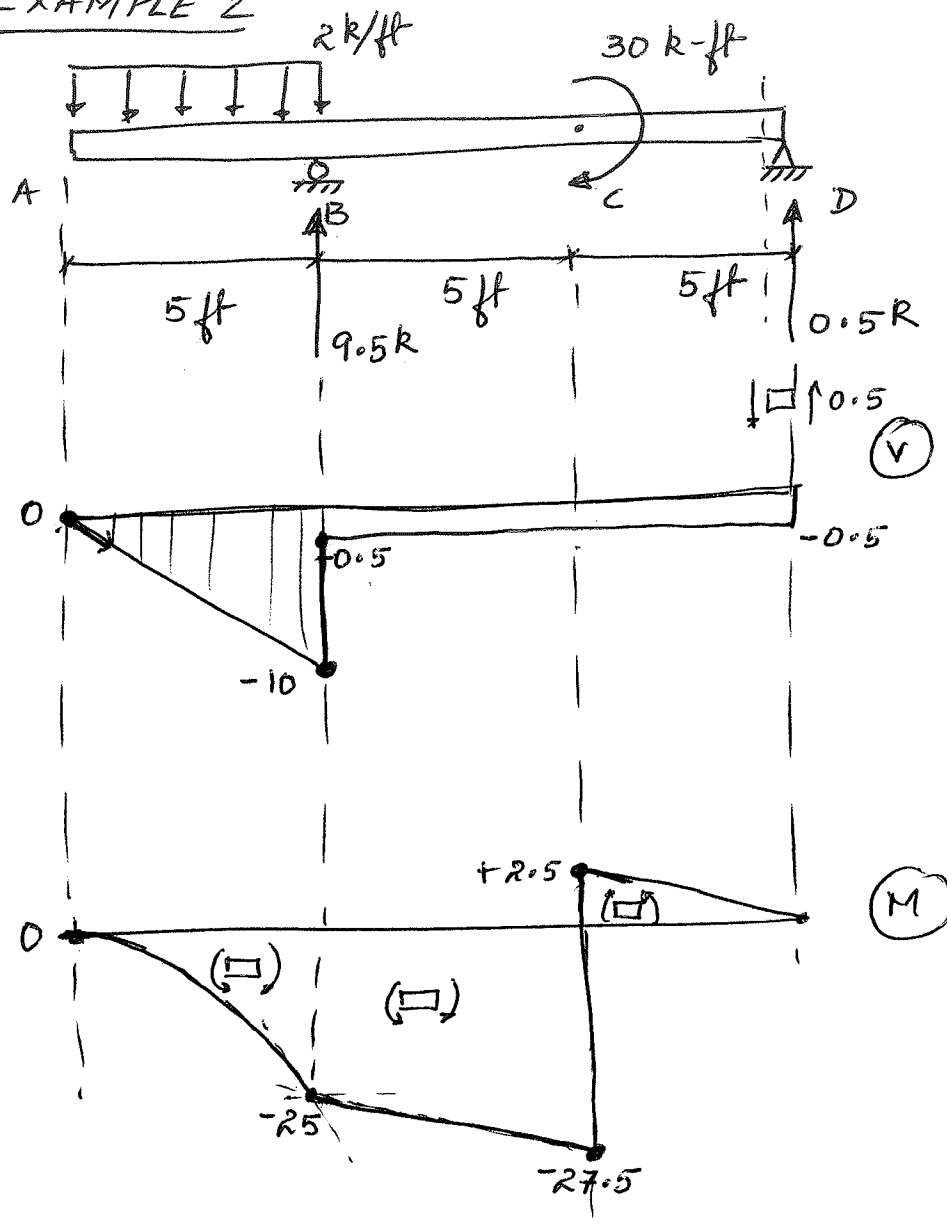
$$(4) \Delta M /_A^B = \int_A^B V(x) dx$$

$$(5) + \uparrow \xrightarrow{\text{conc.}} \text{along the length} \quad (V)$$

$$(6) + \curvearrowright \xrightarrow{\text{conc.}} \text{along the length} \quad (M)$$

$$(7) M_{\max} \text{ or } \min \rightarrow V=0$$

EXAMPLE 2

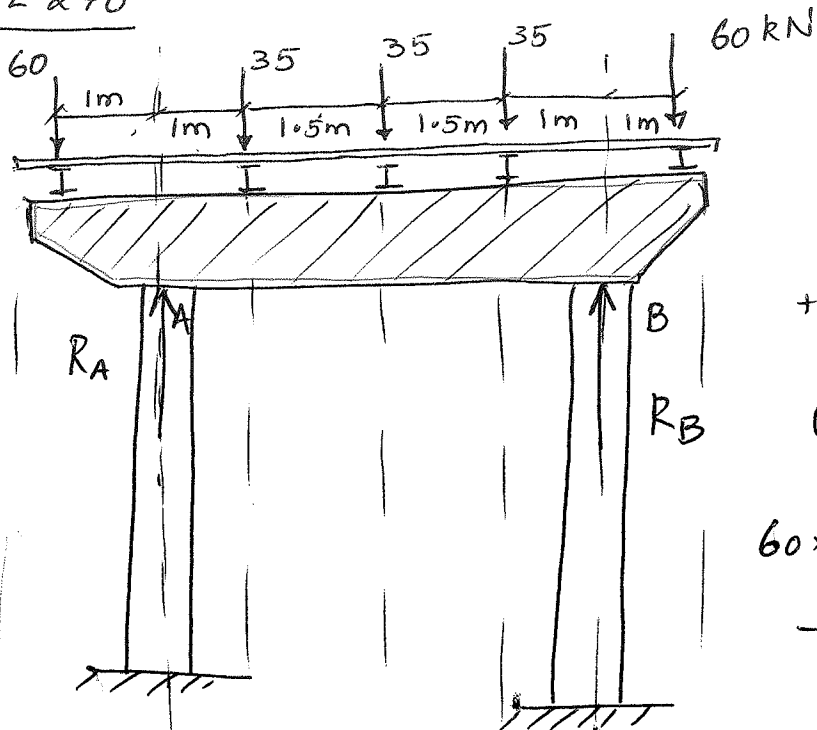


$$\Delta V \Big|_A^B = -2 \times 5 = -10$$

$$\Delta M \Big|_A^B = \frac{1}{2} \times 5 \times (-10) = -25$$

$$\Delta M \Big|_B^C = -0.5 \times 5 \text{ ft} = -2.5$$

$$\Delta M \Big|_C^D = -0.5 \times 5 \text{ ft} = -2.5$$



$$+\uparrow \sum F_y = 0$$

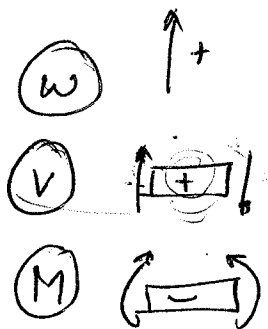
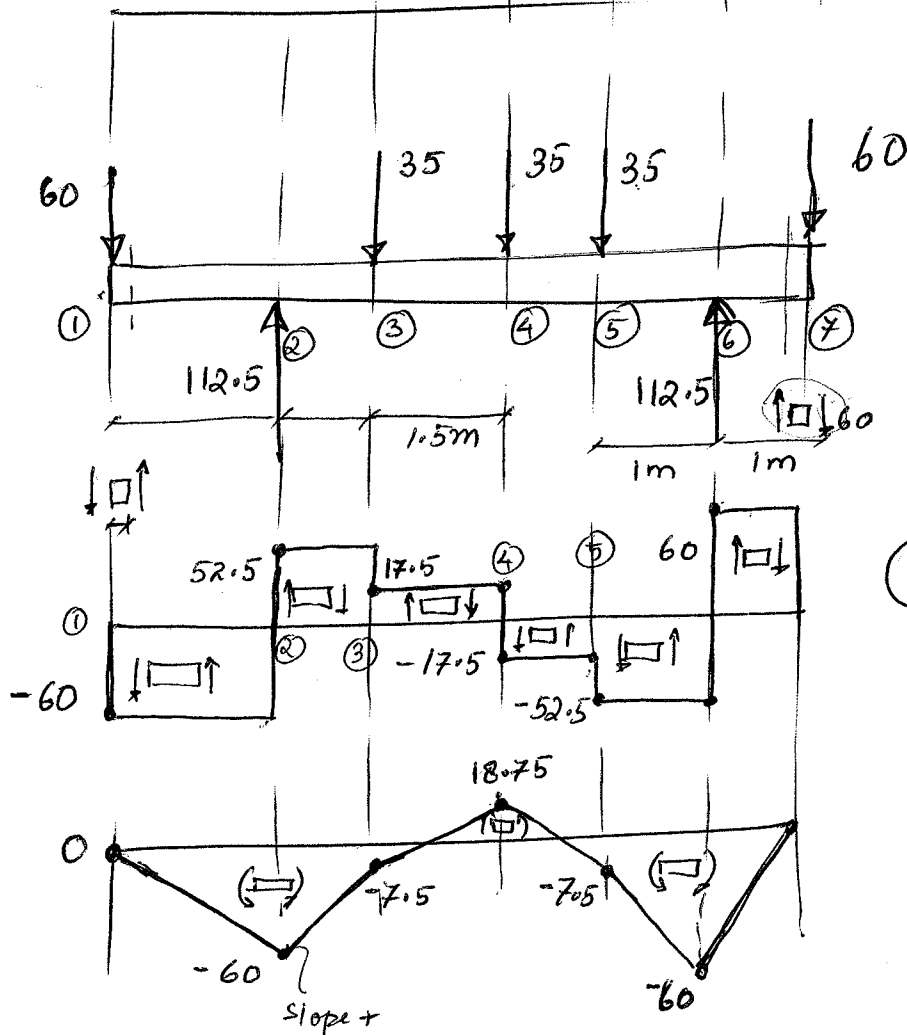
$$(\circlearrowleft) \sum M_A = 0$$

$$60 \times 1m - 35 \times 1m - 35 \times 2.5m - 35 \times 4m - 60 \times 6m + R_B \times 5m = 0$$

$$\therefore R_B = 112.5 \text{ kN}$$

$$R_A = 112.5 \text{ kN}$$

$$60 + 60 + 35 \times 3 = 225 \text{ kN}$$



$$\Delta M \Big|_4^5 = -17.5 \times 1.5 = -26.25$$

$$\Delta M \Big|_3^4 = 17.5 \times 1.5 = +26.25$$

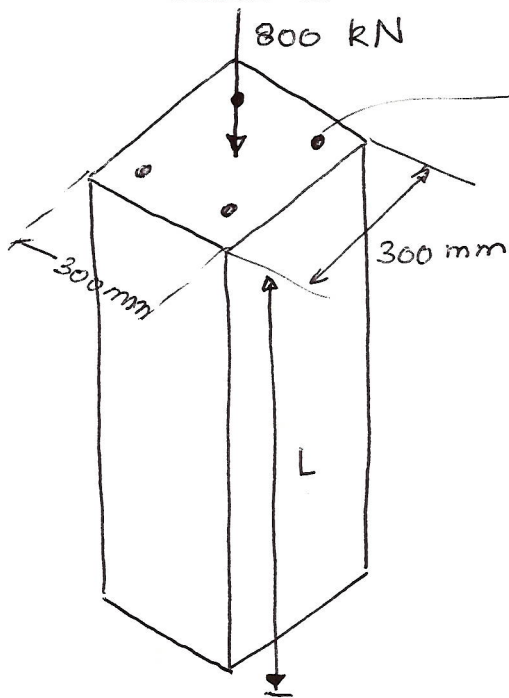
$$\Delta M \Big|_1^2 = -60$$

$$\Delta M \Big|_2^3 = +52.5$$

Exam Review

(94)

EXAMPLE ①



diameter
= 10 mm

calculate the steel bars
& concrete

$$\sigma_s \leq \sigma_{all}$$

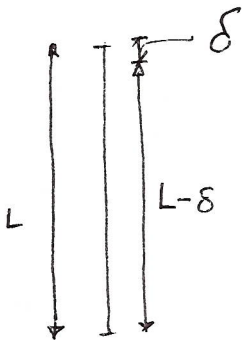
$$\sigma_c \leq \sigma_{all}$$

$$E_s = 200 \text{ GPa} = 200,000 \text{ MPa}$$

$$E_c = 25 \text{ GPa} = 25,000 \text{ MPa}$$

$$A_c = 300 \text{ mm} \times 300 \text{ mm} = 90,000 \text{ mm}^2$$

$$A_s = \frac{\pi \times d^2}{4} \times 4 = \frac{\pi \times 100}{4} \times 4$$
$$= 100 \pi \text{ mm}^2$$



$$\delta_s = \delta_c = \delta \quad \text{--- ①}$$

$$P = P_s + P_c \quad \text{--- ②}$$

Is there a relationship between P_s & δ_s

$$\frac{P_s}{A_s} = \sigma_s = E_s \times \frac{\delta_s}{L} \quad \therefore P_s = \frac{E_s A_s}{L} \times \delta_s$$

$$P_c = \frac{E_c A_c}{L} \times \delta_c$$

$$\left. \begin{aligned} \therefore P_s &= \frac{E_s A_s}{L} \times \delta \\ P_c &= \frac{E_c A_c}{L} \times \delta \end{aligned} \right\} \longrightarrow \text{from eq ①}$$

plug into Eq (2)

$$P = P_s + P_c$$

$$P = \left(\frac{E_s A_s}{L} + \frac{E_c A_c}{L} \right) \delta$$

$$\therefore \delta = \frac{P \times L}{E_s A_s + E_c A_c}$$

$$\delta_s = \delta$$

$$\therefore P_s = \frac{E_s A_s}{L} \times \frac{P \cancel{L}}{A_s E_s + E_c A_c}$$

$$\therefore \sigma_s = \frac{P_s}{A_s} = \frac{P \times E_s}{E_s A_s + E_c A_c}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{P \times E_c}{E_s A_s + E_c A_c}$$

$$\delta_c = \delta$$

$$\therefore P_c = \frac{E_c A_c}{L} \times \delta$$

$$P_c = \frac{E_c A_c}{L} \times \frac{P \cancel{L}}{\cancel{L} E_s A_s + E_c A_c}$$

$$\sigma_c = P_c / A_c$$

$$= \frac{P \times E_c}{E_s A_s + E_c A_c}$$

$$\text{Ans: } \sigma_s = \frac{800,000 \times 200,000}{100 \pi \times 200,000 + 90,000 \times 25,000} = 69.17 \text{ N/mm}^2$$

$$= 69.17 \text{ MPa}$$

$$\sigma_c = \frac{800,000 \times 25,000}{100 \pi \times 200,000 + 90,000 \times 25,000} = 8.65 \text{ MPa}$$

- Failure stress for steel = 410 MPa
- Failure stress for concrete = 20.7 MPa

F.O.S for axial force design = 2.0

$$\therefore \sigma_{all}|_{\text{steel}} = \frac{410}{2} = 205 \text{ MPa}$$

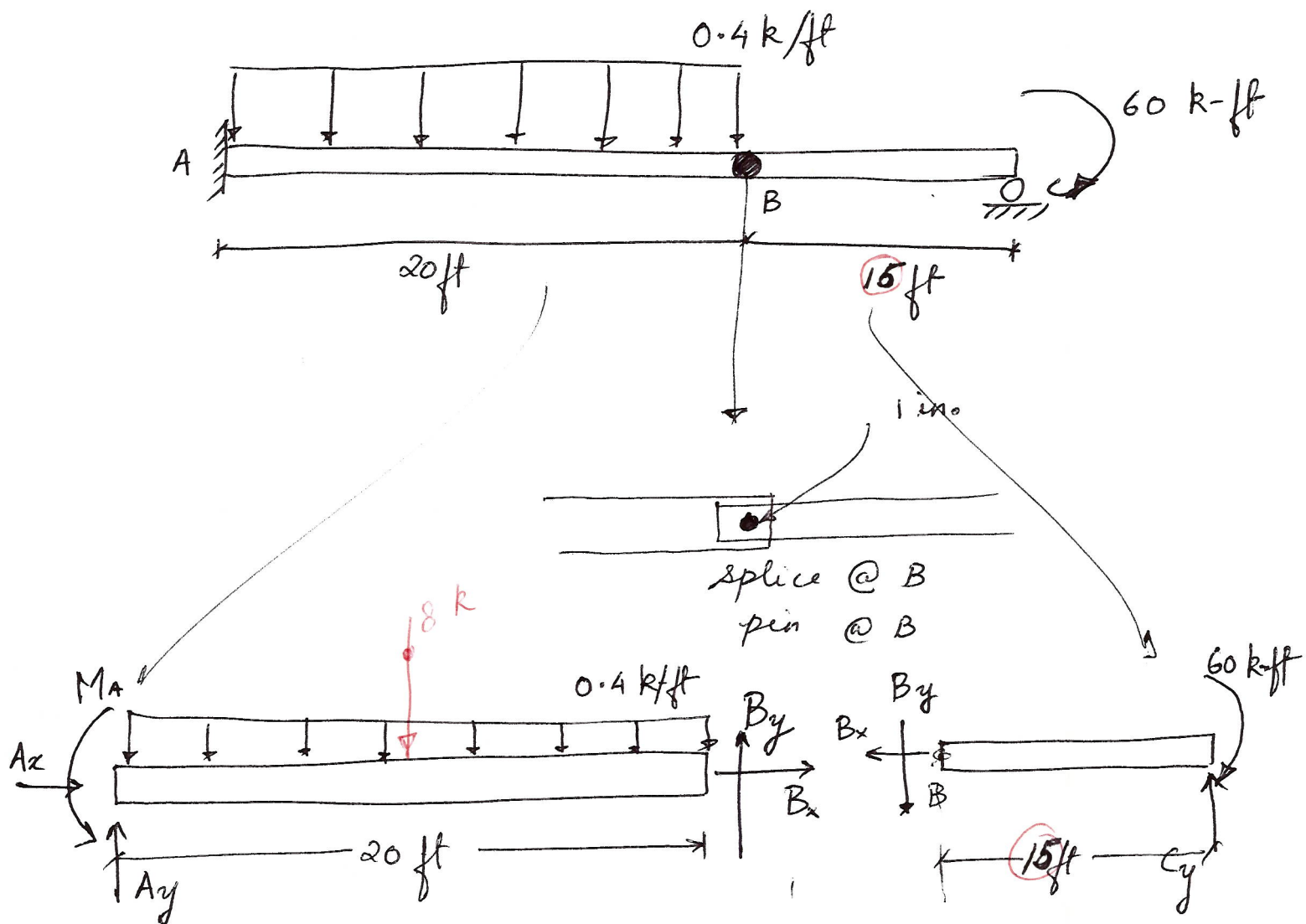
$$\sigma_{all}|_{\text{conc}} = \frac{20.7}{2} = 10.35 \text{ MPa}$$

$$\sigma_s|_{\text{calc}} = 69.17 \text{ MPa} < \sigma_{all}|_s \rightarrow \underline{\underline{\text{OK}}}$$

$$\sigma_c|_{\text{calc}} = 8.65 \text{ MPa} < 10.35 \rightarrow \text{OK}$$

Calculate Reactions & Design the Connection

(97)



$$+\uparrow \sum F_y = 0 \therefore A_y - 8 + B_y = 0$$

$$\therefore A_y - 8 + 4 = 0$$

$$\therefore A_y = 4 \text{ k}$$

$$(+\sum M_A = 0 \therefore M_A - 8 \times 10 + B_y \times 20 = 0$$

$$\therefore M_A - 80 + 4 \times 20 = 0$$

$$\therefore M_A = 0$$

$$+\rightarrow \sum F_x = 0$$

$$\therefore A_x + B_x = 0$$

$$\therefore A_x = 0$$

$$(+\sum M_B = 0$$

$$-60 + C_y \times 15 = 0$$

$$\therefore C_y = 4 \text{ kip}$$

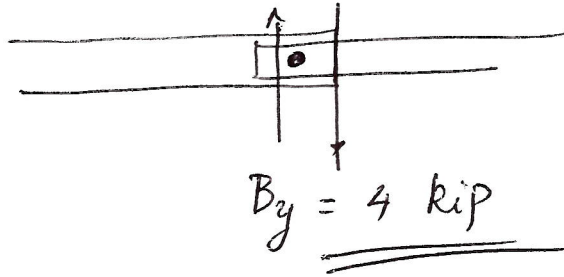
$$+\uparrow \sum F_y = 0$$

$$\therefore -B_y + C_y = 0$$

$$\therefore B_y = C_y = 4 \text{ k}$$

$$+\rightarrow \sum F_x = 0$$

$$\therefore -B_x = 0$$



diameter of bolt = 1 in.

$$\tau = \frac{V}{A} = \frac{4 \text{ kip}}{\pi \times d^2/4} = \frac{16}{\pi \times d^2}$$

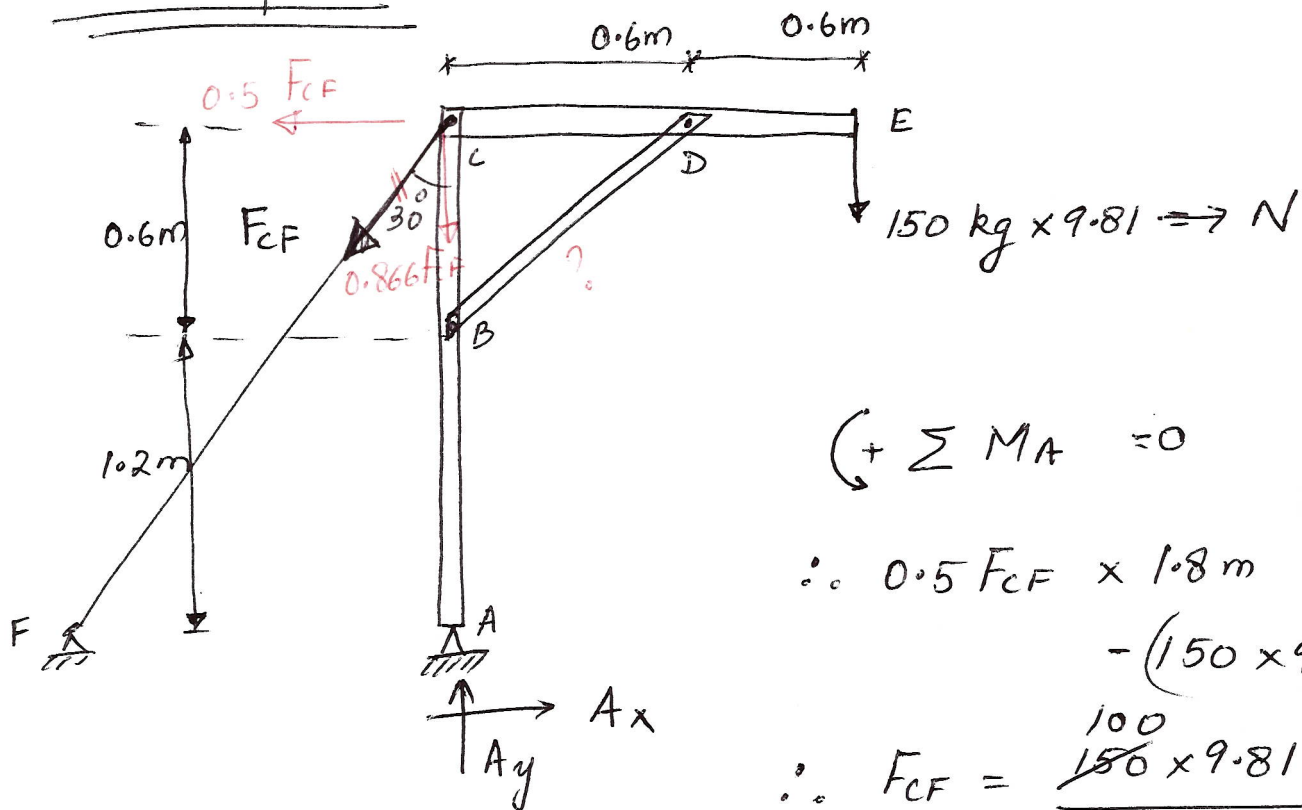
$$= \frac{16}{\pi \times 1^2} \approx 5.0 \text{ ksi}$$

$\tau_{all} = ?$

$$\frac{\tau_{fail}}{F.O.S} = \frac{60 \text{ ksi}}{2} = 30 \text{ ksi}$$

$$\underline{\underline{\tau_{calc} < \tau_{all}}} \quad \underline{\underline{OK!}}$$

Example 3



$$(+ \sum M_A = 0$$

$$\therefore 0.5 F_{CF} \times 1.8 \text{ m} - (150 \times 9.81) \times 1.2 \text{ m} = 0$$

$$\therefore F_{CF} = \frac{150 \times 9.81 \times 1.2}{0.5 \times 1.8}$$

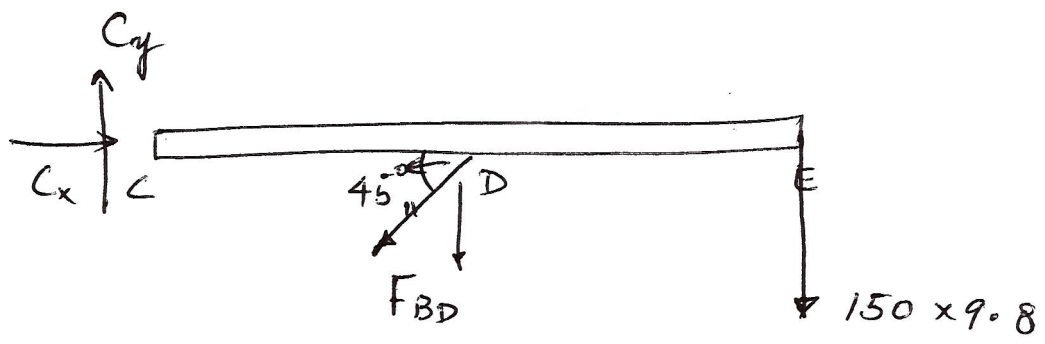
$$\checkmark F_{CF} = 1962 \text{ N}$$

$$+ \rightarrow \sum F_x = 0 \therefore A_x - 0.5 F_{CF} = 0$$

$$\therefore A_x = 981 \text{ N}$$

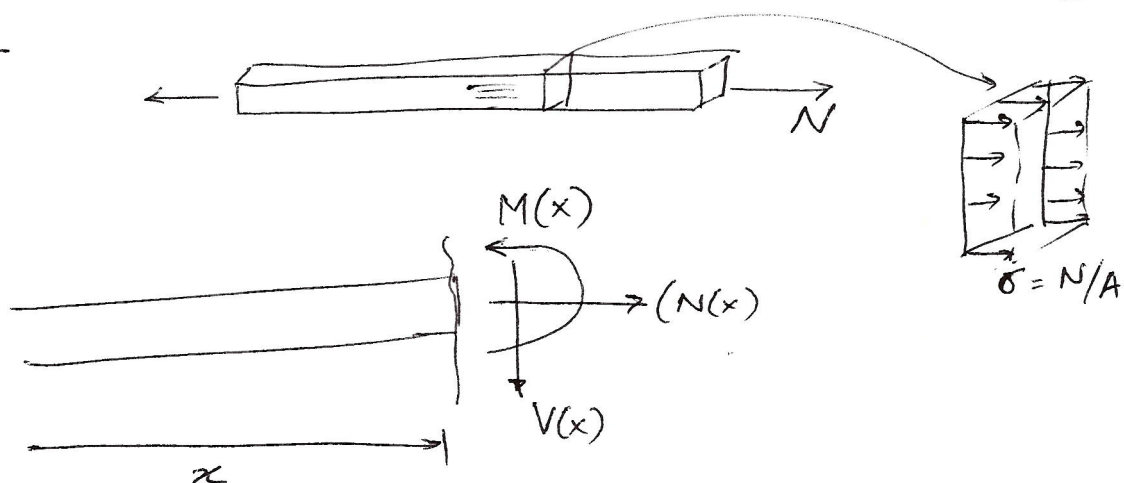
$$+ \uparrow \sum F_y = 0 \therefore A_y - 0.866 (1962) - 150 \times 9.81 = 0$$

$$\therefore A_y = 3170.64 \text{ N}$$



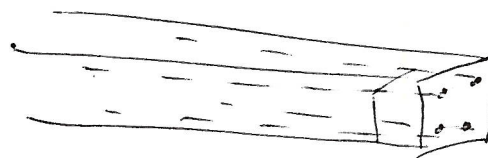
$$\sigma = \frac{N}{A}$$

normal stress

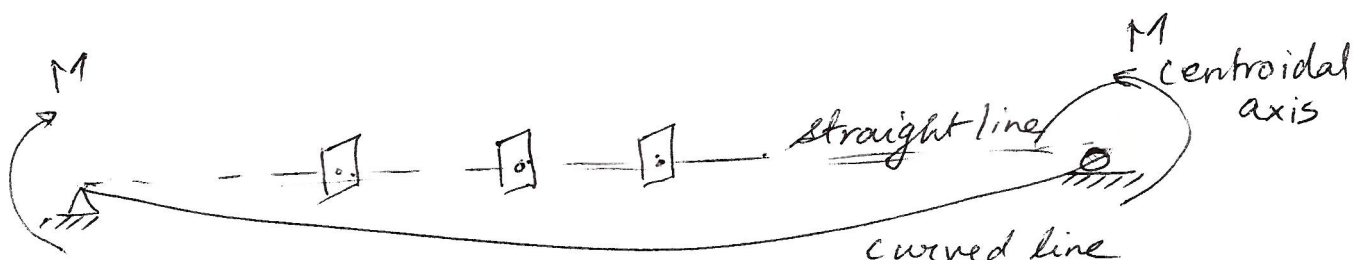


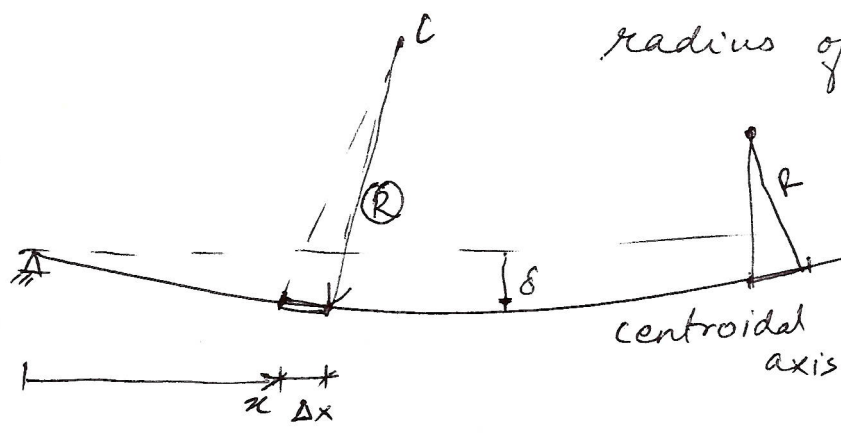
$$\tau_{avg} = \frac{V}{A} \quad \text{---} \rightarrow \text{bolts, pin for fasteners}$$

shear stress



M $\rightarrow$ what kind of stresses?

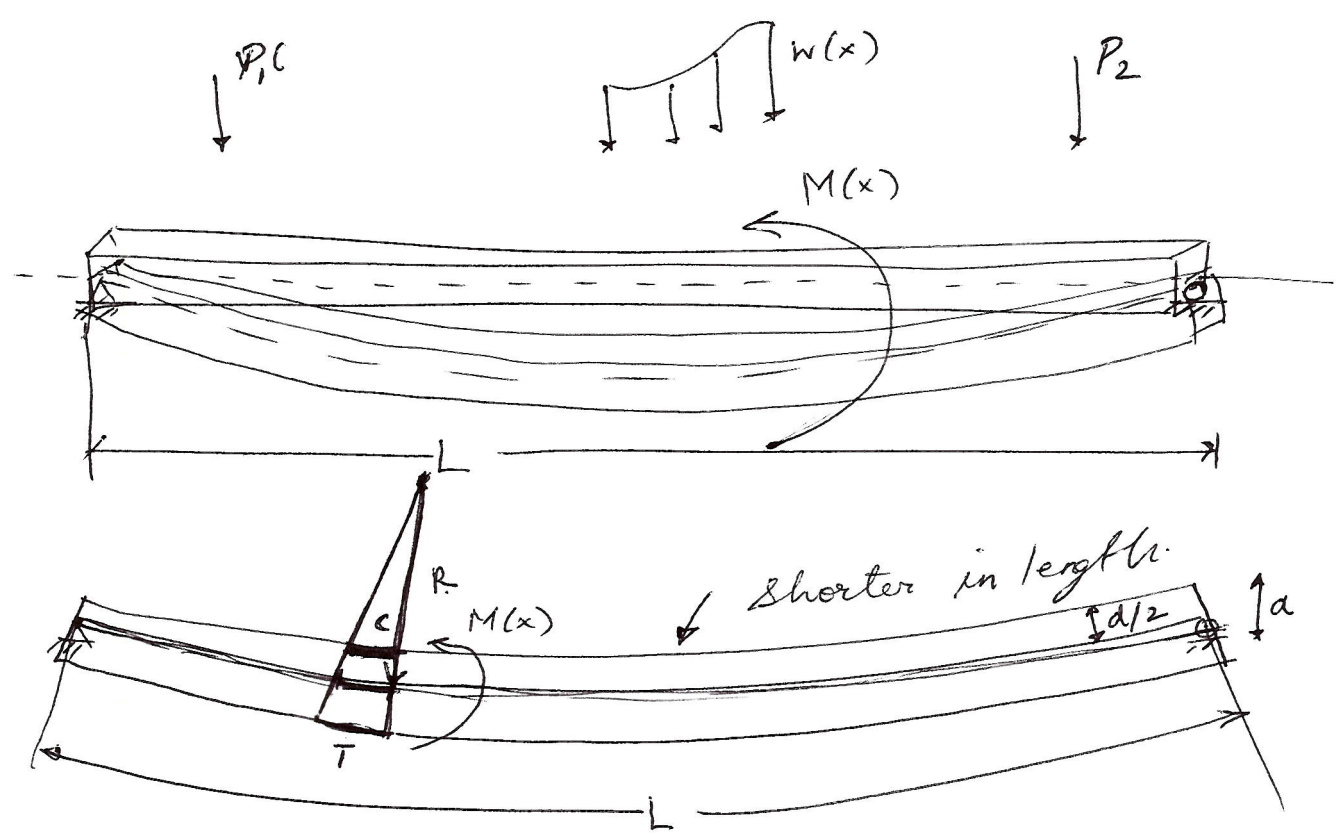
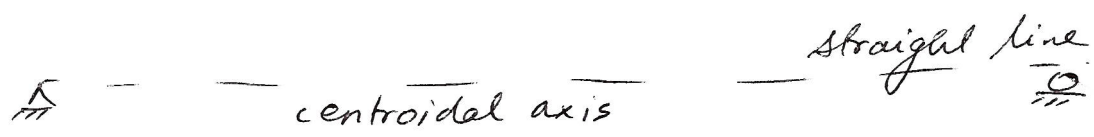




radius of curvature = R or ρ

$1/\text{radius of curvature} = \text{curvature}$

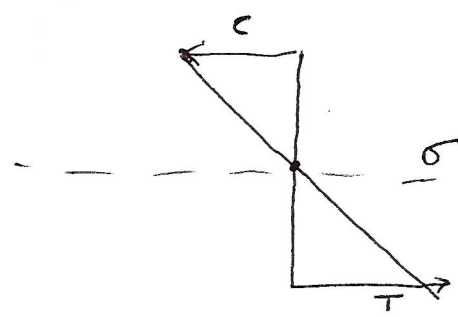
curved line! $= \frac{1}{R} = \frac{1}{\rho}$



compression on top
tension on bottom

$$\epsilon = \frac{\Delta L}{L} = 0$$

Zero strain
Zero stress



$\sigma \rightarrow$ normal stress
due to bending

Neutral axis

→ axis that does not change in length during bending

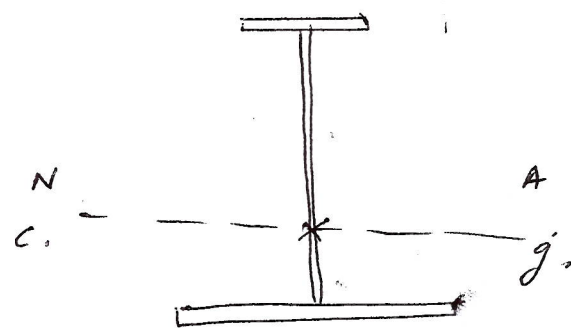
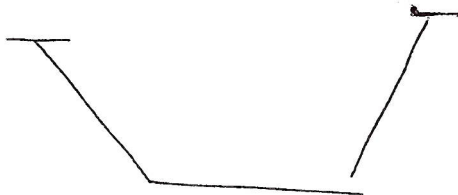
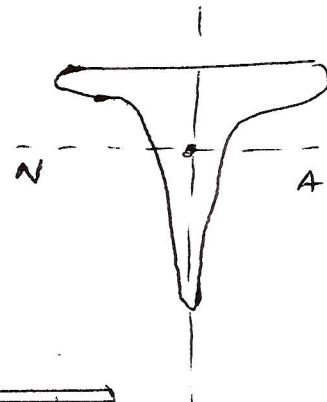
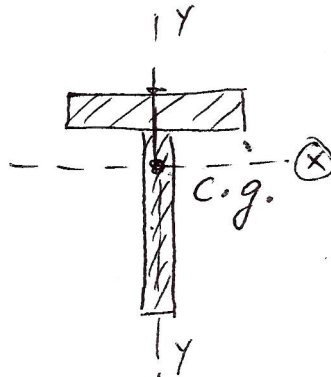
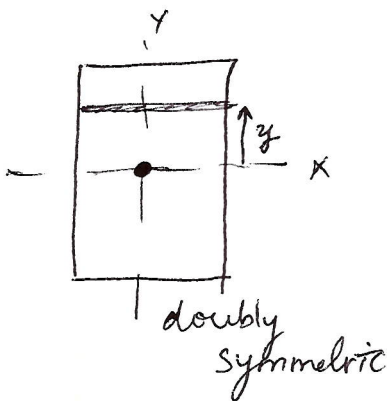
falls in line with the centroidal axis

for elastic behavior

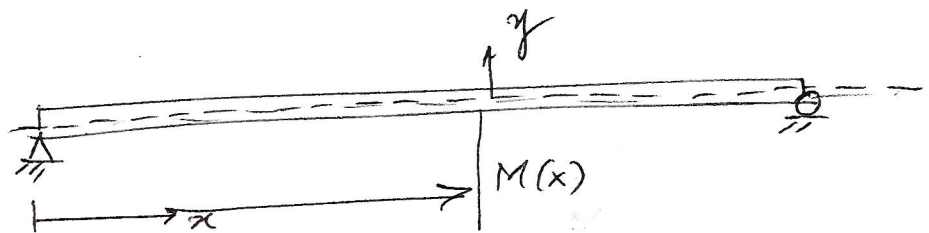
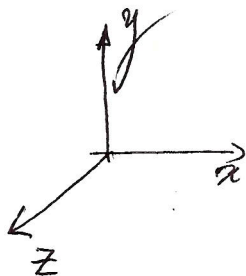
& one material in the section

uniaxial

$$\sigma = E \cdot \epsilon$$



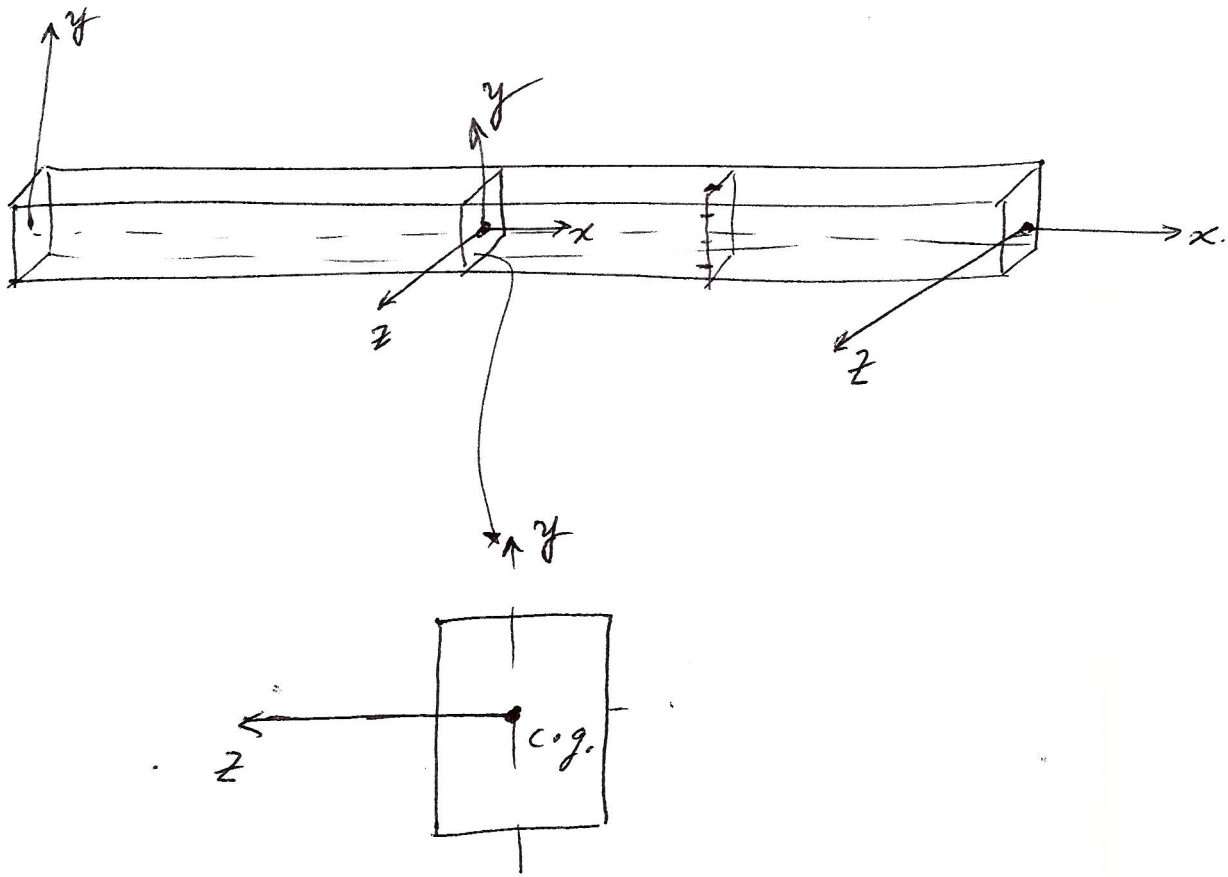
y is +ve upwards



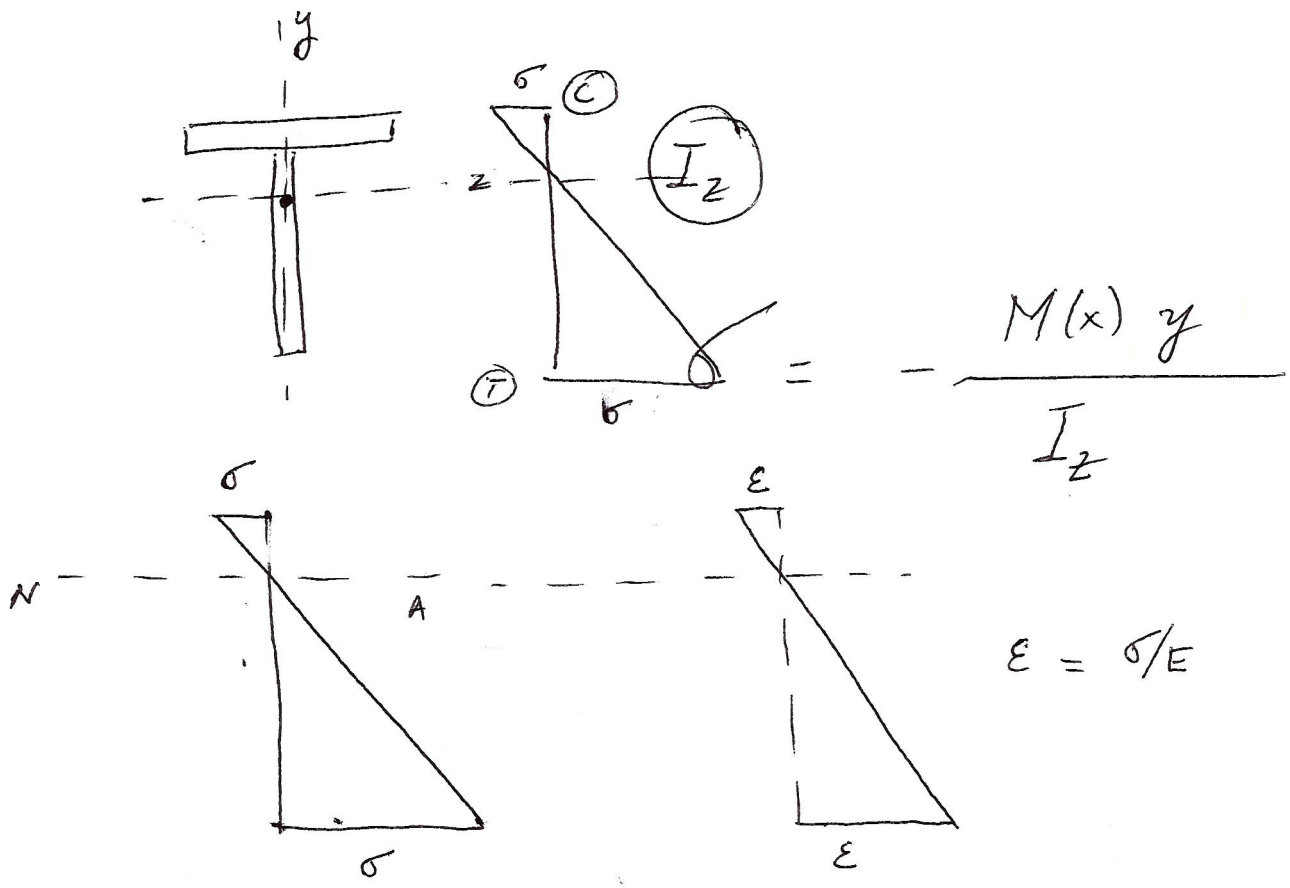
$$\sigma = -\frac{M(x) \times y}{I_z}$$

If M(x) is (+ve) M(x)

compression



$I_z \rightarrow$ moment of inertia about z -axis



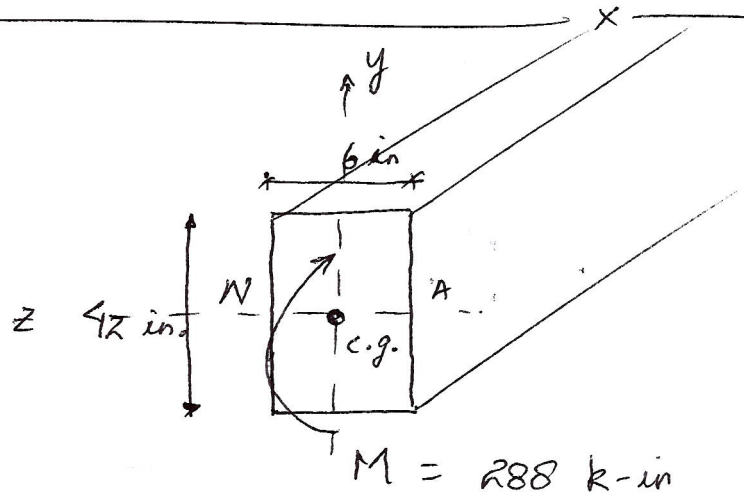
$$\sigma = \frac{-M(x) \times y}{I_z}$$

EXAMPLE ①

$$I_z = \frac{b \times h^3}{12}$$

$$= \frac{6 \times 12^3}{12}$$

$$= 864 \text{ in}^4$$



doubly symmetric
one material.

$I_z \rightarrow$ geometric property of the cross-section ✓

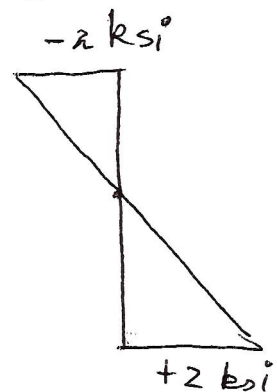
$$\therefore \sigma = \frac{-M \times y}{I_z}$$

$y \rightarrow +6 \text{ in.} \rightarrow \text{max. compressive stress}$

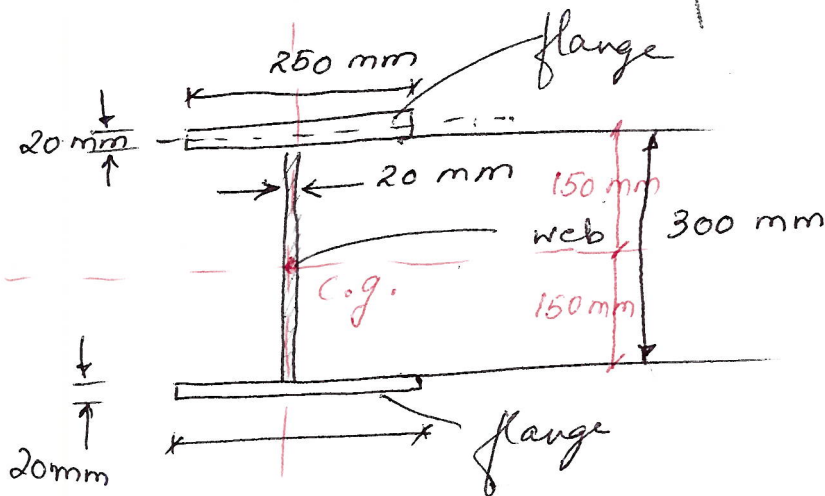
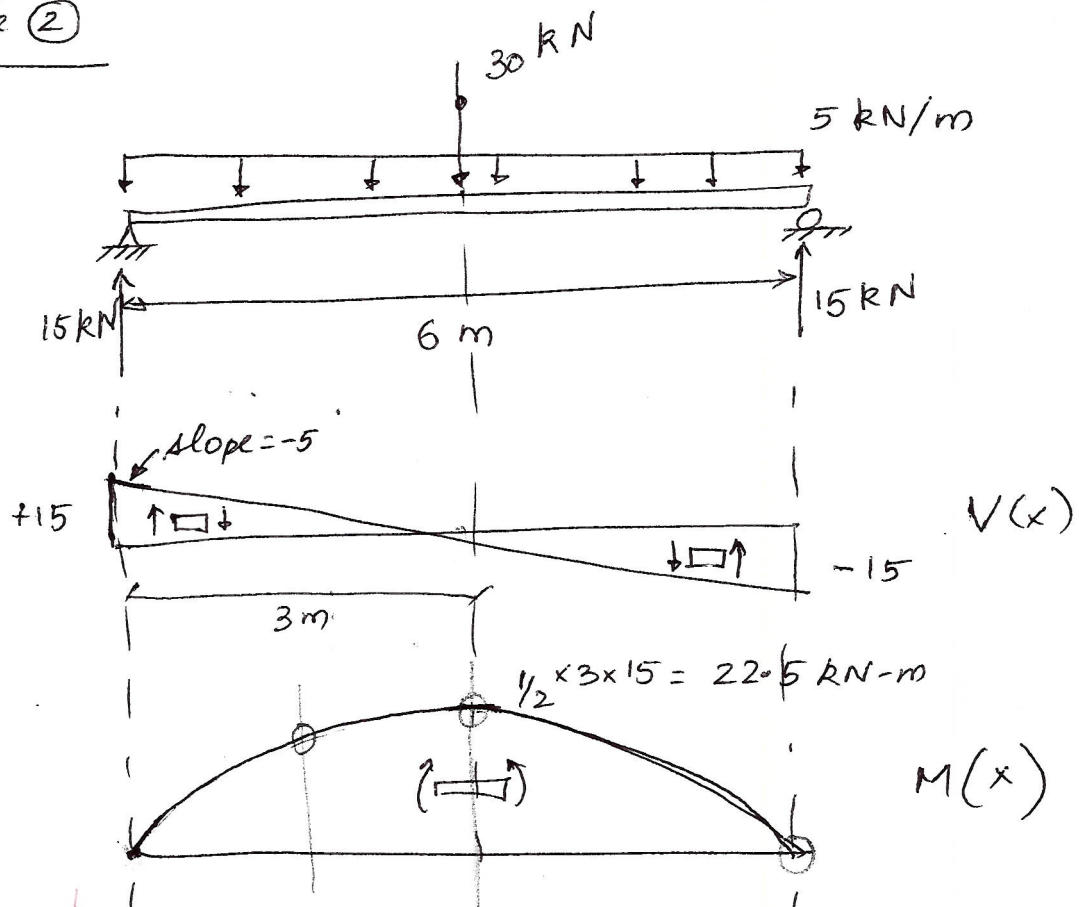
$y \rightarrow -6 \text{ in.} \rightarrow \text{max. tensile stress}$

$$\therefore \sigma = \frac{-288 \times 6}{864 \text{ in}^4} = -2 \text{ ksi}$$

$$\sigma = \frac{-288 \times (-6)}{864} = +2 \text{ ksi}$$



Example ②

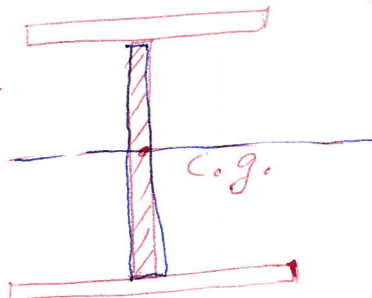


$$M(x) = +22.5 \text{ kN-m}$$

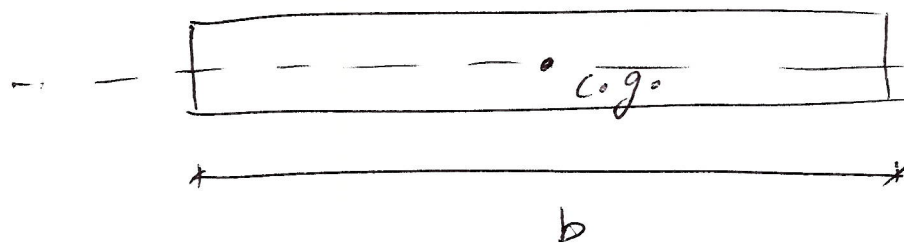
$$I_z = ?$$

$$\sigma = -\frac{M(x) y}{I_z}$$

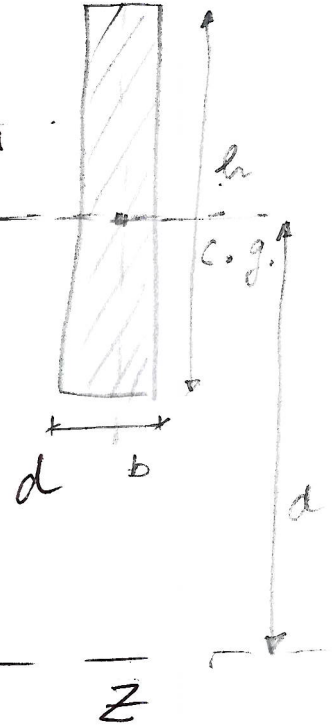
$$I_z = I_z|_{\text{web}} + (I_z|_{\text{flange}}) \times 2$$



$$I_z|_{\text{web}} = \frac{20 \times (300)^3}{12} \text{ mm}^4$$



$$I_{c.g.} = \frac{b \times h^3}{12}$$



$$I_z = \frac{b \times h^3}{12} + \underbrace{A d^2}_{b \times h}$$

$$I_z \Big|_{\text{flange}} = \frac{250 \times 20^3}{12} + (250 \times 20) \times [160]^2$$

$$I_z = I_z \Big|_{\text{web}} + \left(I_z \Big|_{\text{flanges}} \right) \times 2$$

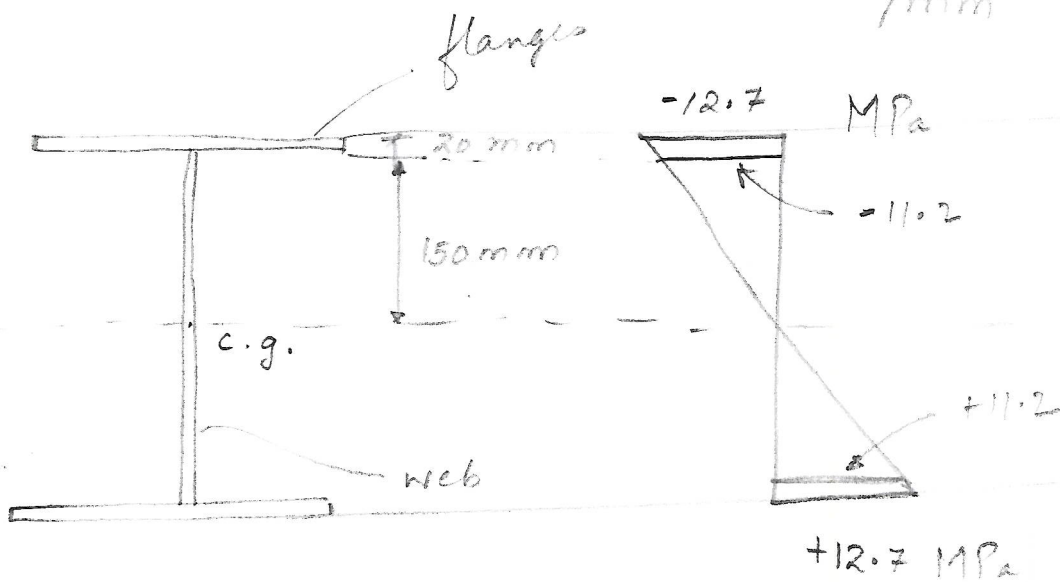
$$= 301.3 \times 10^6 \text{ mm}^4$$

$$\sigma = - \frac{M \times y}{I_z} \quad \text{KN-m} \quad \text{mm}$$

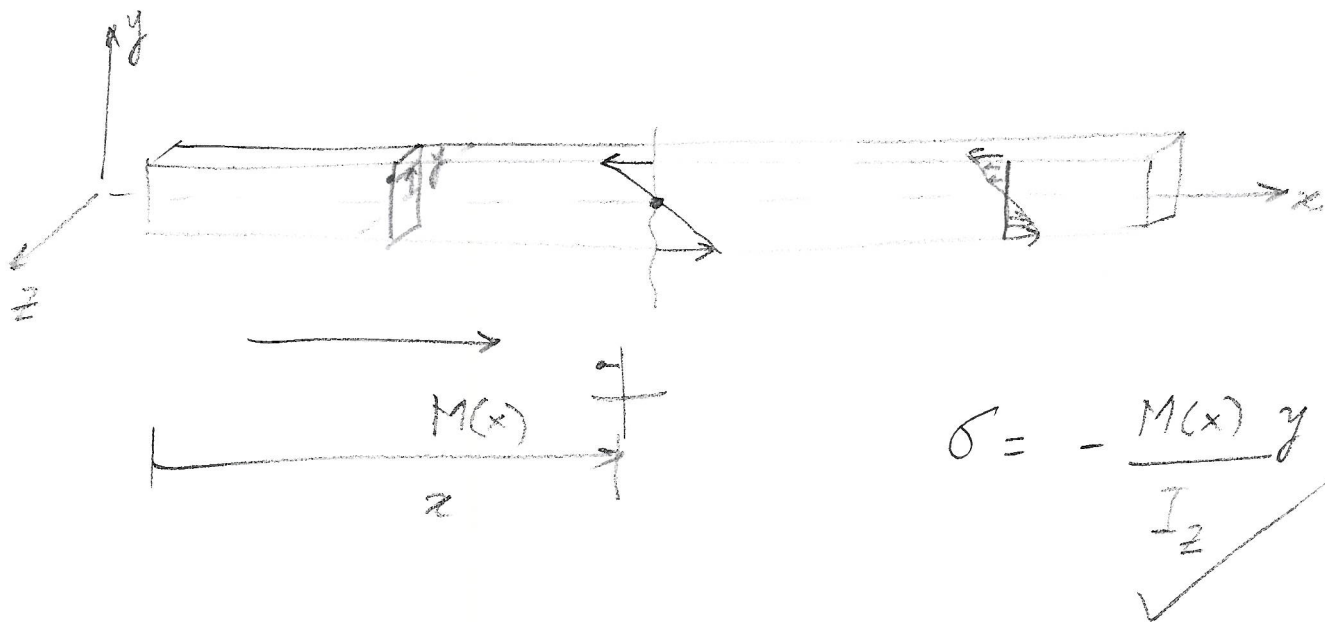
$$= - \frac{22.5 \times 1000 \text{ KN-m} \times 1.70 \text{ mm}}{301.3 \times 10^6 \text{ mm}^4}$$

$$\sigma = -12.7 \text{ MPa}$$

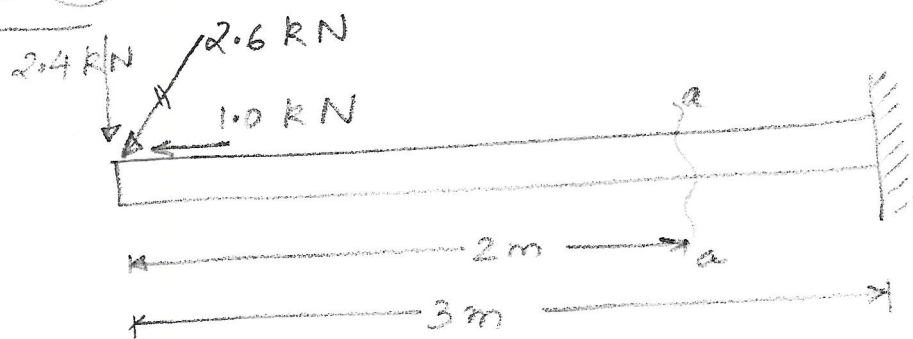
N/mm² or ~~kg/cm²~~



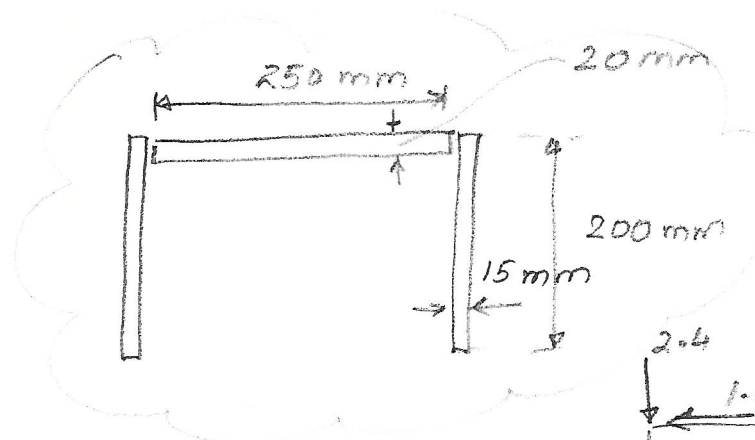
$$\sigma = - \frac{M y}{I_z} = \frac{-22.5 \times 10^3 \times 150}{301.3 \times 10^6} = -11.2 \text{ MPa}$$



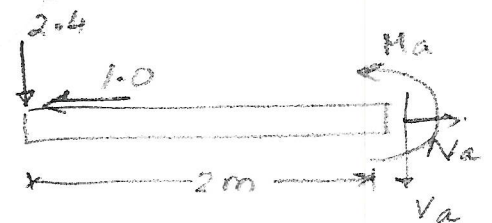
EXAMPLE (3)



Section
a-a



At section a-a



$$N_a = 1.0 \text{ kN}$$

$$M_a = -4.8 \text{ kN}\cdot\text{m}$$

$$V_a = -2.4 \text{ kN}$$

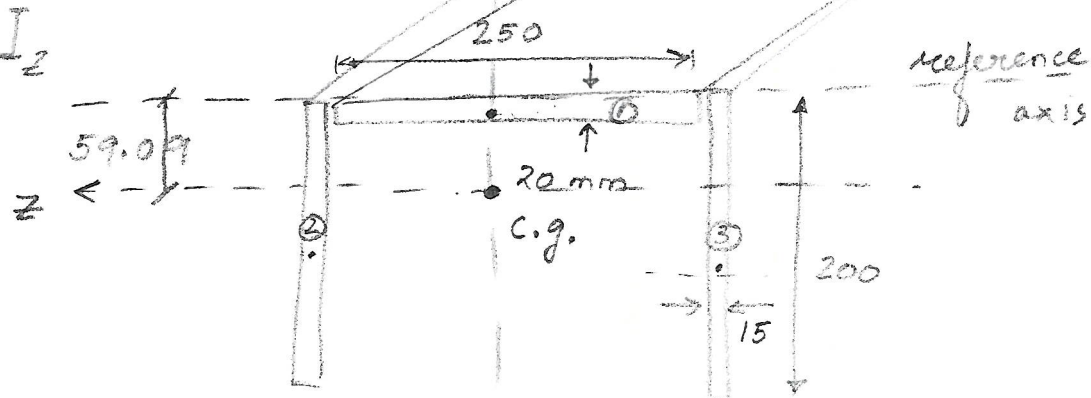
$$\leftarrow \boxed{+} \rightarrow 1.0$$

$$\left(\boxed{-} \right)$$

$$\boxed{-} \uparrow$$

✓

$$\sigma = \frac{-M(x) y}{I_z}$$



Need to find C.G.

because NA for elastic
one material } → coincides with C.G.

How to find C.G.?

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

① → represents each plate

$$A_1 = 250 \times 20$$

$$A_2 = 200 \times 15$$

$$A_3 = 200 \times 15$$

$$y_1 = 10$$

$$y_2 = 100$$

$$y_3 = 100$$

$$\therefore \bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = 59.09 \text{ mm}$$

$$I_z = ?$$

$$I_z = ?$$

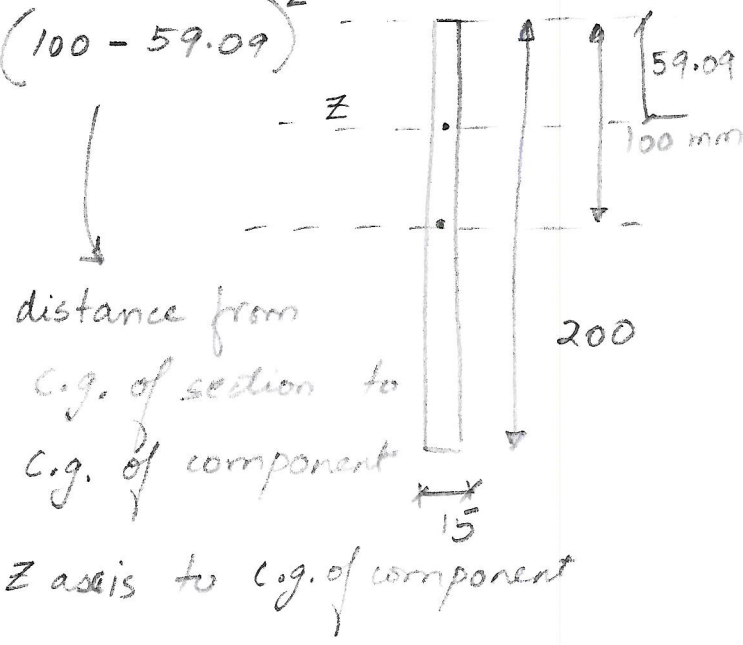
$$= I_1|_z + I_2|_z + I_3|_z$$



$$I_1|_z = 250 \times \frac{20^3}{12} + 250 \times 20 \times 49.09^2$$

$$I_2|_z = 15 \times \frac{200^3}{12} + 15 \times 200 (100 - 59.09)^2$$

$$I_3|_z = \text{--- " ---}$$

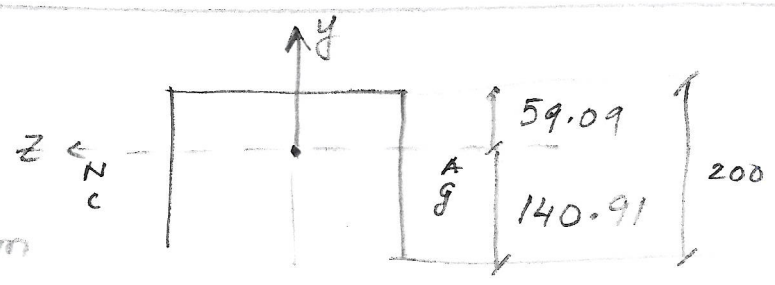


$$I_z = 42.26 \times 10^6 \text{ mm}^4$$

What is y?

$$y_{\text{top}} = +59.09 \text{ mm}$$

$$y_{\text{bottom}} = -140.91 \text{ mm}$$



$$\sigma_{top} = \frac{-M(x) y_{top}}{I_z}$$

$$= \frac{-(-4.8) \text{ kN-m} \times 59.09 \text{ mm}}{42.26 \times 10^6 \text{ mm}^4}$$

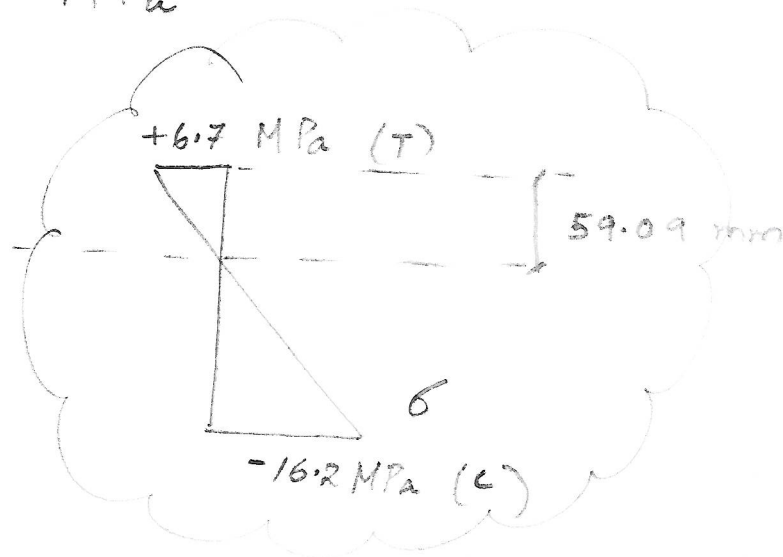
$$= \frac{+4.8 \times 10^6 \text{ N-mm} \times 59.09 \text{ mm}}{42.26 \times 10^6 \text{ mm}^4}$$

$$= 6.7 \text{ MPa}$$

$$\sigma_{bottom} = \frac{-M(x) y_{bottom}}{I_z}$$

$$= \frac{-(-4.8) \times 10^6 \times [140.9]}{42.26 \times 10^6}$$

$$= -16.2 \text{ MPa}$$

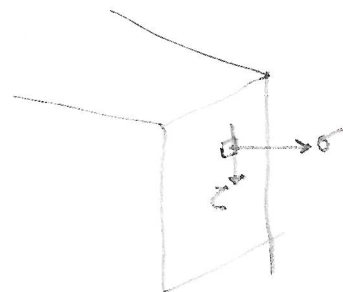


Additional stresses due to N_a

$$N_a = 1.0 \text{ kN} = 1000 \text{ N}$$

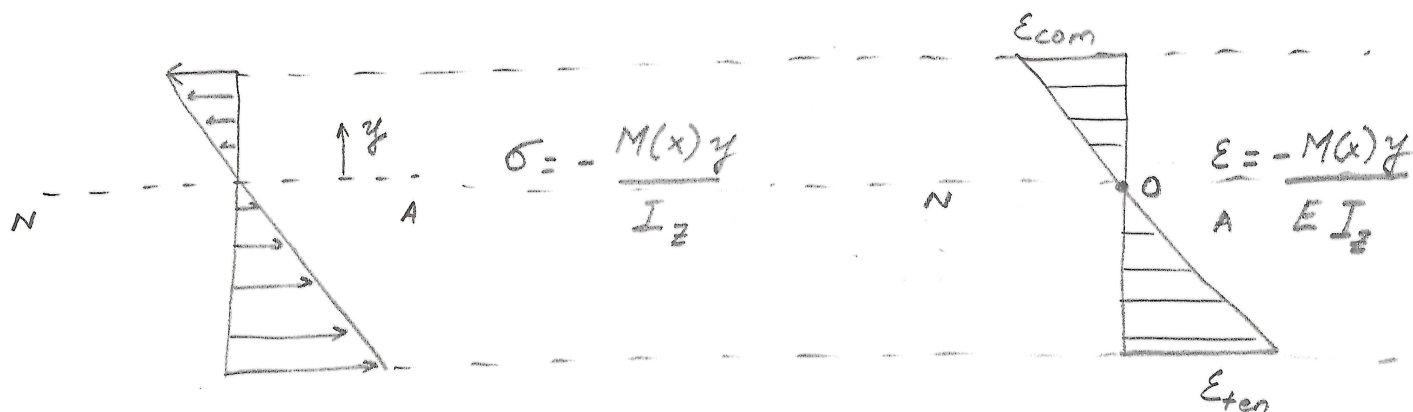
$$A = A_1 + A_2 + A_3 = \text{mm}^2$$

$$\sigma = \frac{N_a}{A}$$



So far, ONE MATERIAL IN THE CROSS-SECTION
Sections made of more than one material

COMPOSITE BEAMS



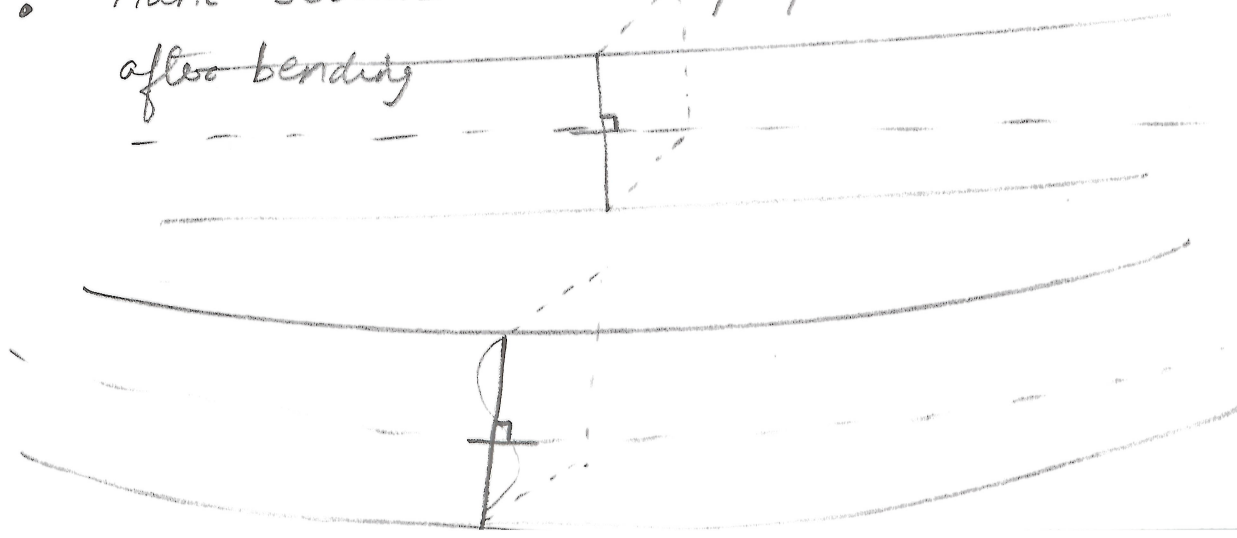
$\therefore$ According to Hooke's Law $\sigma = E \cdot \epsilon$ or $\epsilon = \sigma/E$

Linear strain diagram $\rightarrow$ more fundamental than
linear stress diagram

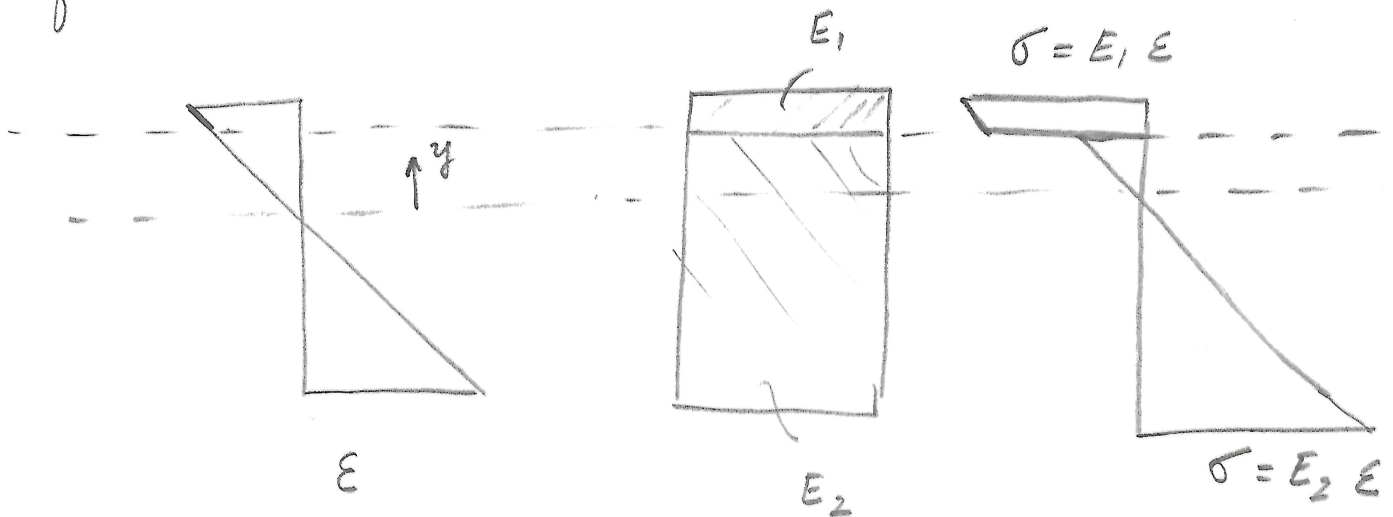
- Plane sections remain plane before and after

BENDING

- Plane sections remain perpendicular to neutral axis after bending



If strain is linear, so what?



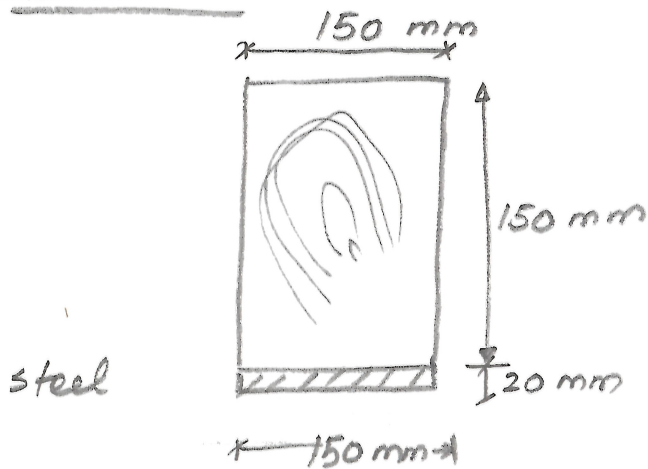
discontinuity in
my σ -diagram!

What should I do?

to calculate stresses in a composite beam.

Transform the composite section into a
simple section!

- ① Transform material ① $\longrightarrow$ ②
- ② Calculate stresses in simple beam
- ③ Transform stresses back to the material ①

EXAMPLE

$$E_{\text{wood}} = 12 \text{ GPa}$$

$$E_{\text{steel}} = 200 \text{ GPa}$$

$$M = +2 \text{ kN-m}$$

$$= 2 \times 10^6 \text{ N-mm}$$

Q. Calculate stresses due to bending?

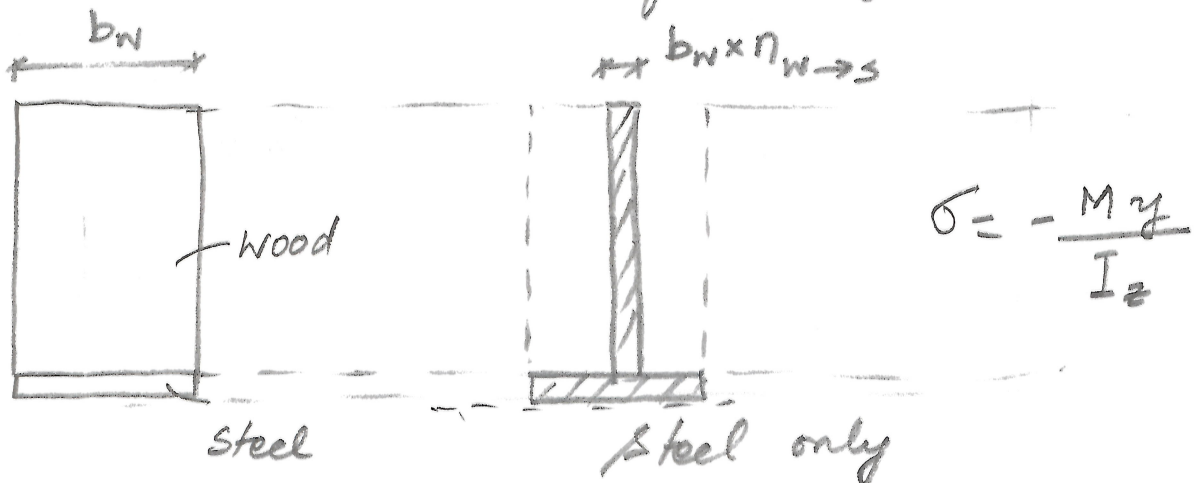
Transform from 2 materials to 1

① Transform wood into steel

① Calculate the modular ratio

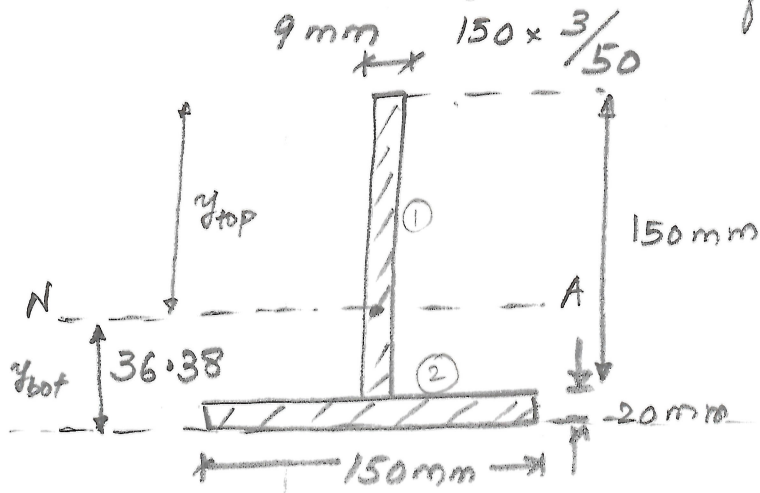
$$\eta_{N \rightarrow S} = \frac{E_N}{E_S} = \frac{12}{200} = \frac{3}{50} = \frac{1}{16.67}$$

② ~~reduce~~ multiply b or width of wood by $\eta_{N \rightarrow S}$



$$\sigma = -\frac{My}{I_z}$$

Calculate the c.g. and I_z of transformed section



$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

$$\bar{y} = 36.38 \text{ mm}$$

$$= \frac{(150 \times 9) \times (20 + 75) + (50 \times 20) \times 10}{(150 \times 9) + (50 \times 20)}$$

$$I_z = 9.358 \times 10^6 \text{ mm}^4 \rightarrow \text{using parallel axis theorem}$$

Calculate the stresses due to M

$$\sigma = -\frac{M \times y}{I_z}$$

@ the top

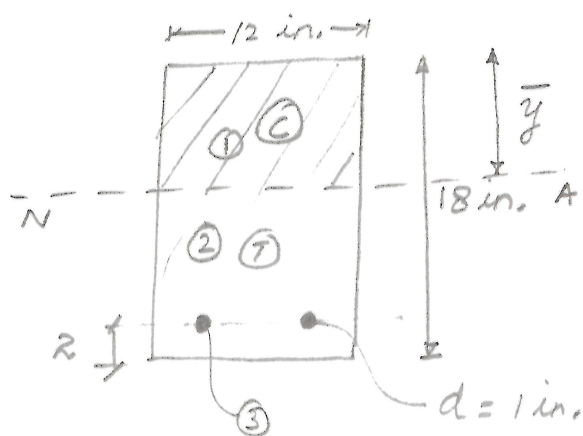
$$\sigma = -\frac{2 \times 10^6 \text{ N-mm} \times (170 - 36.38)}{9.358 \times 10^6}$$

$$\sigma_{top} = -28.6 \text{ N/mm}^2 = -28.6 \text{ MPa}$$

$$\sigma_{wood} = \frac{\sigma_{calc / transformed} \times n_{N \rightarrow S}}{n_{N \rightarrow S}} = \frac{-28.6 \times 1}{16.67} = -1.71 \text{ MPa}$$

- ① Transform the softer material $\rightarrow$ stiffer material
- ② $n_{1 \rightarrow 2} = \frac{E_1}{E_2} \rightarrow$ If you are transforming softer material then $n_{1 \rightarrow 2} < 1$
- ③ Multiply the width of softer material $b_1 \times n_{1 \rightarrow 2}$
- ④ Calculate $\bar{y}$, I_z of the transformed section
- ⑤ Calculate stresses
- ⑥ Transform stress in the softer material $\sigma_{calc} \times n_{1 \rightarrow 2}$

Composite Concrete-Steel beam



$$+M = 60 \text{ k-ft} = 60 \times 12 = 720 \text{ k-in.}$$

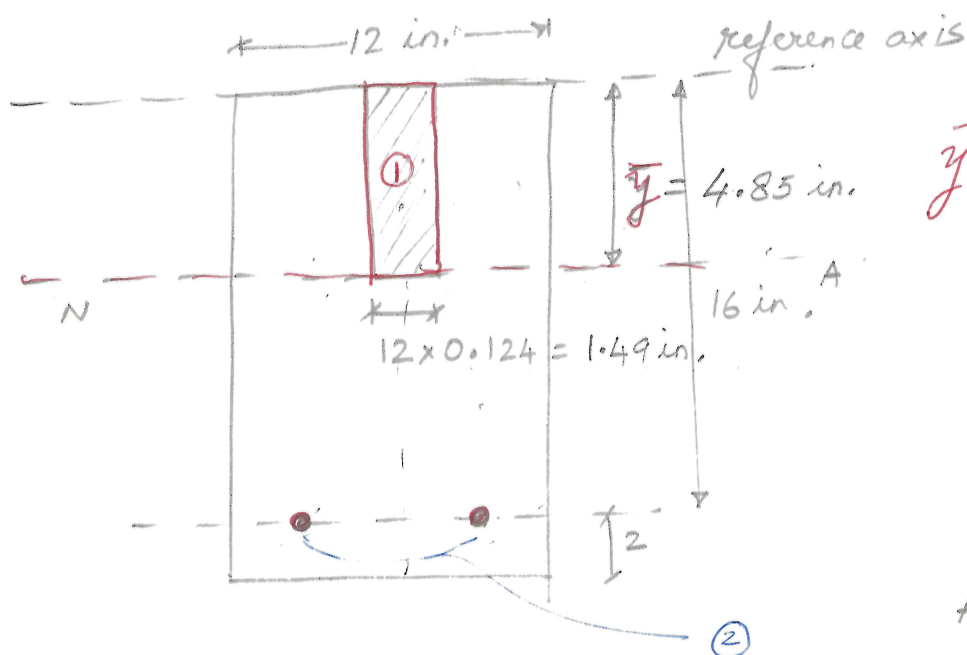
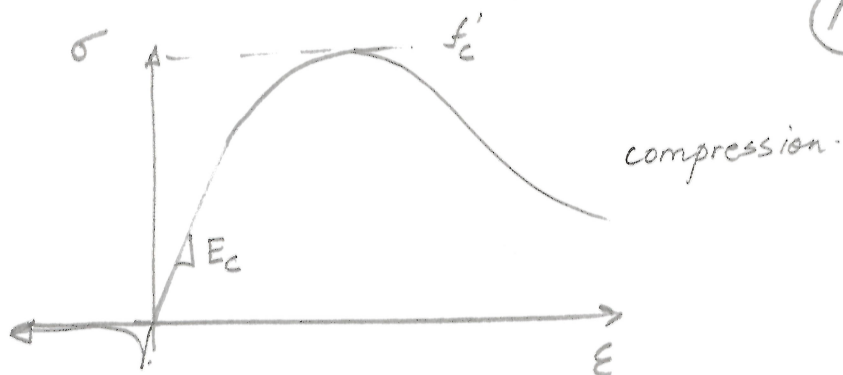
$$\begin{aligned} & \left(\begin{array}{c} + \\ \square \\ - \end{array} \right) \begin{array}{c} C \\ T \end{array} \\ & E_s = 29000 \text{ ksi} \\ & E_c = 3600 \text{ ksi} \end{aligned}$$

$$E_c^{\text{comp}} = 3600 \text{ ksi}$$

$$E_c^{\text{ten}} = 0$$

$$n_{1 \rightarrow 3} = \frac{3600}{29000} = 0.124$$

$$n_{2 \rightarrow 3} = \frac{0}{29000} = 0$$



Cracked-transformed section

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

$$A_1 = 1.49 \times \bar{y}$$

$$y_1 = \frac{\bar{y}}{2}$$

$$A_2 = \frac{\pi d^2}{4} \times 2_{\text{bars}} = \frac{\pi \times 1^2 \times 2}{4} = 1.57 \text{ in}^2$$

$$y_2 = 16 \text{ in.}$$

$$\bar{y} = \frac{1.49 \bar{y} \times \frac{\bar{y}}{2} + 1.57 \times 16}{1.49 \bar{y} + 1.57}$$

$$\bar{y} = \frac{0.745 \bar{y}^2 + 25.13}{1.57 + 1.49 \bar{y}}$$



$$1.57 \bar{y} + 1.49 \bar{y}^2 - 0.745 \bar{y}^2 - 25.13 = 0$$

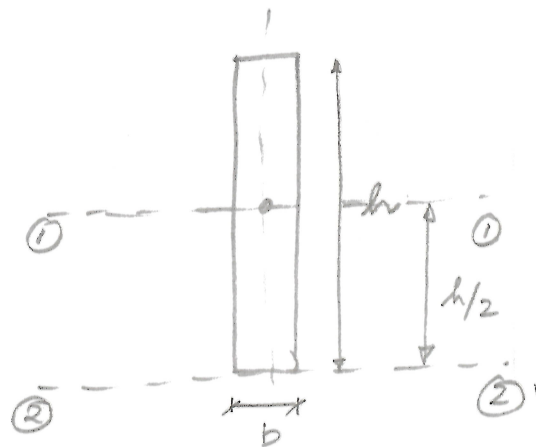
$$\therefore 0.745 \bar{y}^2 + 1.57 \bar{y} - 25.13 = 0$$

$$\therefore \bar{y} = \frac{-1.57 \pm \sqrt{(1.57)^2 + 4(25.13)(0.745)}}{2 \times 0.745}$$

$$= \frac{-1.57 + 8.79}{2 \times 0.745}$$

$$\bar{y} = 4.85 \text{ in.}$$

$$\begin{aligned} I_z \Big|_{\text{center}} &= \frac{1}{3} \times 1.49 \times (4.85)^3 \\ &+ 1.57 \times (16 - 4.85)^2 \\ &= 56.62 + 195.32 \\ &= 251.94 \text{ in}^4 \end{aligned}$$



$$I_1 = \frac{bh^3}{12}$$

$$I_2 = \frac{bh^3}{12} + bh \times \left(\frac{h}{2}\right)^2$$

$$= \frac{bh^3}{12} + \frac{bh^3}{4} = \frac{3bh^3 + bh^3}{12}$$

$$= \frac{4bh^3}{12} = \frac{bh^3}{3}$$

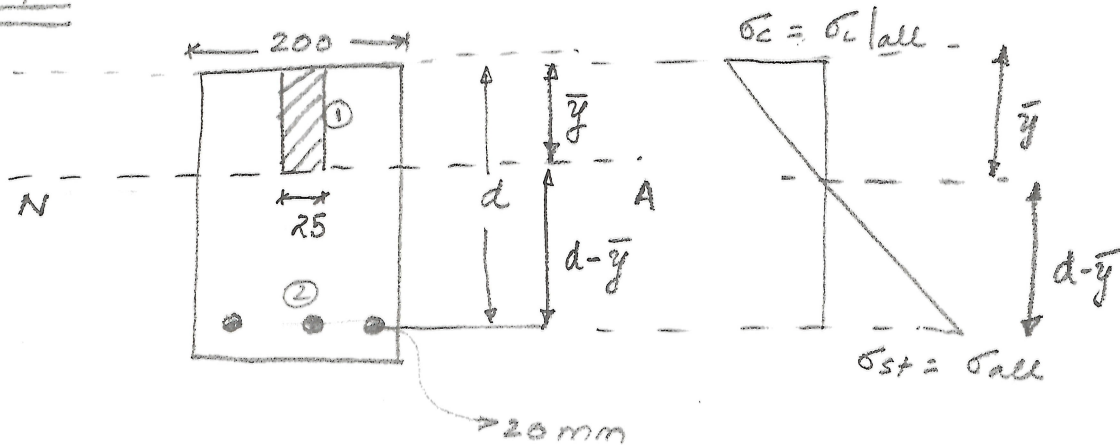
$$\sigma = - \frac{M \times y}{I_z \Big|_{\text{center}}}$$

$$\sigma_s = - \frac{60 \times 12 \text{ k-in} \times (16 - 4.85)}{251.94 \text{ in}^4} = 31.87 \text{ ksi} \checkmark$$

$$\sigma_c = - 720 \times 4.85 / 251.94 = -13.86 \text{ ksi} \checkmark$$

Take $\sigma_c \big|_{x(\bar{c} \rightarrow s)} = -13.86 \times 0.124 = \underline{\underline{-1.72 \text{ ksi}}}$ ✓

Example



$$\left. \begin{array}{l} E_c = 25 \text{ GPa} \\ E_s = 200 \text{ GPa} \end{array} \right\} n_{c \rightarrow s} = \frac{E_c}{E_s} = \frac{25}{200} = \frac{1}{8} = 0.125 \checkmark$$

$$\sigma_c|_{all} = 12.5 \text{ MPa}$$

$$\sigma_s|_{all} = 200 \text{ MPa}$$

$$\left(\begin{array}{l} A_1 = 25 \times \bar{y} \\ y_1 = \bar{y}/2 \end{array} \right.$$

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

$$\left(\begin{array}{l} A_2 = \frac{\pi \times d^2}{4} \times 3_{bars} \\ = \frac{\pi \times 20^2}{4} \times 3 = 300\pi \text{ mm}^2 \end{array} \right.$$

$$y_2 = d$$

$$\bar{y} = \frac{25 \times \frac{\bar{y}^2}{2} + 300\pi d}{25\bar{y} + 300\pi}$$

Stalled because I need d & $\bar{y}$ relationship

$$\sigma_c = - \frac{M \times \bar{y}}{I_z} \times n_{c \rightarrow s} = 12.5 \text{ MPa}$$

calc

$$\sigma_t = - \frac{M \times (d - \bar{y})}{I_z} = 200 \text{ MPa}$$

$$\therefore \frac{\bar{y} \times n_{c \rightarrow s}}{(d - \bar{y})} = \frac{12.5}{200} \quad \left| \begin{array}{l} \text{Since} \\ \frac{\sigma_c}{\sigma_t} = \frac{12.5}{200} \end{array} \right.$$

$$\therefore \frac{\bar{y}}{d - \bar{y}} = 0.5$$

$$\therefore 2\bar{y} = d - \bar{y} \quad \therefore 3\bar{y} = d$$

$$\therefore \bar{y} = \frac{12.5 \bar{y}^2 + 900\pi \bar{y}}{25\bar{y} + 300\pi} \quad \rightarrow \text{Substituted } d = 3\bar{y}$$

$$\therefore 25\bar{y}^2 + 300\pi \bar{y} = 12.5 \bar{y}^2 + 900\pi \bar{y}$$

$$\therefore 12.5 \bar{y}^2 - 600\pi \bar{y} = 0$$

$$\therefore \bar{y} = \frac{600\pi}{12.5} = 150.8 \text{ mm}$$

$$\therefore d = 3\bar{y} = \underline{\underline{452.4 \text{ mm}}}$$

What is M

$$\sigma_c = - \frac{M \times \bar{y}}{I_z} \times n_{c \rightarrow s}$$

$$12.5 = - \frac{M \times 150.8}{I_z} \times n_{c \rightarrow s}$$

$$\therefore M = \underline{\hspace{2cm}}$$

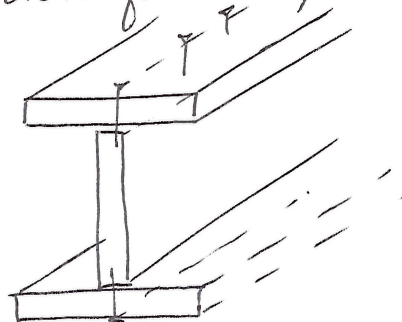
CE270

• Flexure

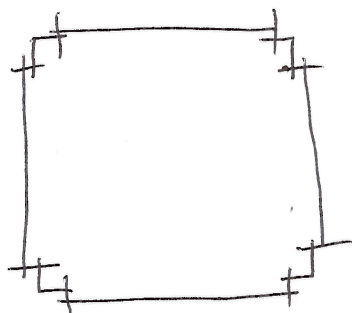
- (1) $\rightarrow V(x), M(x), N(x)$ by drawing FBD & Eq. of Eq
- (2) $\rightarrow$ graphical approach V, M diagrams directly
- (3) $\rightarrow$ Calculation of stresses due to bending
- (4) $\rightarrow$ Composite beam stresses
 $\rightarrow$ Reinforced concrete beams

• Shear

- (1) calculation of shear stresses
- (2) design of connectors for composite beams

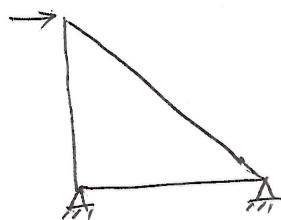


built-up
cross sections



built-up
cross-sections

• Calculation of Shear Stresses



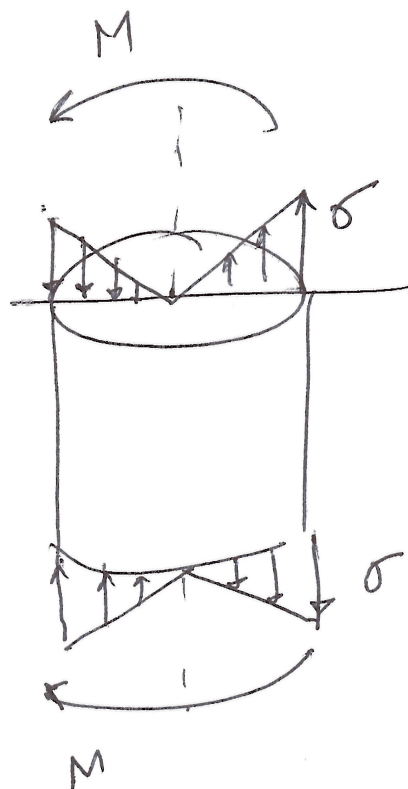
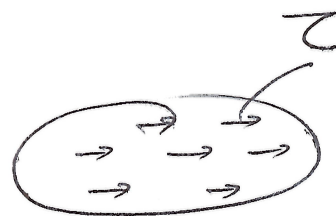
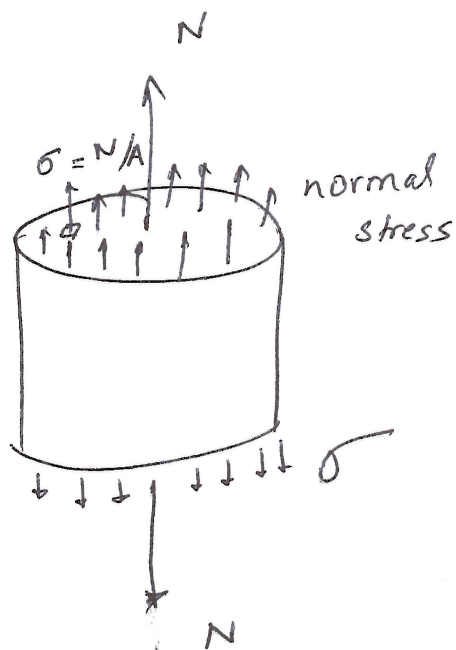
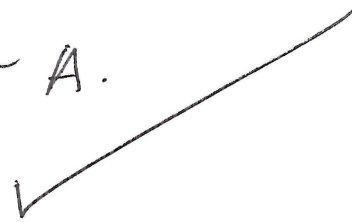
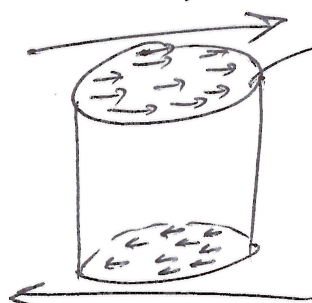
fasteners $\rightarrow$ bolts $\rightarrow$ single
 $\rightarrow$ double

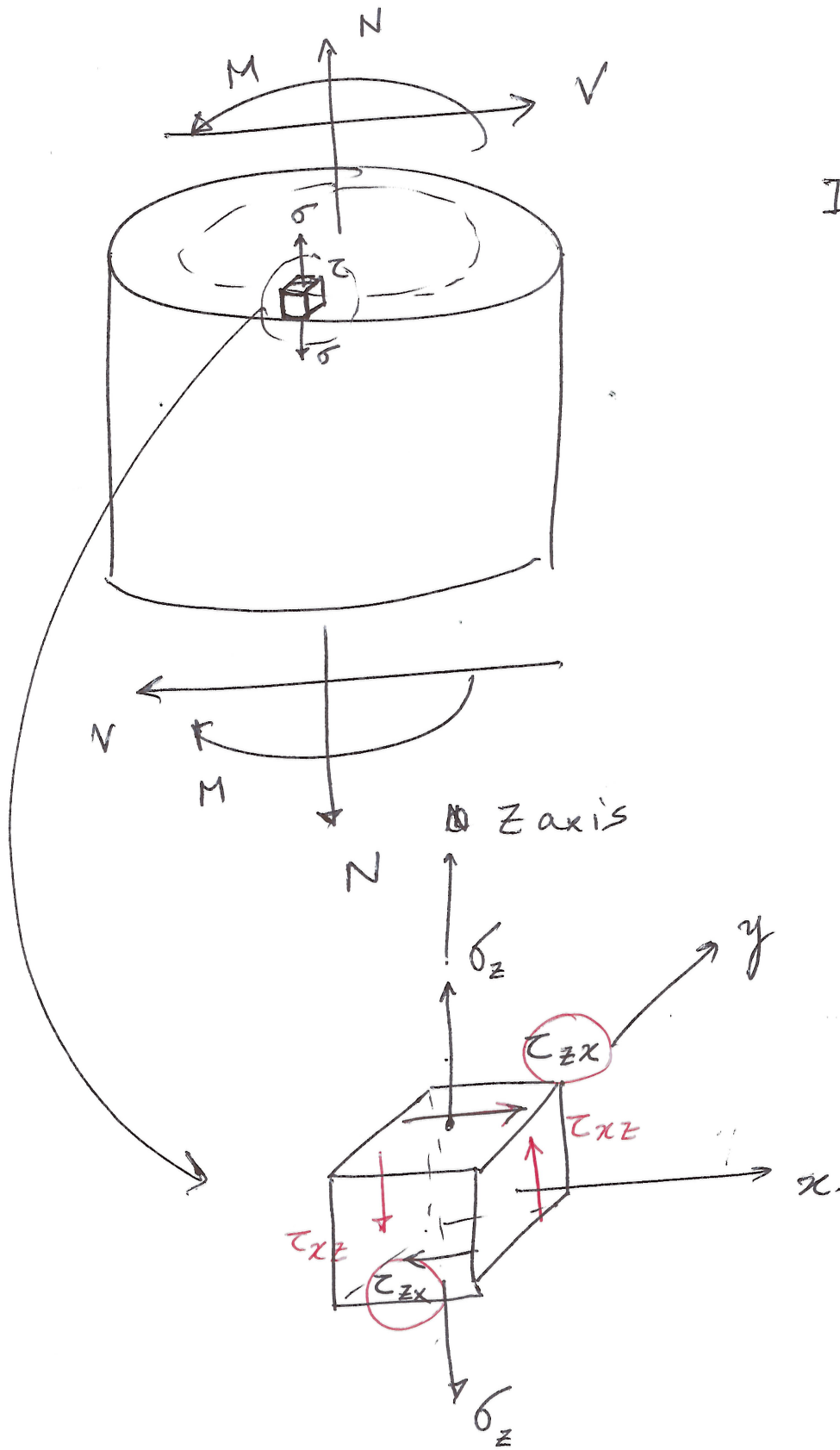


$\tau = V/A$
single

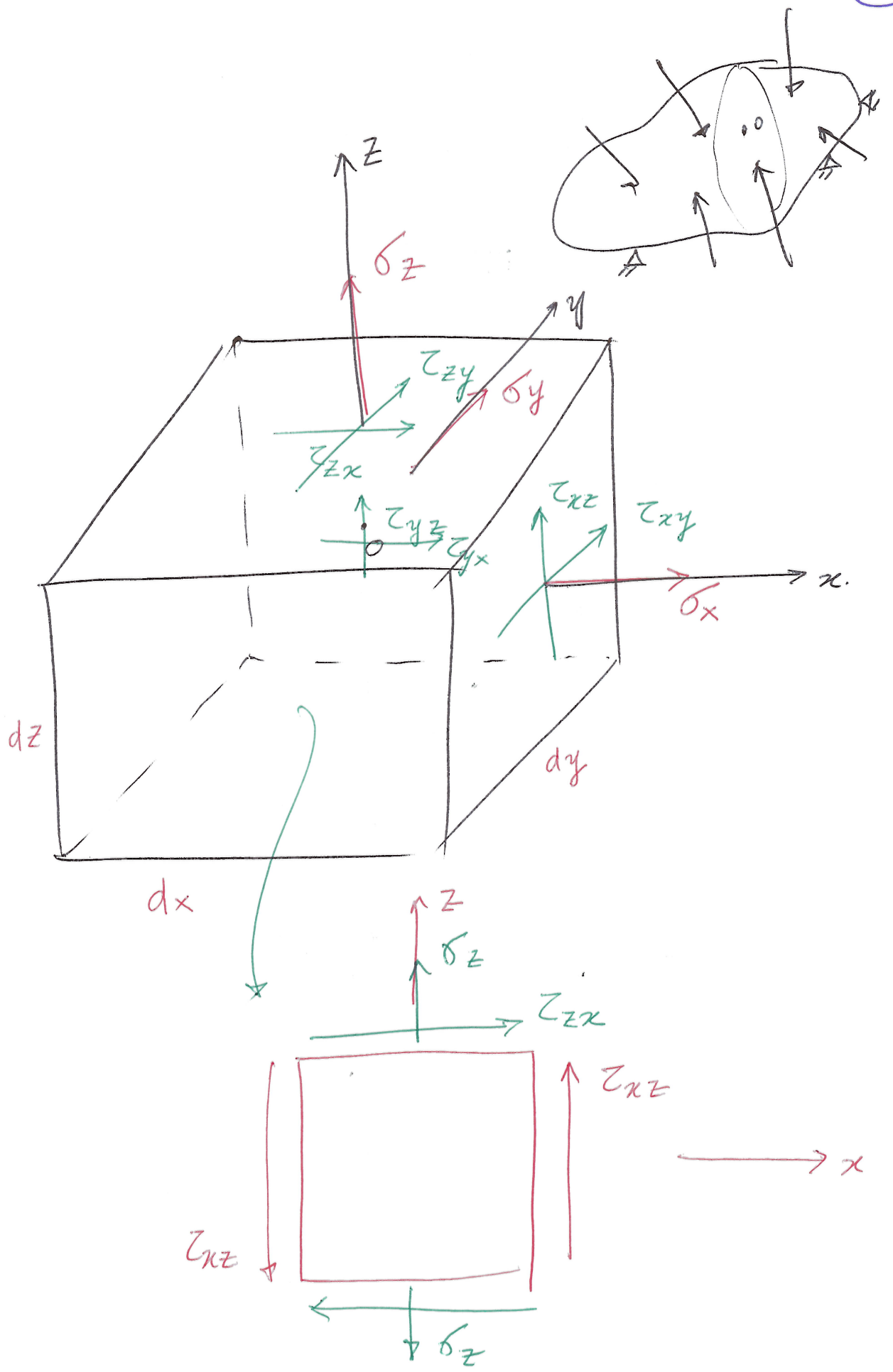
$\tau = V/2A$
double

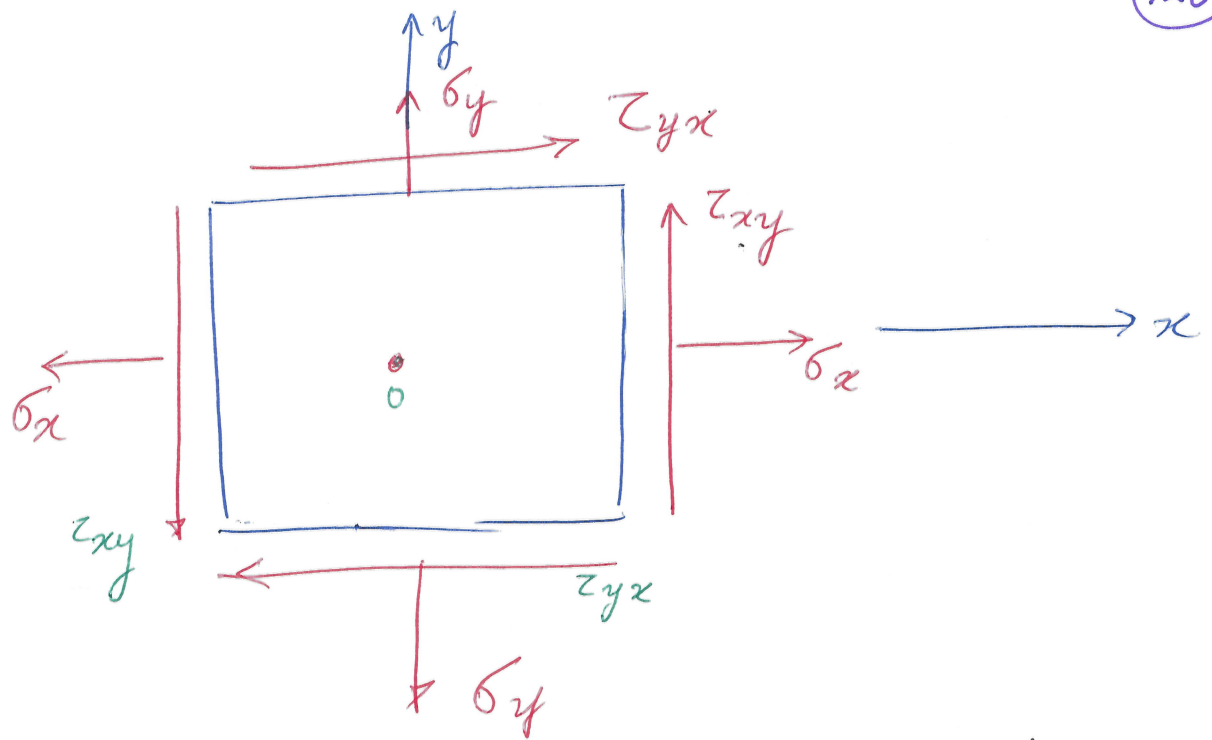
$$\tau_{avg} = V / A \uparrow N$$





Internal forces
 $N, V \& M.$





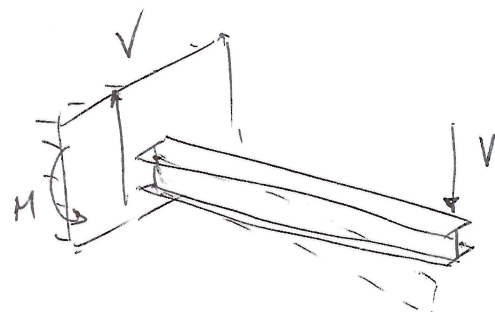
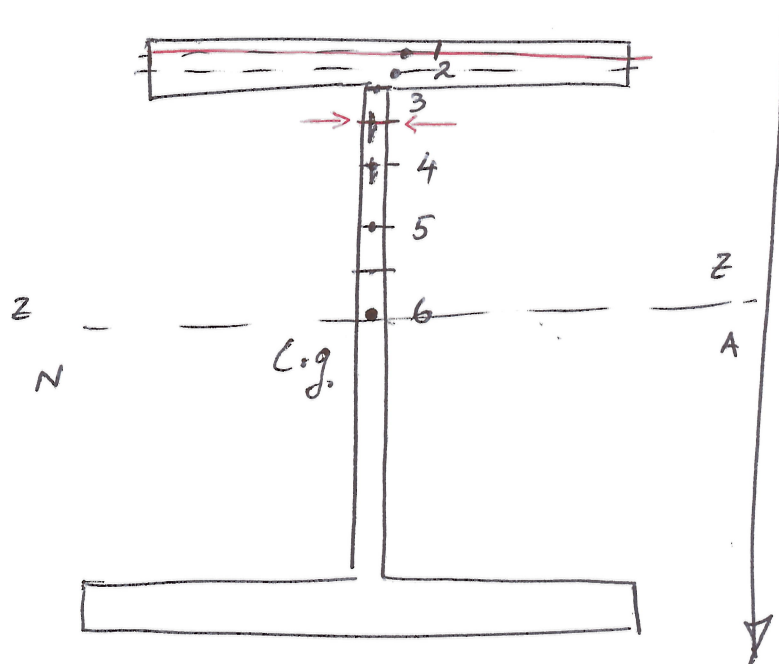
- ① Shear stress acts parallel to surface
- ② Shear stress never appears alone.
always with its counterpart ✓

Calculating Shear Stress with an Example

What is Q ?

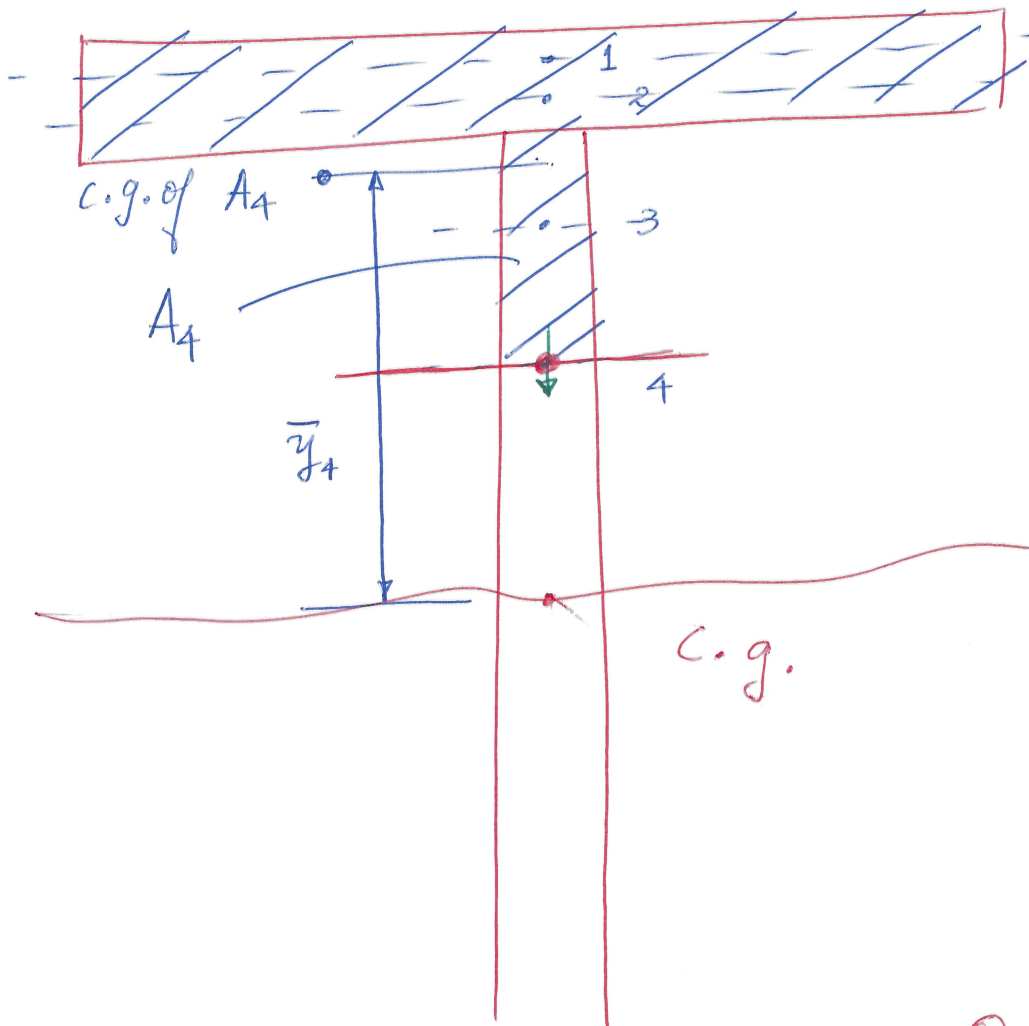
$$\tau = \frac{V}{I} \times Q$$

$$I \times t$$



- τ_1 thickness @ 1
- τ_2
- τ_3 thickness @ 3
- τ_4
- τ_5
- τ_6

What is Q



$$Z_1 = \frac{V}{I} \times Q_1$$

$$Z_2 = \frac{V}{I} \times Q_2$$

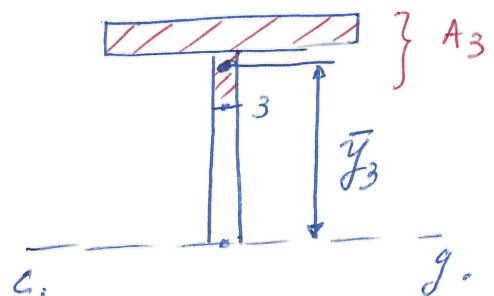
$$Z_3 = \frac{V}{I} \times Q_3$$

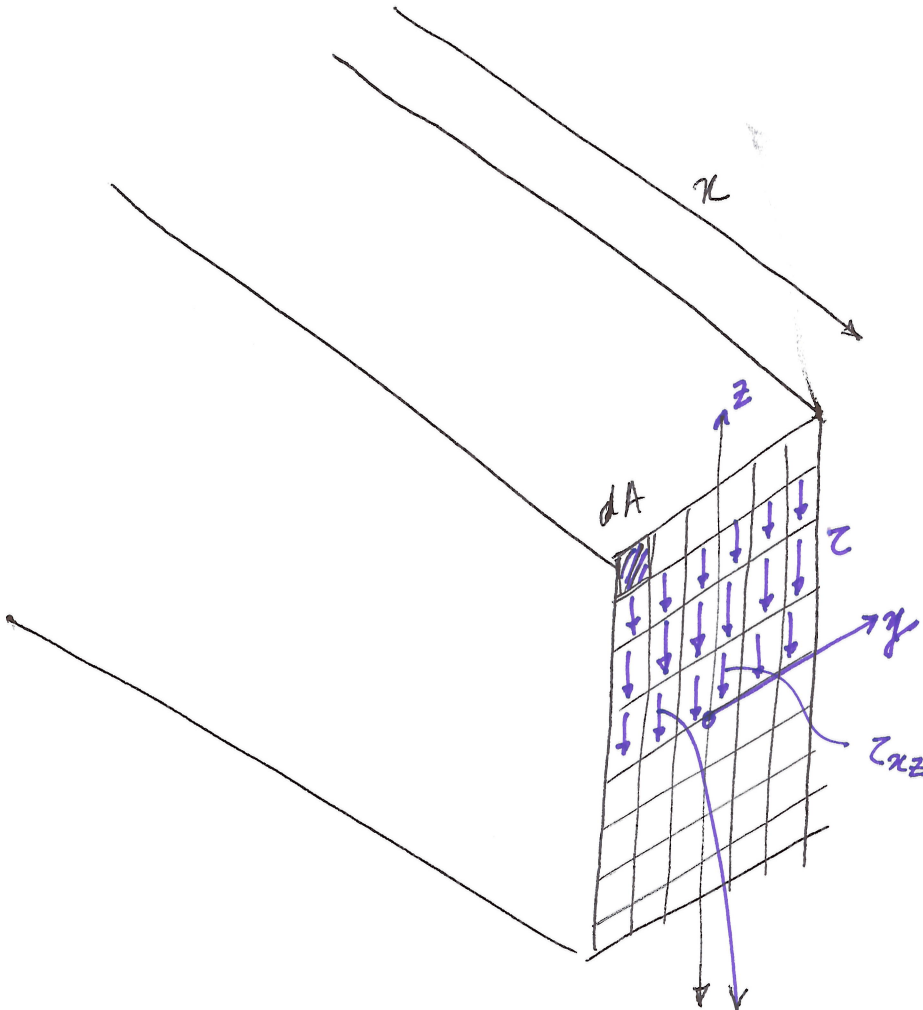
$$Z_4 = \frac{V}{I} \times Q_4$$

What are Q_1, Q_2, Q_3, Q_4

$$Q_4 = A_4 \times \bar{y}_4$$

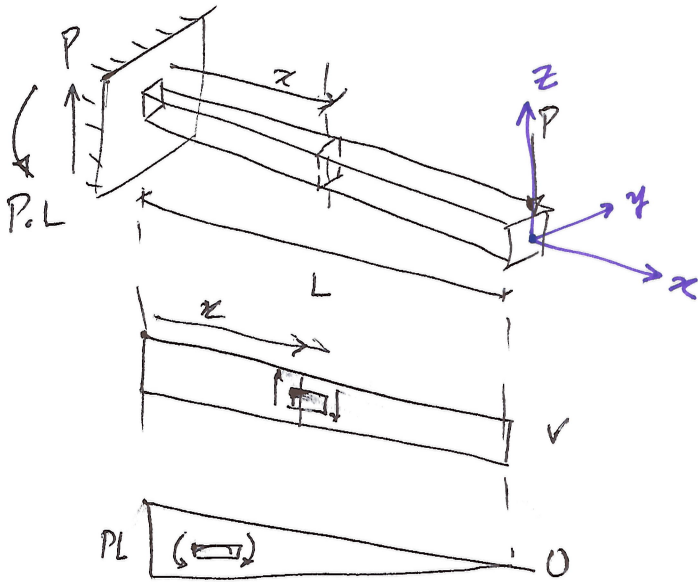
$$Q_3 = A_3 \times \bar{y}_3$$





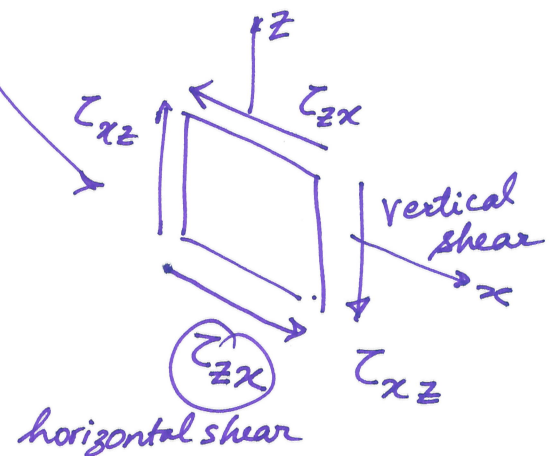
$$A = \int dA$$

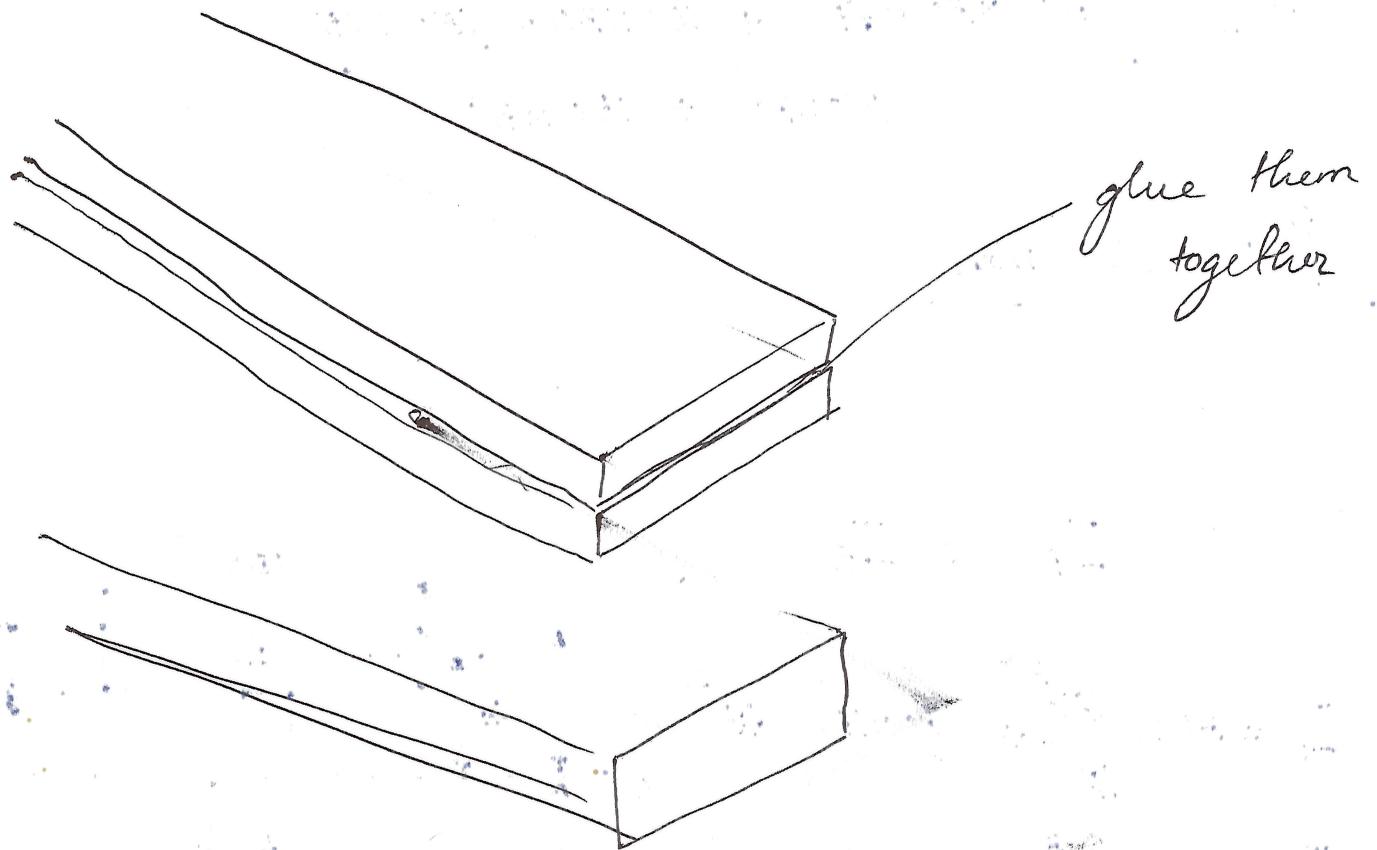
$$V = \int z \cdot dA$$



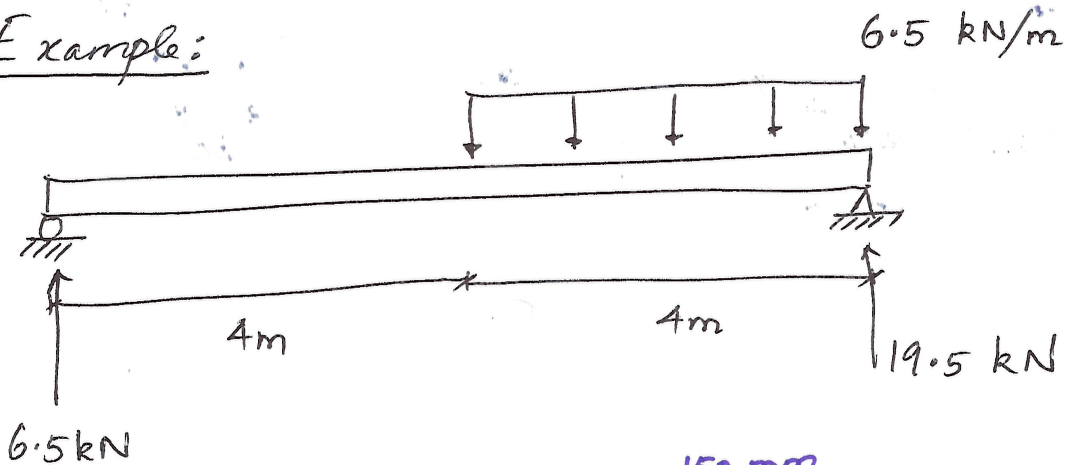
$$\tau_{xz}$$

τ_{zx} comes into existence with τ_{xz}

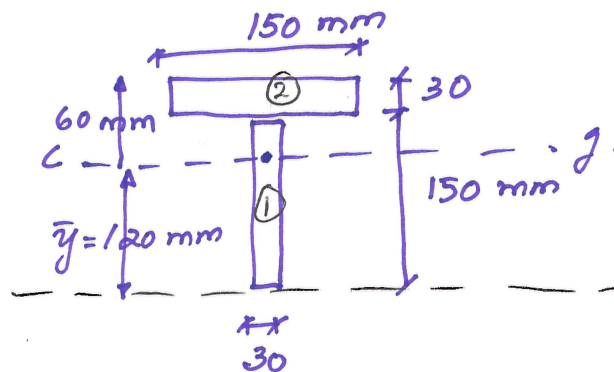




Example:



cross-section



$\bar{y}$

I_{NA}

V

Q

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{\sum A_i y_i}{\sum A_i}$$

$$\bar{y} = \frac{\overset{A_1}{150 \times 30} \times \overset{y_1}{75} + \overset{A_2}{150 \times 30} \times \overset{y_2}{(150+15)}}{150 \times 30 + 150 \times 30}$$

$$\bar{y} = 120 \text{ mm}$$

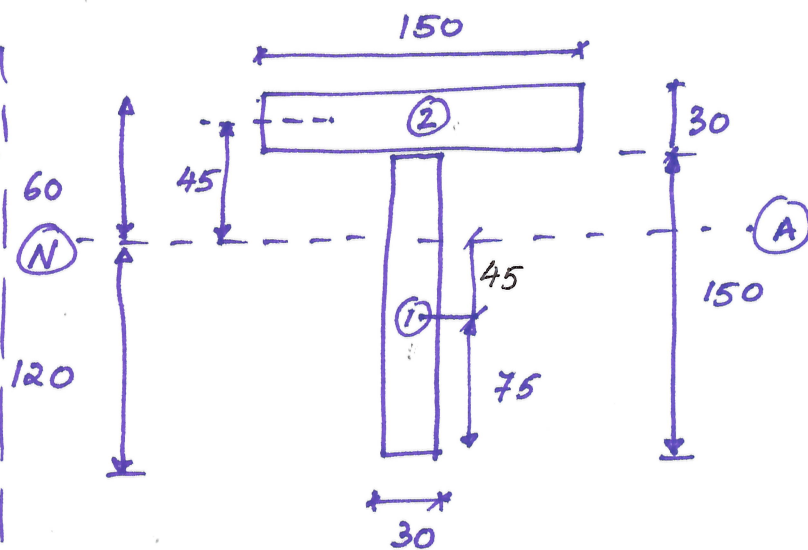
Calculate the section properties needed

$$I_{NA} = I_1 + I_2$$

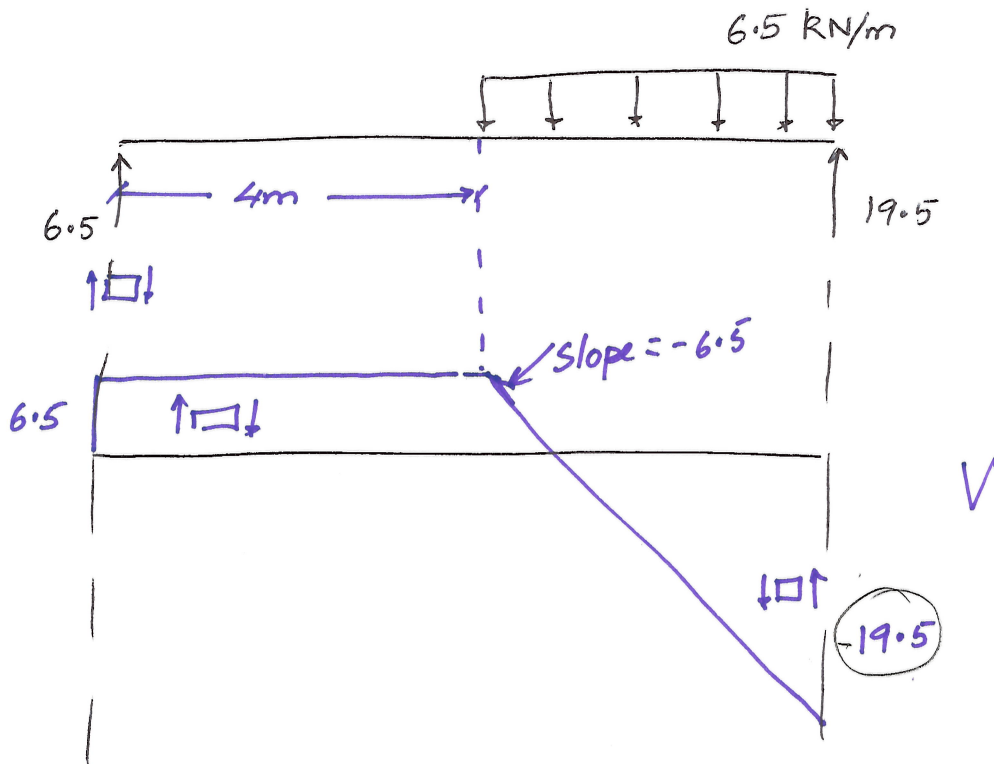
$$I_1 = 30 \times \frac{150^3}{12} + (30 \times 150) \times 45^2$$

$$I_2 = 150 \times \frac{30^3}{12} + (30 \times 150) \times 45^2$$

$$I_{NA} = 27 \times 10^6 \text{ mm}^4$$



$$\tau = \frac{V}{I} \times \frac{Q}{t}$$

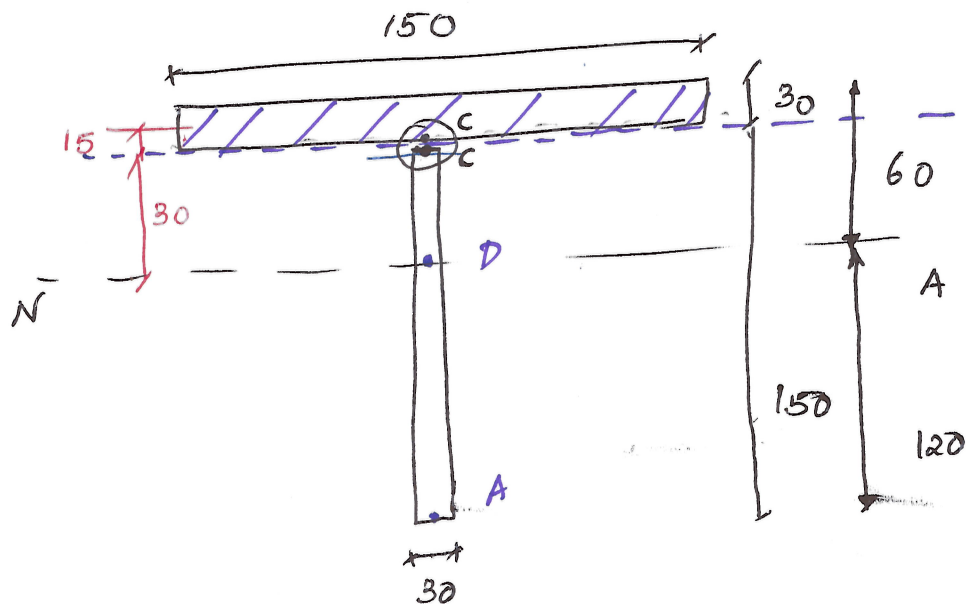


Shear stresses $\rightarrow$ max. when V is max.

$$V = -19.5 \text{ kN} \rightarrow \text{max shear.}$$

$$\sigma = -\frac{M}{I} y$$

Calculate shear stress @ the interface



$$\tau_c = \frac{V}{I} \times \frac{Q_c}{t_c}$$

$27 \times 10^6 \text{ mm}^4$

$$Q_c = A_c \times \bar{y}_c \quad \left| \begin{array}{l} \text{portion above } c. \\ \bar{y}_c = 45 \end{array} \right.$$

$\bar{y}$ = distance from the NA to centroid of A_c

$$(t_c) = 30 \text{ mm}.$$

$$A_c = 150 \times 30 \quad \checkmark$$

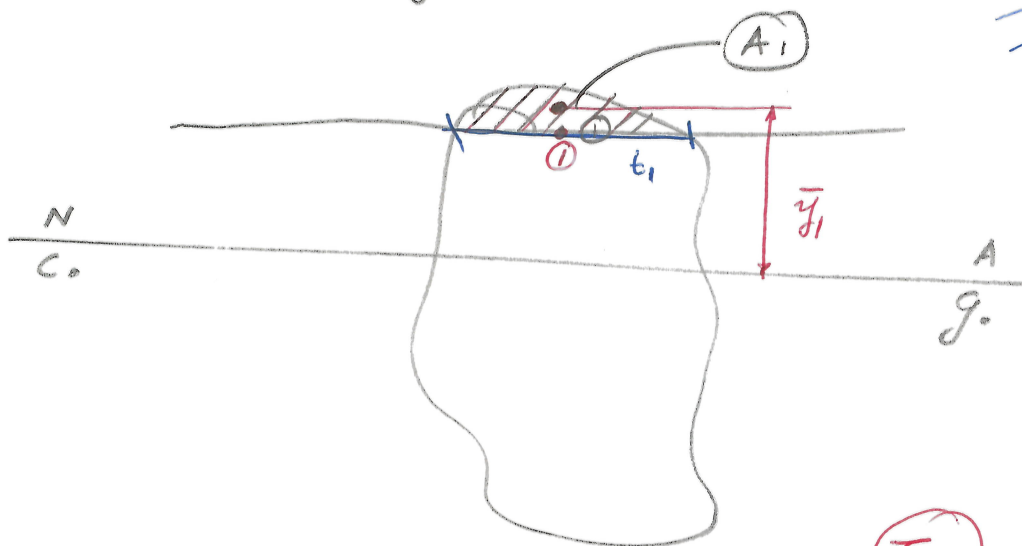
$$Q_c = 202.5 \times 10^3 \text{ mm}^3$$

$$\therefore \tau_c = \frac{19.5 \text{ kN} \times 202.5 \times 10^3}{27 \times 10^6 \text{ mm}^4 \times 30 \text{ mm}}$$

$$= 4.88 \text{ N/mm}^2$$

$$= 4.88 \text{ MPa}$$

Consider any section

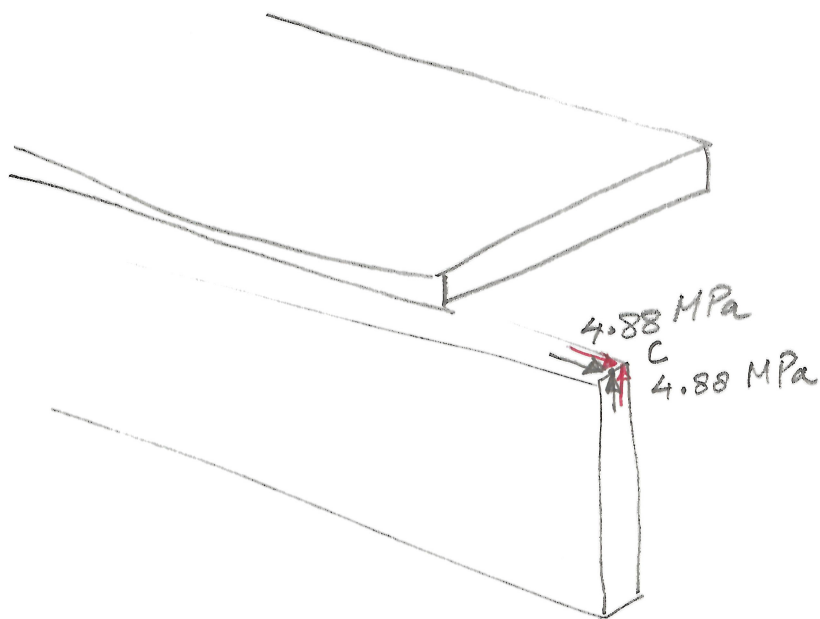
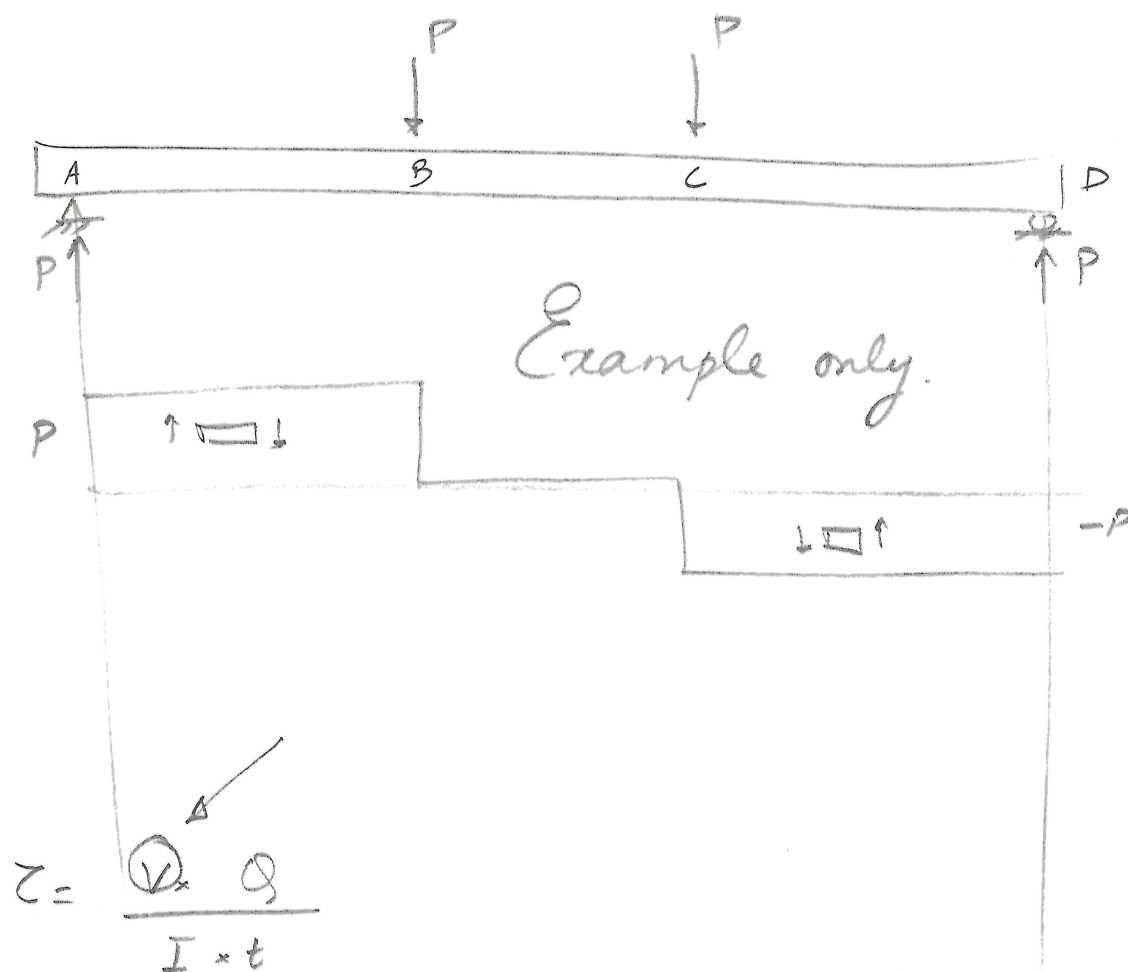


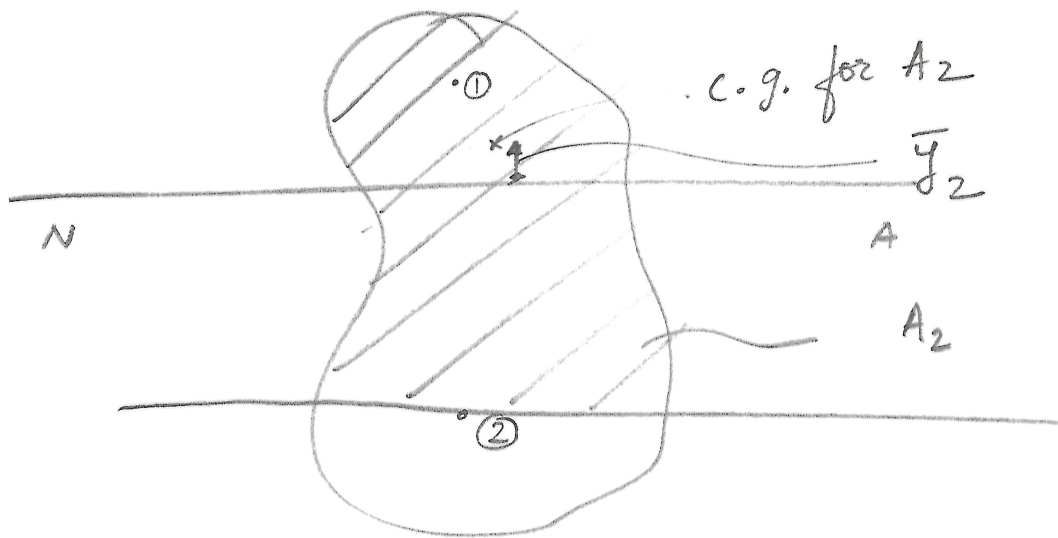
A_1 = area above ①

$\bar{y}_1$ = distance from NA to centroid of A_1

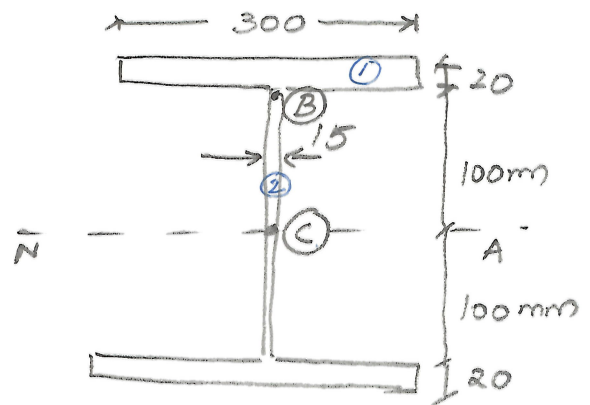
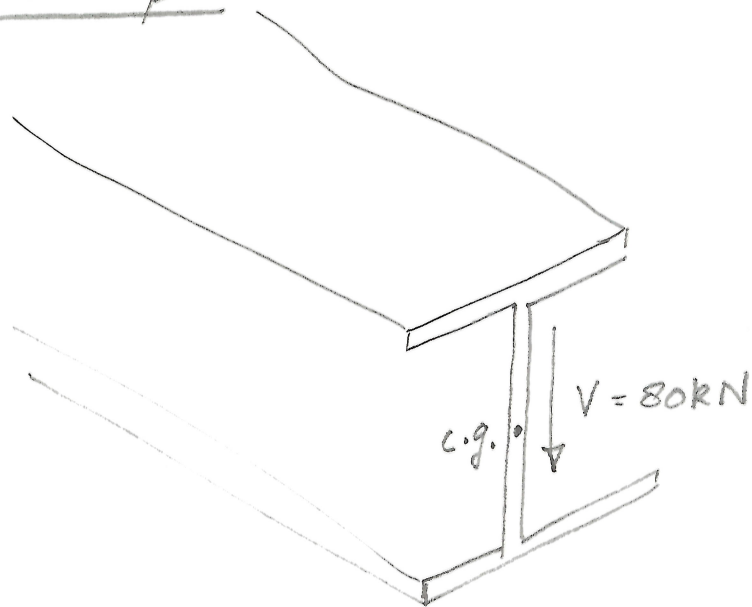
$$Q_1 = A_1 \times \bar{y}_1$$

$$(\tau_1) = \frac{V}{I_{NA}} \times \frac{Q_1}{t_1}$$





Example.



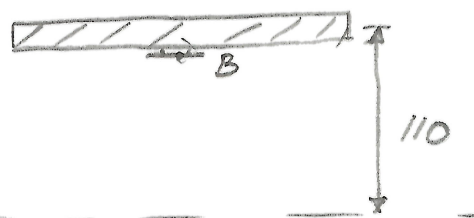
Doubly symmetric section
 $\therefore$ c.g. & NA coincide.

$$I_{NA} = I_{(2)} + 2 I_{(1)}$$

$$= 15 \times \frac{200^3}{12} + 2 \left[\frac{300 \times 20^3}{12} + (300 \times 20) [100 + 10]^2 \right]$$

$$= 155.6 \times 10^6 \text{ mm}^4$$

$$\tau_B = ?$$



$$Q_B = A_B \times \bar{y}_B$$

$$= (300 \times 20) \times 110$$

$$= 60 \times 10^4 \text{ mm}^3$$

$$t_B = 15 \text{ mm}$$

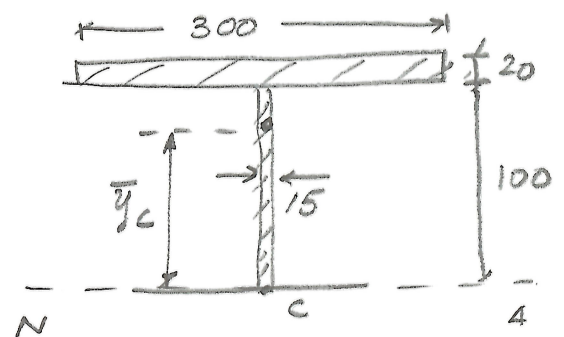
$$\therefore \tau_B = \frac{V \times Q_B}{I_{NA} \times t_B}$$

$$= \frac{80 \times 1000 \text{ N} \times 60 \times 10^4 \text{ mm}^3}{155.6 \times 10^6 \text{ mm}^4 \times 15 \text{ mm}}$$

$$\tau_B = 22.6 \text{ N/mm}^2 = \boxed{22.6 \text{ MPa} = \tau_B}$$

$$\tau_c = \frac{V \times Q_c}{I_{NA} \times t_c}$$

$$Q_c = A_c \times \bar{y}_c$$



$$\therefore \bar{y}_c = \frac{(300 \times 20) \times 110 + (100 \times 15) \times 50}{(300 \times 20) + (100 \times 15)}$$

$$A_c = (300 \times 20) + (15 \times 100)$$

$$\therefore Q_c = A_c \times \bar{y}_c = 73.5 \times 10^4 \text{ mm}^3$$

$$\tau_c = \frac{80 \times 1000 \text{ N} \times 73.5 \times 10^4 \text{ mm}^3}{155.6 \times 10^6 \text{ mm}^4 \times 15 \text{ mm}}$$

$$\boxed{\tau_c = 25.2 \text{ MPa}}$$



$$\tau_B = \frac{V \times Q_B}{I \times t_B}$$

flange

$$= \frac{80 \times 1000 \times 60 \times 10^4 \text{ mm}^3}{155.6 \times 10^6 \text{ mm}^4 \times 300 \text{ mm}}$$

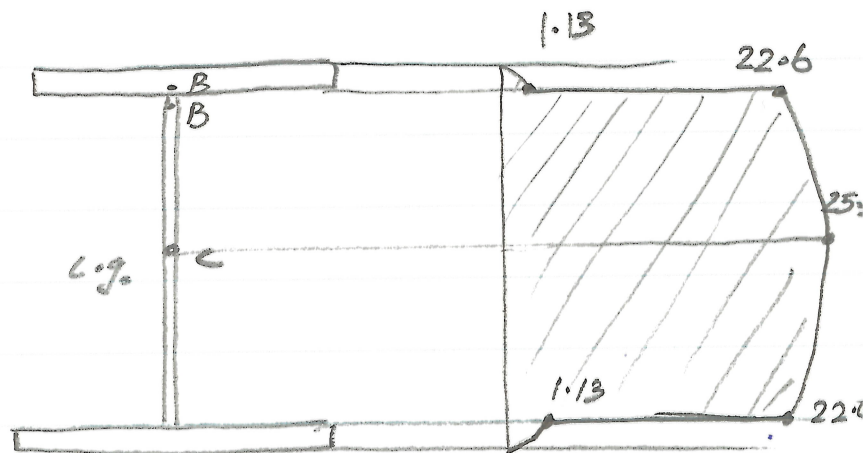
 Q_B

$$t_B = 300 \text{ mm}$$

$$= 1.13 \text{ MPa}$$

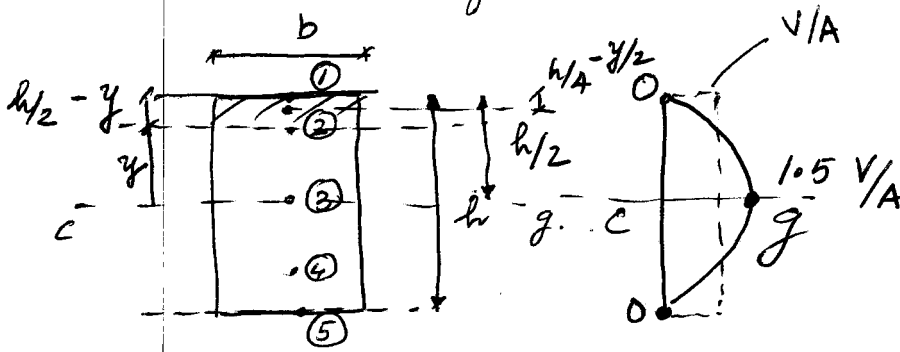
 τ

cross-section



Variation of Shear Stress over cross-section

139



$$\tau = \frac{V}{I} \times \left(\frac{Q}{t} \right)$$

$$= \frac{V}{I b} \times Q$$

What is Q

$$= A \bar{y}$$

↓
distance from
c.g. of section to
c.g. of int area

$$Q_1 = ? \quad 0 \times h/2$$

$$= 0$$

$$\tau_1 = 0$$

$$Q_5 = ? \quad b h \times 0$$

$$\tau_5 = 0$$

$$Q_2 = A_2 \times \bar{y}_2$$

$$A_2 = b \times \left(\frac{h}{2} - y \right)$$

$$\bar{y}_2 = \frac{h}{2} - \left(\frac{h}{4} - y/2 \right)$$

$$\bar{y}_2 = \frac{h}{4} + y/2$$

$$\therefore Q_2 = b \times \left(\frac{h}{2} - y \right) \left(\frac{h}{4} + y/2 \right)$$

$$\tau_3 = \frac{V}{I b} \times Q_3$$

$$Q_3 = A_3 \times \bar{y}_3$$

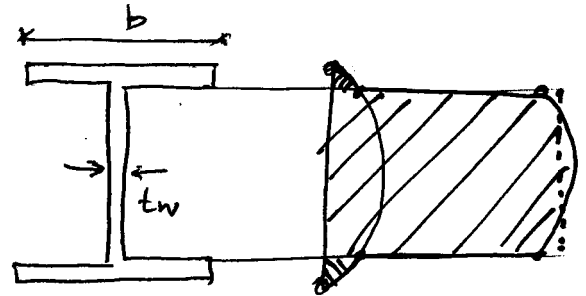
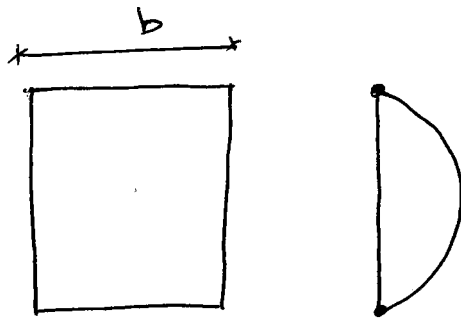
$$= \left(b \times \frac{h}{2} \right) \times \frac{h}{4}$$

$$= b \times \frac{h^2}{8}$$

$$\tau_3 = \frac{V}{\frac{b h^3}{12}} \times \frac{b h}{8}$$

$$= \frac{V \times \cancel{12} \times 1.5}{b h \cancel{8}}$$

$$= \frac{V}{A} \times 1.5$$



$$Z = \frac{V Q}{I \times t}$$

$$V = 80 \text{ kN} = 80,000 \text{ N}$$

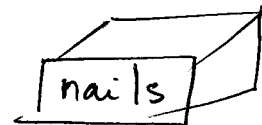
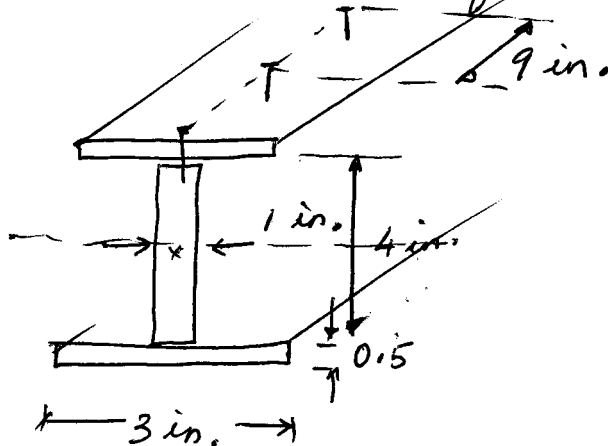
$$A_{web} = 200 \times 15 \text{ mm}^2 = 3000 \text{ mm}^2$$

$$\tau_{avg} = \frac{V}{A_{web}} = \frac{80000}{3000} = 26.6 \text{ N/mm}^2$$

Example

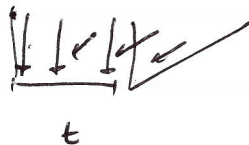
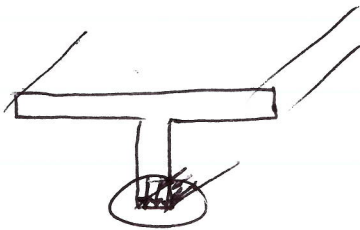
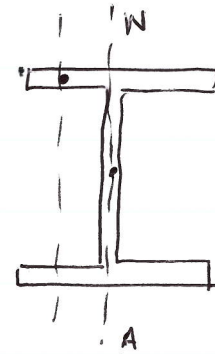
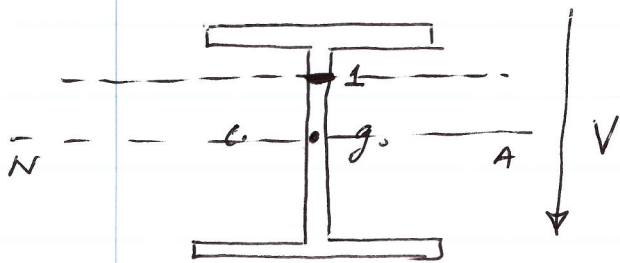
Shear Flow $q = \frac{V Q}{I}$

It is a force per unit length $= (Z \times t) = q$



nails have a shear strength of 40 lb

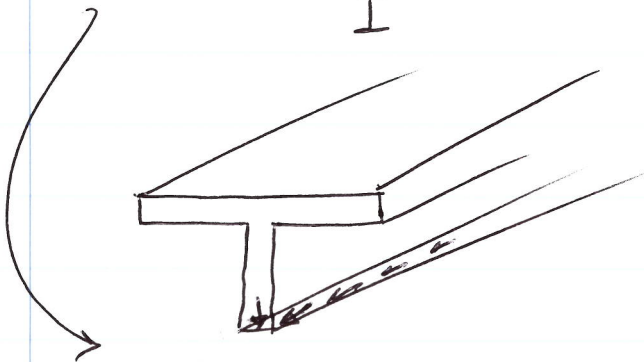
Q. what is the maximum shear force this built-up section can resist?



τ is uniform over t .

$$\tau = \frac{V}{I} \times \left(\frac{Q}{t} \right)$$

$$(\tau \times t) = \frac{V}{I} Q$$



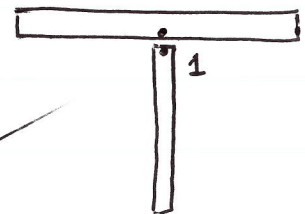
$$\textcircled{Q} \rightarrow \text{Shear flow} = \frac{V}{I} \times \textcircled{Q}$$

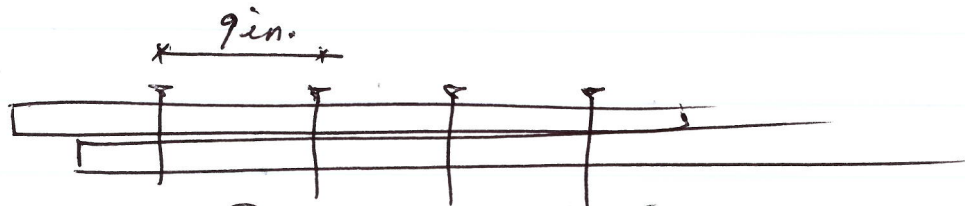
$$\tau = \frac{V}{I} \times \left(\frac{Q}{t} \right)$$

τ units of force/in²



$(\tau \times t) \rightarrow \text{force/in.}$
or force/mm.





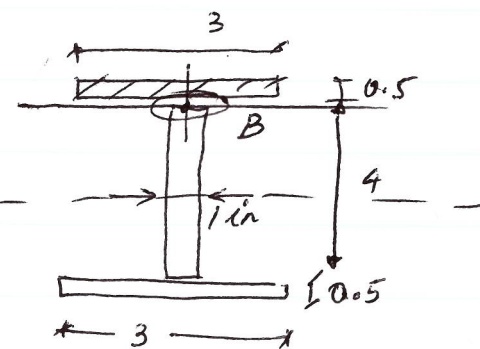
$$q_{res.} = \frac{40 \text{ lb}}{\text{spacing}} = \frac{40}{9} \text{ lb/in}$$

What is the max V that this system can carry.

$$q_B = \frac{V}{I_{NA}} \times Q_B$$

$$I_{NA} = \left[\frac{3 \times 0.5^3}{12} + 3 \times 0.5 \times (2 + 0.25)^2 \right] \times 2 + 1 \times 4^3 / 12$$

$$= 20.58 \text{ in}^4$$



$$Q_B = A_B \times \bar{y}_B = (3 \times 0.5) \times (2.25) = 3.375 \text{ in}^3$$

$$q = \frac{V \times 3.375 \text{ in}^3}{20.58 \text{ in}^4} \rightarrow q_{\text{demand}}$$

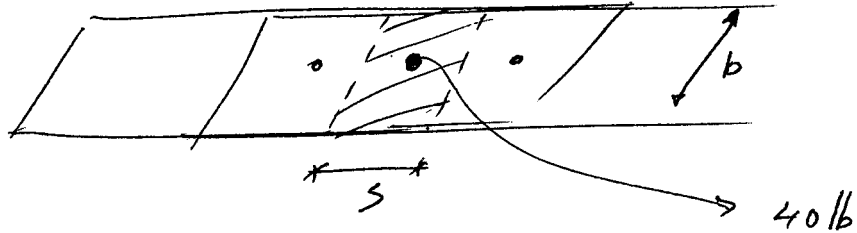
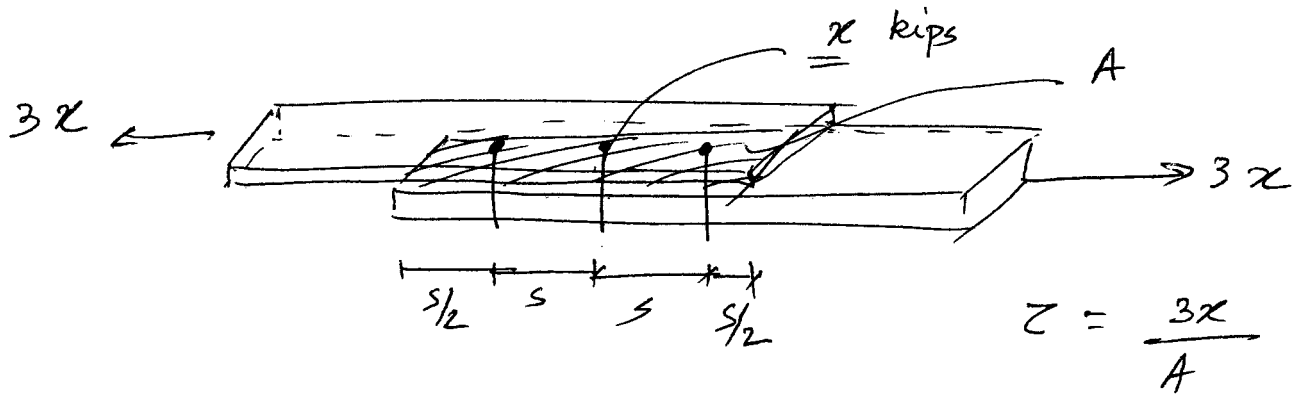
$$q_{\text{resistance}} = \frac{40 \text{ lb}}{9 \text{ in}}$$

$$V \leq 810.3 \text{ lb}$$

$$q_{\text{resis}} \geq q_{\text{demand}}$$

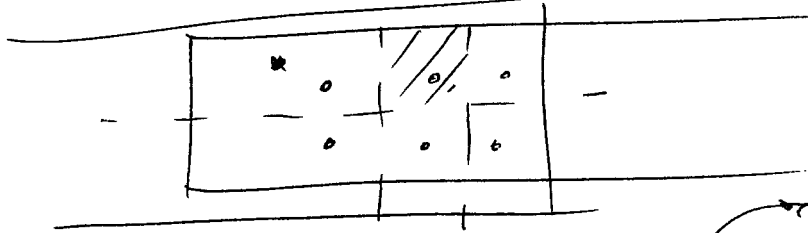
$$\frac{40}{9} \geq \frac{V \times 3.375}{20.58}$$

Why is $q_{resistance} = 40 lb/q$ in
 $= \frac{\text{Shear strength}}{\text{spacing}}$



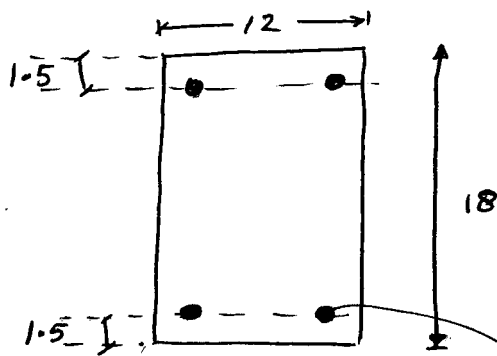
$$z_{resist} = \frac{40}{s \times b}$$

$$q_{resist} = z \times b = \frac{40}{\text{spacing}}$$



bolts, rivets, nails
fasteners having $\rightarrow$ Shear strength force
 $\rightarrow$ shear flow resistance
welds
 $\rightarrow$ Shear stress resistance
glue, bond

Example for Exam ②



- ① What value of M causes yielding of the bottom rebar?
- ② What will be the corresponding stress in concrete & top rebar?

$$f'_c = 5000 \text{ psi}$$

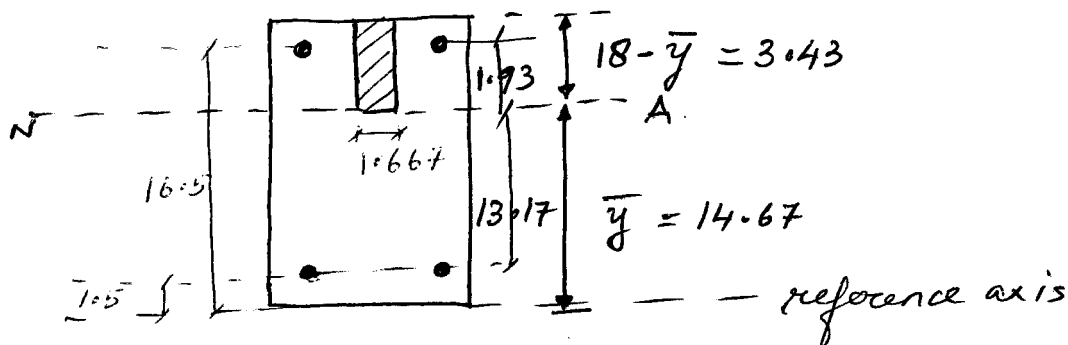
$$f_y = 60 \text{ ksi}$$

$$E_c = 4030 \text{ ksi}$$

$$E_s = 29000 \text{ ksi}$$

$$\sigma = -\frac{My}{I}$$

$$60 = -M \times \frac{y}{I}$$



$$\text{Transform concrete into steel} \rightarrow n_{c \rightarrow s} = \frac{E_c}{E_s} = \frac{4030}{29000}$$

$$b_c \rightarrow b_s \quad 12 \times 0.139 = 1.667 \text{ in} \quad = 0.139$$

$$\#6 \text{ rebar } d_b = \frac{6}{8} \text{ in.} \rightarrow A_b = \frac{\pi d_b^2}{4} = 0.44 \text{ in}^2$$

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{0.88 \times 1.5 + 0.88 \times 16.5}{\sum A_i}$$

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

$$\bar{y} = \frac{(0.88 \times 1.5) + (0.88 \times 16.5) + (1.667 \times 18 - \bar{y}) \times \left(\bar{y} + \frac{18 - \bar{y}}{2}\right)}{0.88 \times \cancel{1.5} + 0.88 \times \cancel{16.5} + 1.667 \times (18 - \bar{y})}$$

$$\bar{y} = \frac{15.84 + 1.667(18 - \bar{y}) \times (9 + \bar{y}/2)}{1.76 + 1.667(18 - \bar{y})}$$

$$\bar{y} = \frac{15.84 + 0.8335 \times (18 - \bar{y})(18 + \bar{y})}{1.76 + 1.667(18 - \bar{y})}$$

$$0.8335 \bar{y}^2 - 31.766 \bar{y} + 285.894 = 0$$

$$\therefore \bar{y} = \frac{31.766 \pm \sqrt{(31.766)^2 - 4(285.894)(0.8335)}}{2 \times 0.8335}$$

$$= \frac{31.766 \pm 7.4771}{1.667} = 14.67 \text{ in.}$$

$$I = \frac{1.667 \times 3.43^3}{3} + (0.88) \times 1.93^2 + 0.88 \times 13.17^2$$

$$I = 162.45 \text{ in}^4$$

$$\sigma = - \frac{M \times y}{I}$$

$$60 \text{ ksi} = - \frac{M \times 13.17}{162.45}$$

$$\therefore \underline{M} = 740.1 \text{ k-in}$$

$$\sigma_{\text{conc}} = - \frac{740.1 \times 3.43 \text{ in}}{162.45} \times n_{c \rightarrow s} = 2.17 \text{ ksi}$$

$$\sigma_{\text{steel}}^{\text{top}} = - \frac{740.1 \times 1.93}{162.45} = - 8.79 \text{ ksi}$$



Subject:

Job:

Job Number:

By:

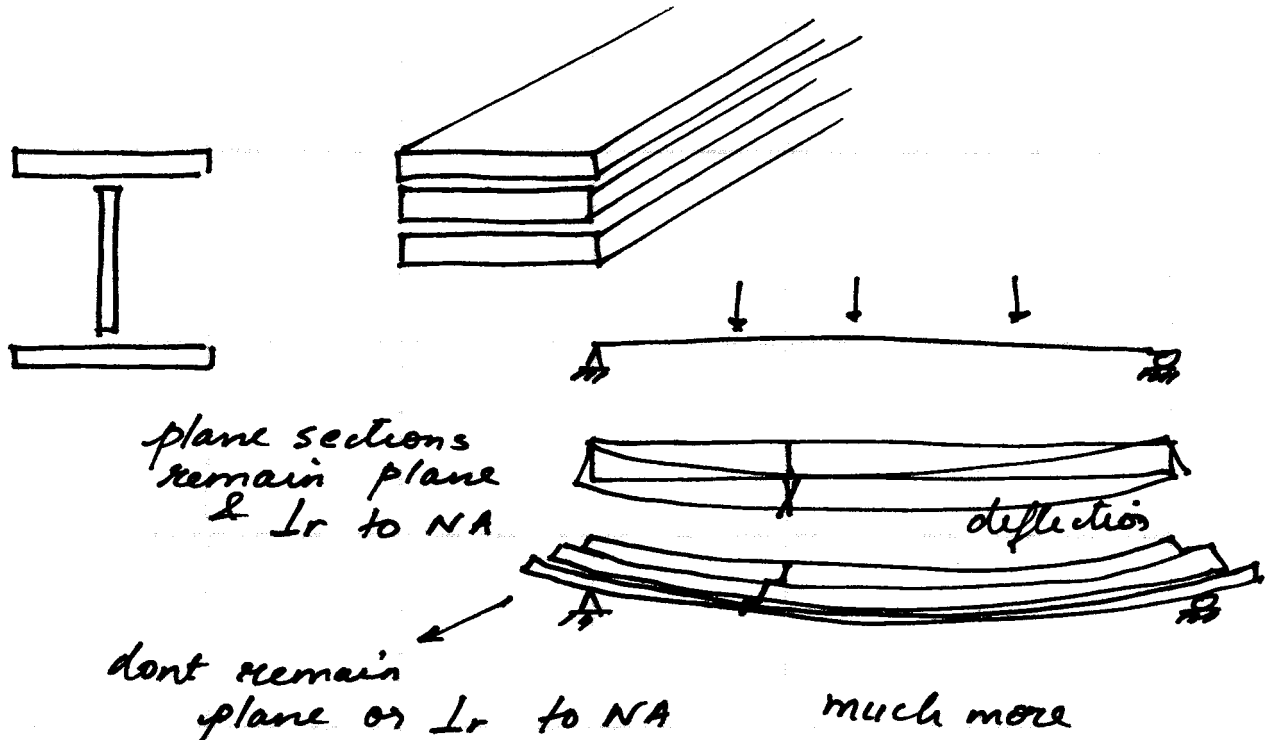
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Date:

Section:

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end slip $\rightarrow$ violated beam theory

full composite action

no composite action

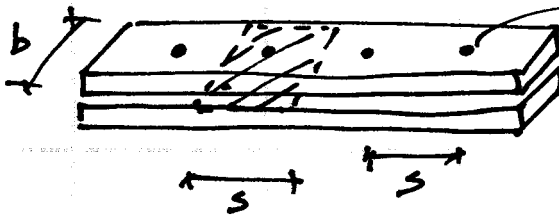
Shear flow $\rightarrow$ resisting the slip across the interface

- resistance to shear force $\rightarrow$ nails, bolts, rivets etc.
- resistance to shear flow $\rightarrow$ glue or weld
- resistance to shear stress $\rightarrow$ glue or adhesives



Subject: _____
 Job: _____

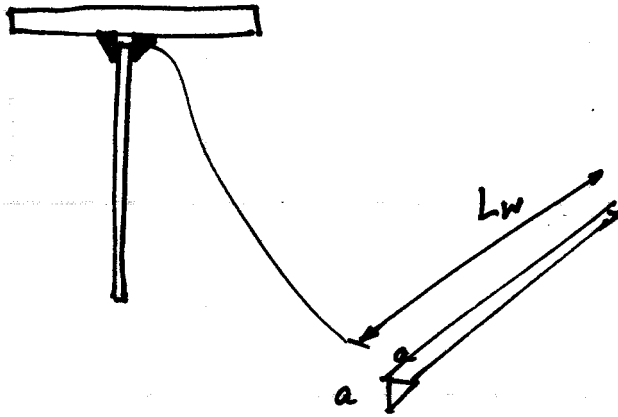
Job Number: _____ Date: 1/4/8
 By: _____ Section: _____
 Checked By: _____ Page _____ of _____



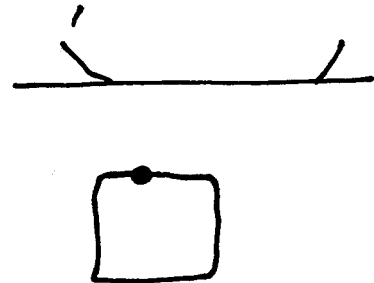
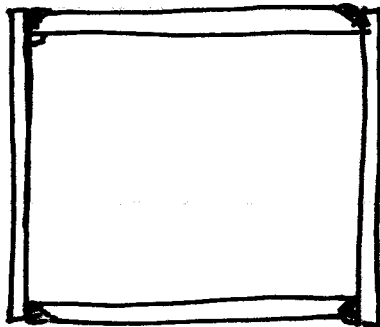
fastener → shear force resistance

Shear flow → shear force resistance
 spacing.

Shear stress resistance → shear flow resistance
 width or transverse spacing



$$0.707 \times a \times (F_u^N \times 0.6)$$





Subject: _____

Job: _____

Job Number: _____

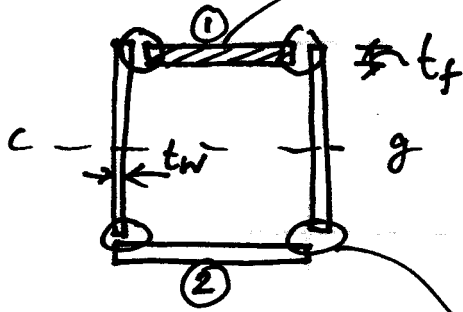
By: _____

Checked By: _____

Date: 1/4/9

Section: _____

Page _____ of _____



$$Z_{\text{demand}} = \frac{V}{I} \times \frac{Q}{t}$$

$Q = A_1 \bar{y}_1$

$t = t_f \times 2$

$$Z_{\text{demand}} = \frac{V}{I} \times \frac{Q}{t}$$

$Q = A_2 \bar{y}_2$

$$t = t_w \times 2$$

$$\begin{aligned} \tau_{\text{glue}}^{\text{all}} &= 400 \text{ lb/in}^2 \\ &= 400 \text{ psi} \end{aligned}$$

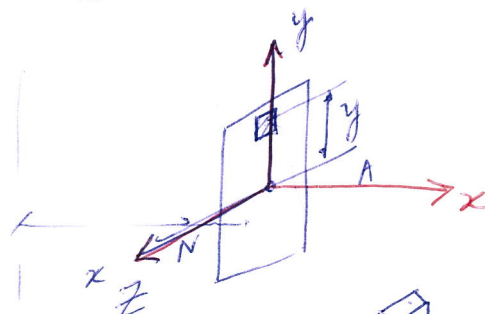
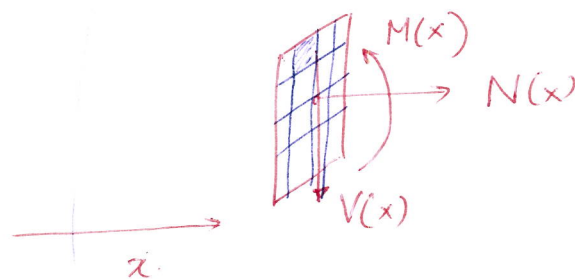
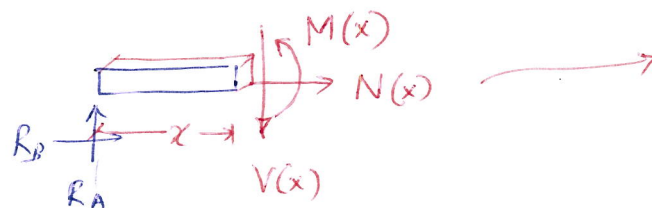
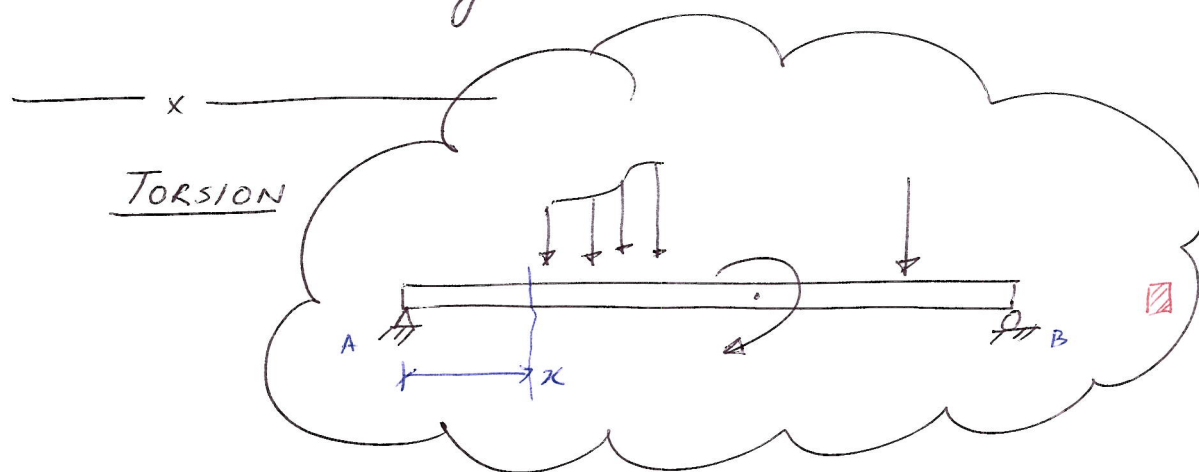
Topics for rest of the semester

✓① Torsion

✓② Elastic Deflections

③ Principal Stresses & Strains

④ Column Buckling



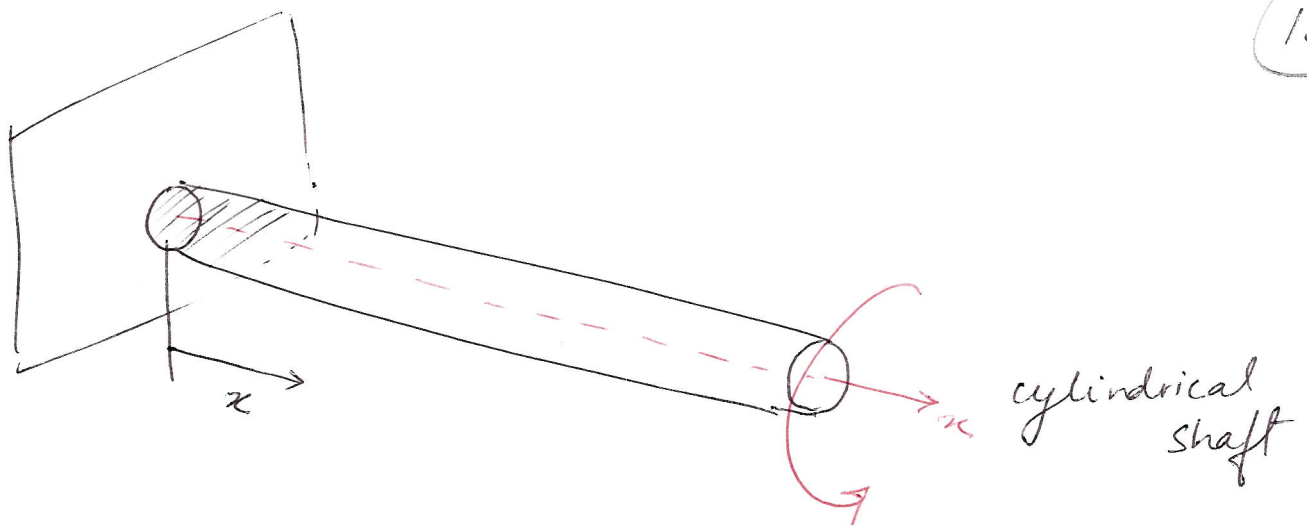
$$\sigma = \frac{N(x)}{A}$$

$$\sigma = -\frac{M(x) \times y}{I_z}$$

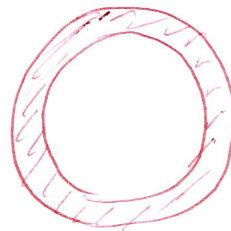
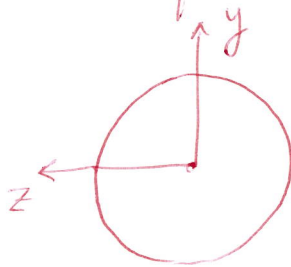
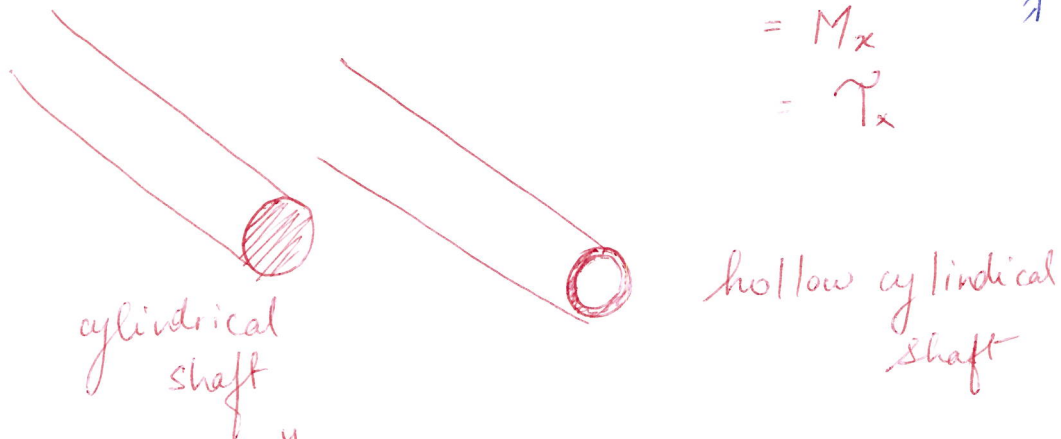
$$\tau_{xy} = \frac{V(x)}{I_z} \times \frac{Q}{t}$$

$$Q = A \times \bar{y}$$

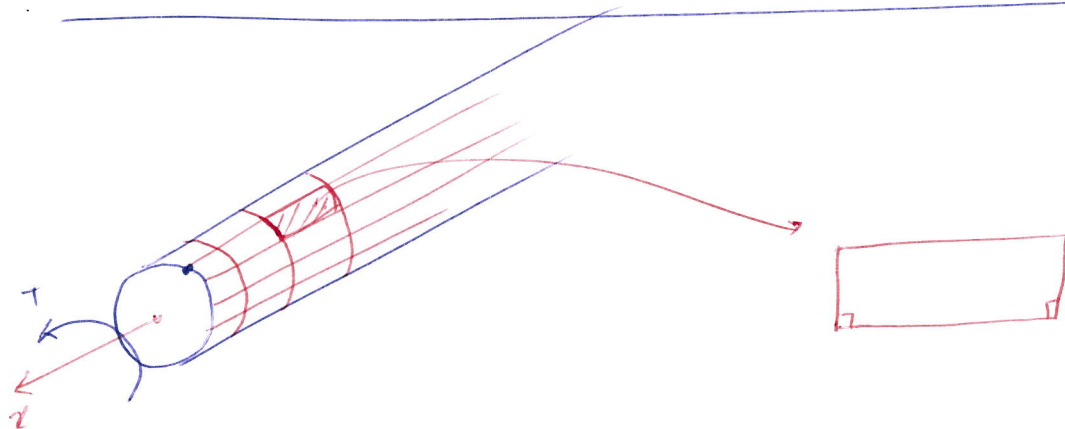
$$\tau_{yx}$$

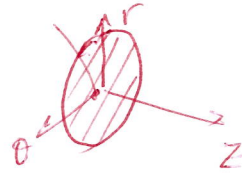
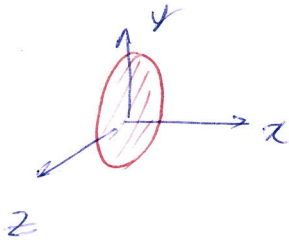


$$\begin{aligned} \text{Torque} &= M_x \\ &= T_x \end{aligned}$$

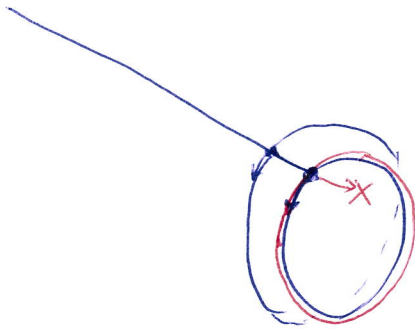


What happens to the structural member

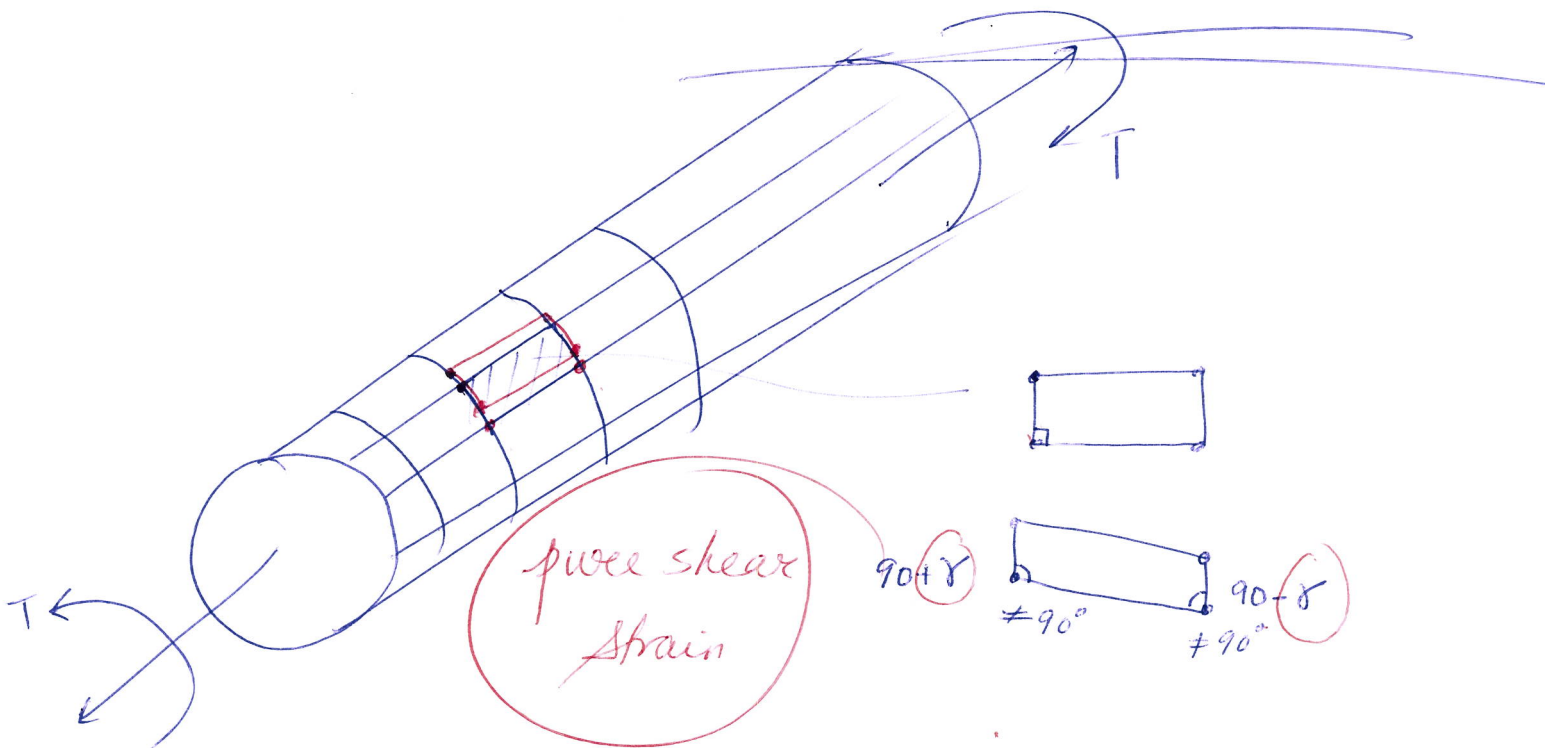




True only for circular sections.



no warping deformation
in the out-of-plane
direction



$$\gamma \longrightarrow \tau$$

Shear strain Shear stress

$$\tau = G \cdot \gamma$$

↓
Shear modulus

$$G = \frac{E}{2(1+\nu)}$$

→ Poisson's ratio

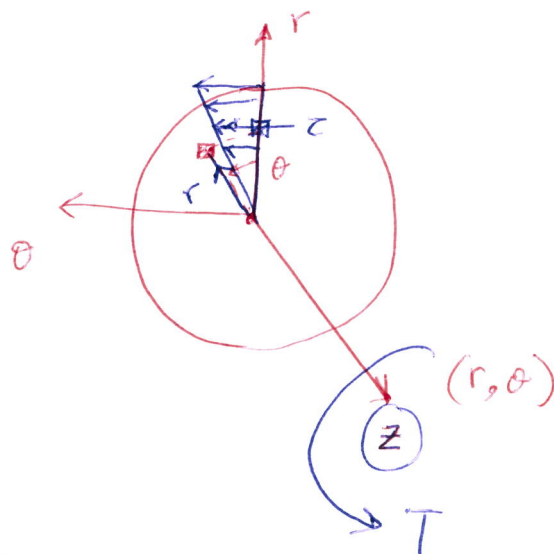
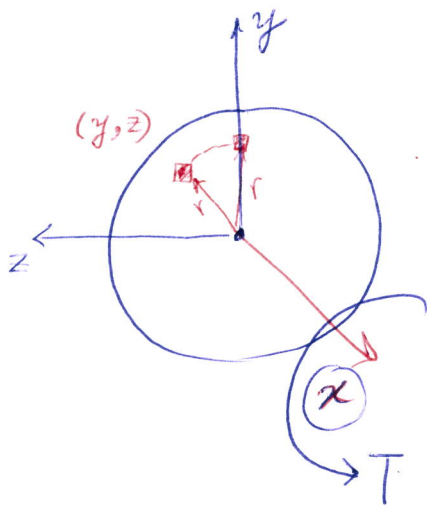
0.3 steel

0.15 concrete

$$\epsilon \longrightarrow \sigma$$

$$\sigma = E \cdot \epsilon$$

↓
Hooke's law



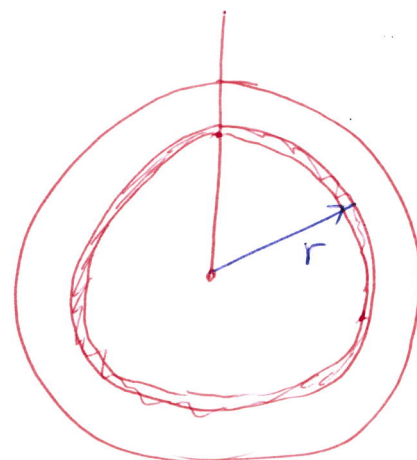
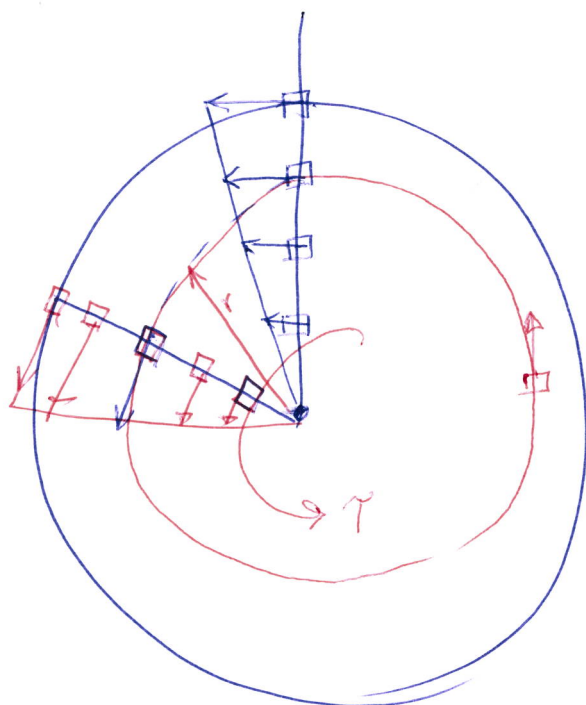
Shear stress due to torsion:

→ variation along length

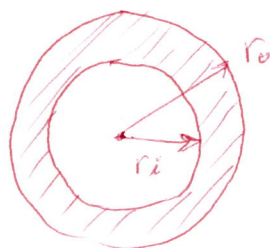
$$\tau = \frac{\tau \times r}{J} \rightarrow \text{variation through section}$$

→ polar moment of inertia

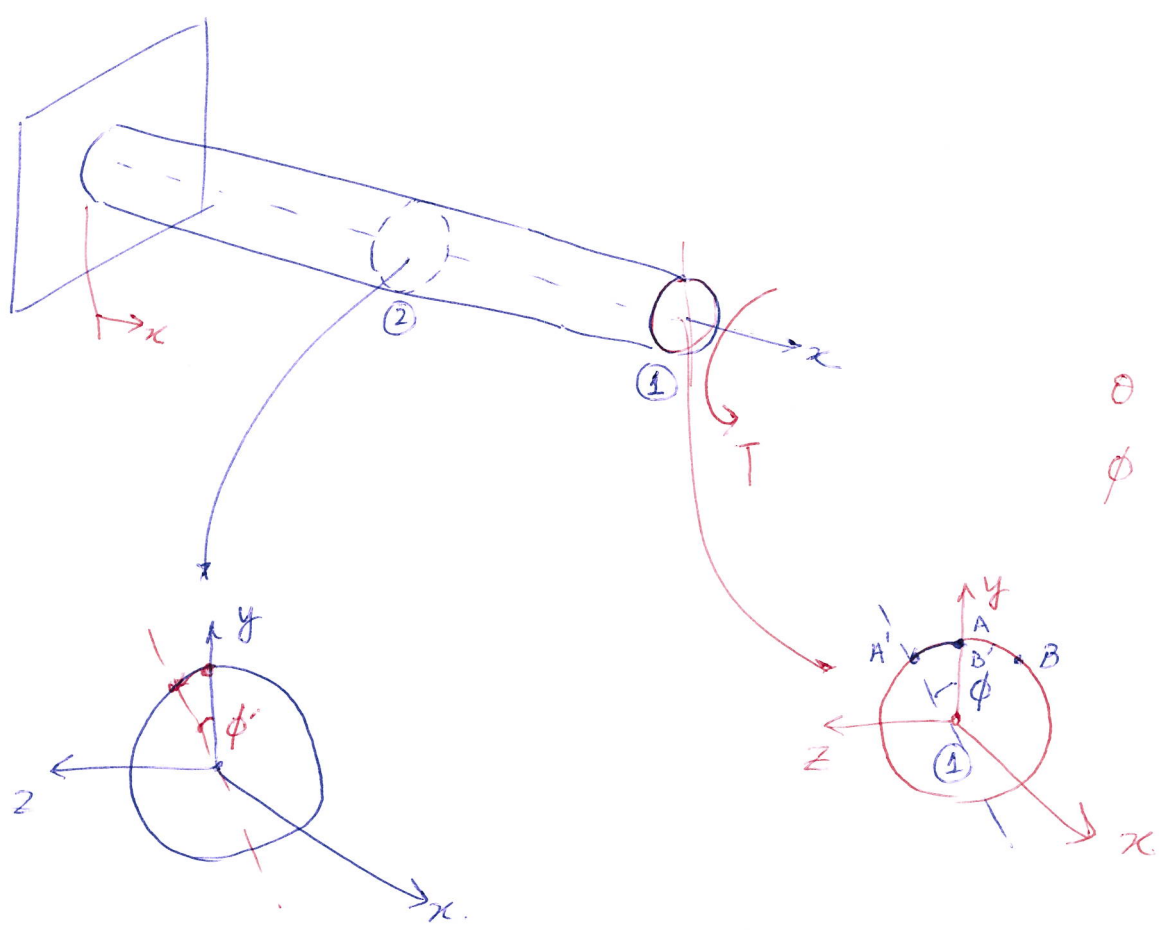
$$J = \frac{\pi}{2} \times r^4 \quad \left| \text{solid} \right.$$



$$\tau = \frac{\tau \times r}{J}$$



$$J = \frac{\pi}{2} \times (r_o^4 - r_i^4) \quad \left| \text{tube} \right.$$

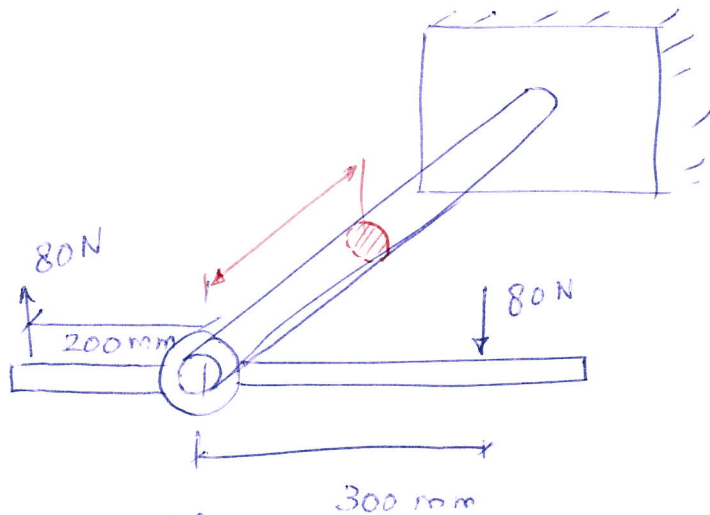


$$\phi \rightarrow f(x)$$

motion & deformation

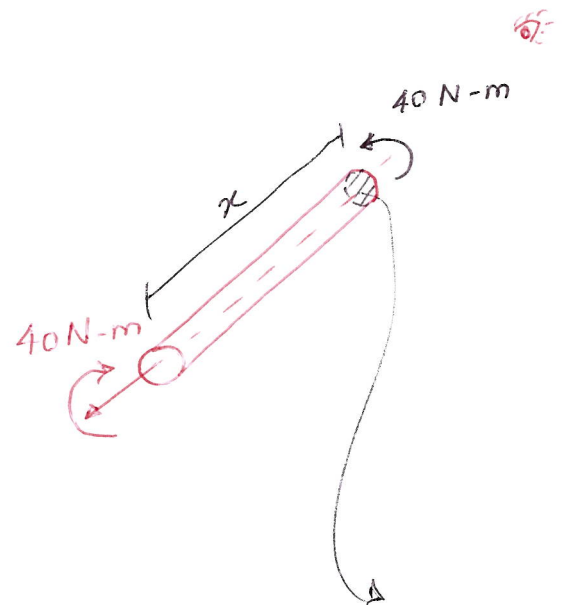
Example:

Pipe $\rightarrow$ inner diameter = 80 mm
outer diameter = 100 mm.



$$\tau = 80 \times 500 \text{ mm} = 40000 \text{ N-mm}$$

$$\tau = 40 \text{ N-m}$$



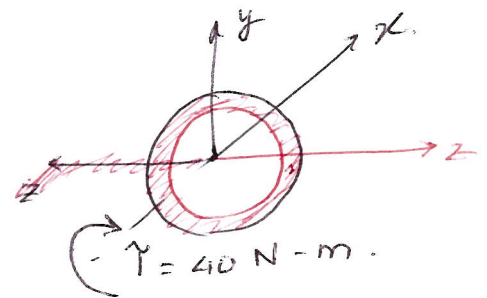
Polar moment of inertia

$$J = \frac{\pi}{2} \times [r_o^4 - r_i^4]$$

$$= \frac{\pi}{2} \times [50^4 - 40^4] \text{ mm}^4$$

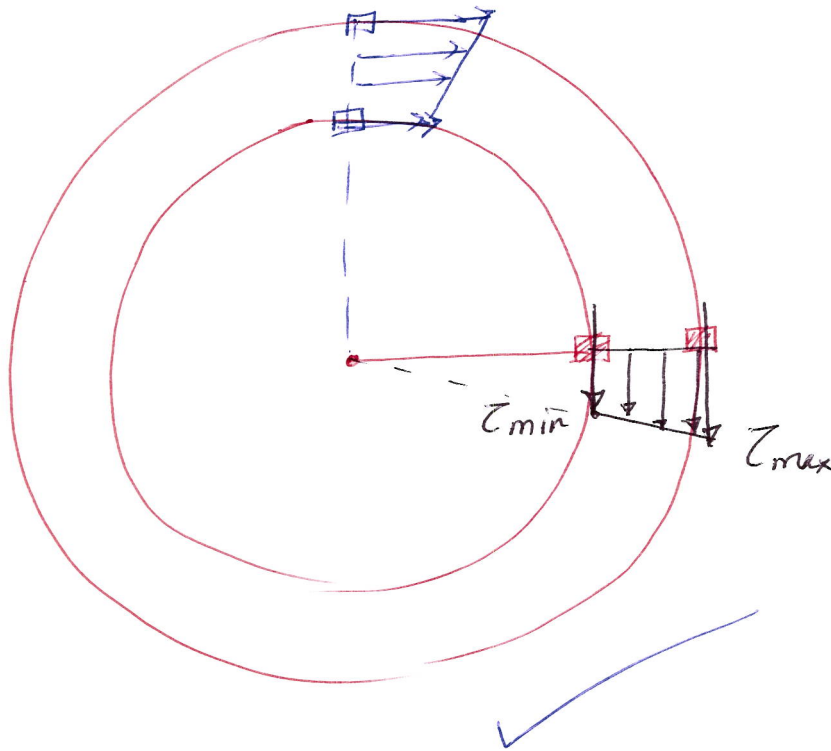
transformed

$$= 5.79 \times 10^{-6} \text{ m}^4$$



$$\tau_{\max} = \frac{\tau \times r_o}{J} = \frac{40 \text{ N-m} \times (0.05 \text{ m})}{5.79 \times 10^{-6} \text{ m}^4} = 0.345 \text{ N/mm}^2 = 0.345 \text{ MPa}$$

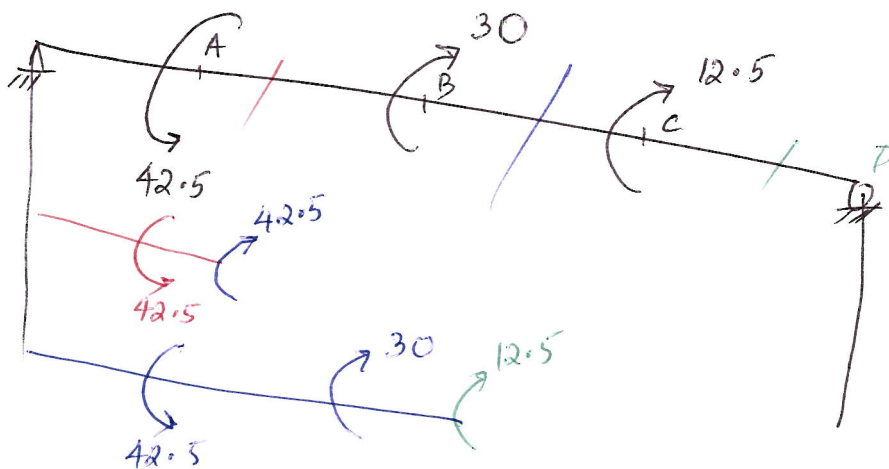
$$\tau_{\min} = \frac{\tau \times r_i}{J} = \frac{40 \text{ N-m} \times 0.04 \text{ m}}{5.79 \times 10^{-6} \text{ m}^4}$$



—————x—————

Variation of τ along member length

$$\tau = \frac{T \times r}{J}$$



CE 270: Intro Structural Mechanics

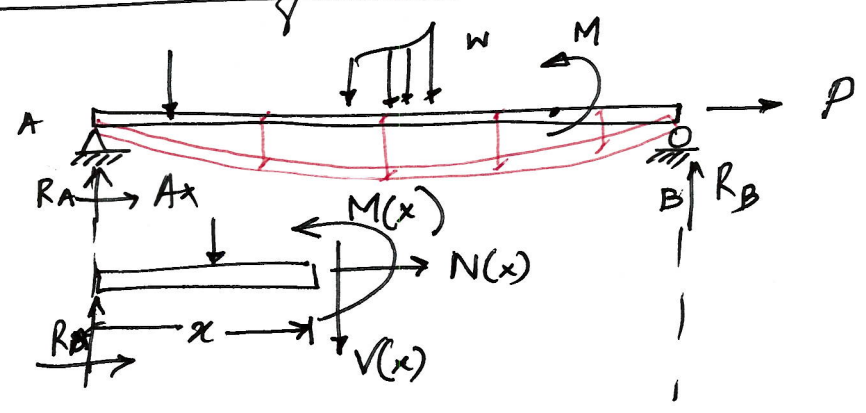
Why calculate deflections?

- to meet the requirements of IBC for design
- required max. deflection depend on finished surface
- plastered surfaces - $L/360$ where $L \rightarrow$ span longest in in.
- unplastered surfaces - $L/240$
- to prevent cracks in finished surfaces
- Serviceability requirements for design

$$F \rightarrow \Delta \quad F \propto \Delta \quad \checkmark$$
$$F = k \cdot \Delta$$

Serviceability load combination
↳ Nominal

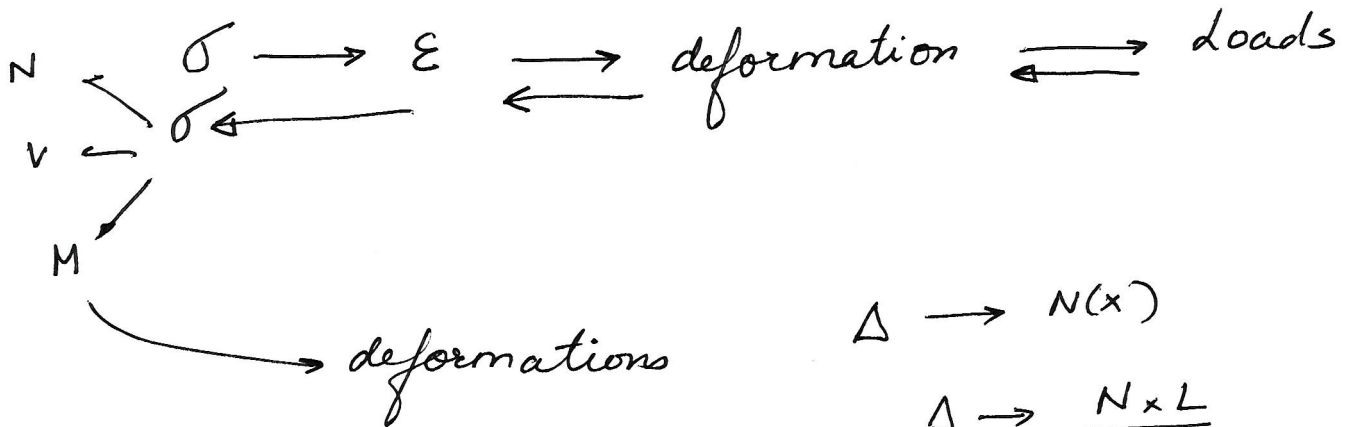
How do I calculate deflections?



$$N(x) \rightarrow \text{Axial Force} \quad \sigma = \frac{N(x)}{A}$$

$$V(x) \rightarrow \text{Shear Force} \quad \tau = \frac{V(x)}{I} \times \left(\frac{Q}{t}\right)$$

$$M(x) \rightarrow \text{Bending Moment} \quad \sigma = -\frac{M(x)}{I} \times y$$



$$\Delta \rightarrow N(x)$$

$$\Delta \rightarrow \frac{N \times L}{AE}$$

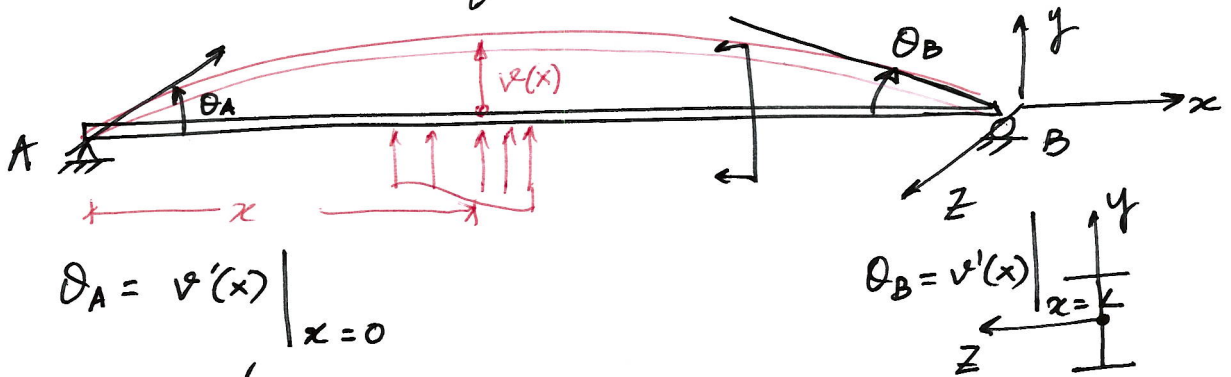
deflection due to $M(x)$ overpowers everything else.

flexure deformation
will dominate as long as

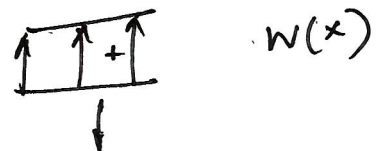
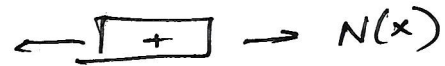
$$\frac{L}{d} \geq 10$$

◎ Focus on flexural deformations only! ✓

How do I calculate deflection due to $M(x)$



Sign convention

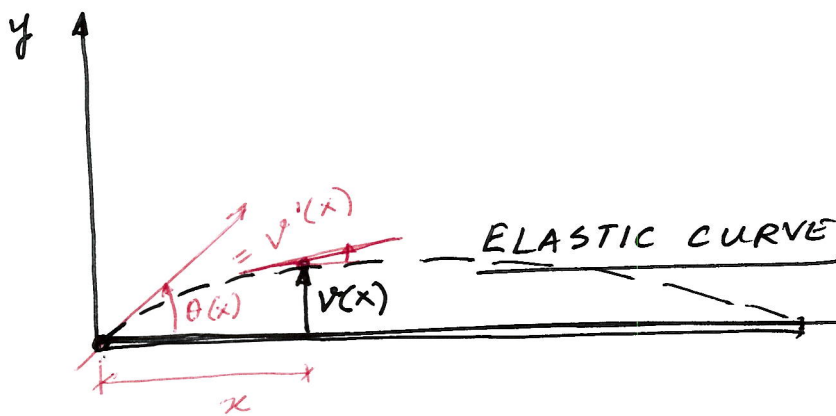


$v(x)$ → deflection
+ve upward

$$\theta(x) = v'(x)$$

→ rotations
along the
length

$$\phi(x) = \theta'(x) = \text{curvature along the length}$$



rotations θ → slope of $v(x)$

$v(x) \longrightarrow$ deflection

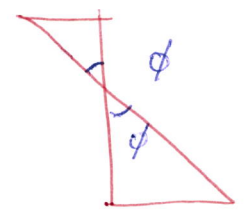
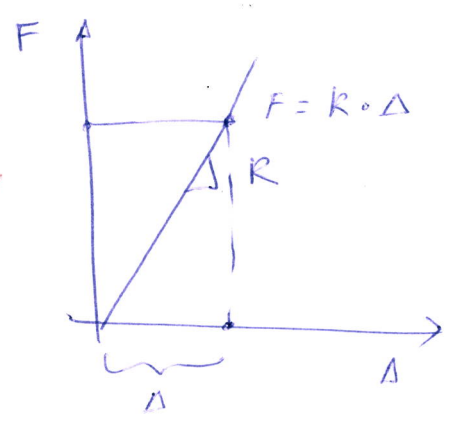
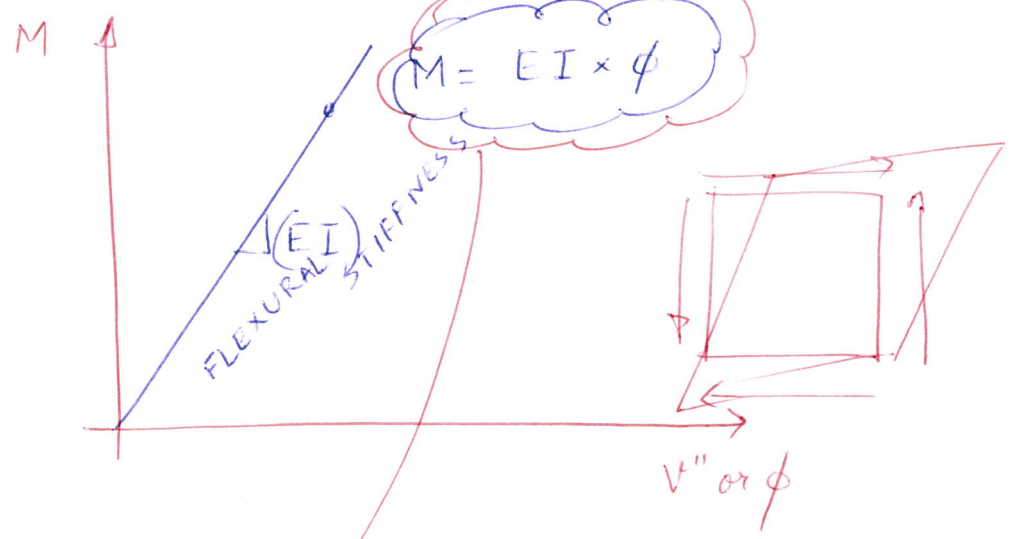
$\dot{v}(x) \longrightarrow$ rotation

$v''(x)$ $\longrightarrow$ curvature $= \frac{M(x)}{EI}$

$$v''(x) = \frac{M(x)}{EI}$$

differential equation of a beam

$$M(x) = v''(x) \times EI$$



Equation from beam theory
plane sections remain plane. E

Support conditions



$$v = 0$$

$$M = 0 \rightarrow \cancel{EI} \phi = 0$$

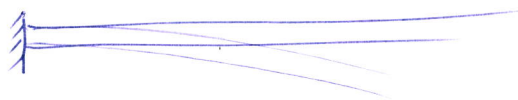
$$v'' = 0$$



$$v = 0$$

$$v'' = 0$$

$$\frac{dM}{dx} = V$$



$$v = 0$$

$$v' = 0$$

$$M = EI v''$$

$$V = EI v'''$$

$$\frac{dV}{dx} = w$$

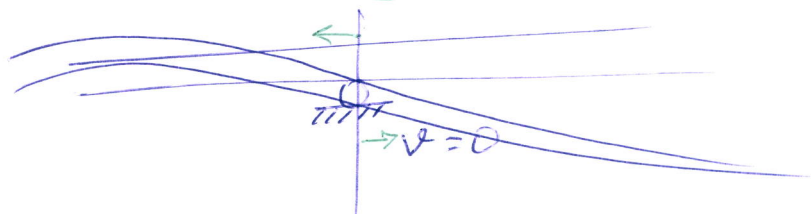
$$w = EI v^{IV}$$

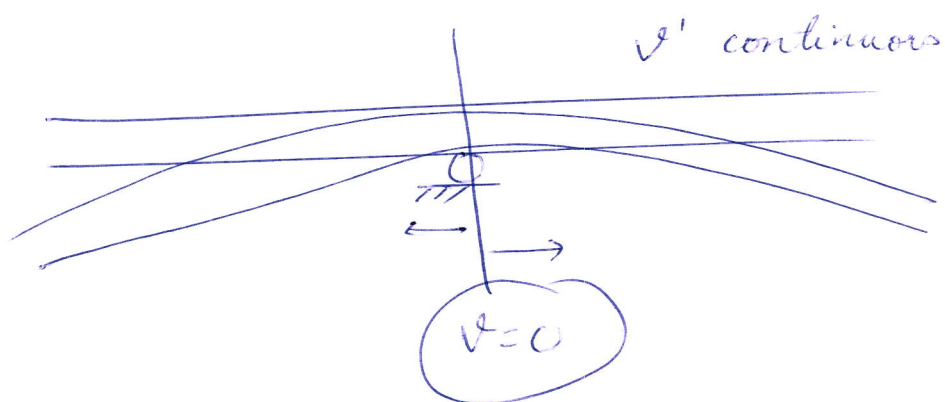
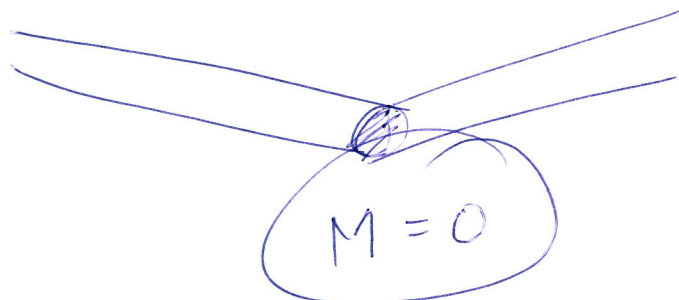


$$v'' = 0 \quad M = 0$$

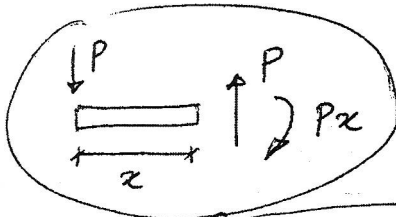
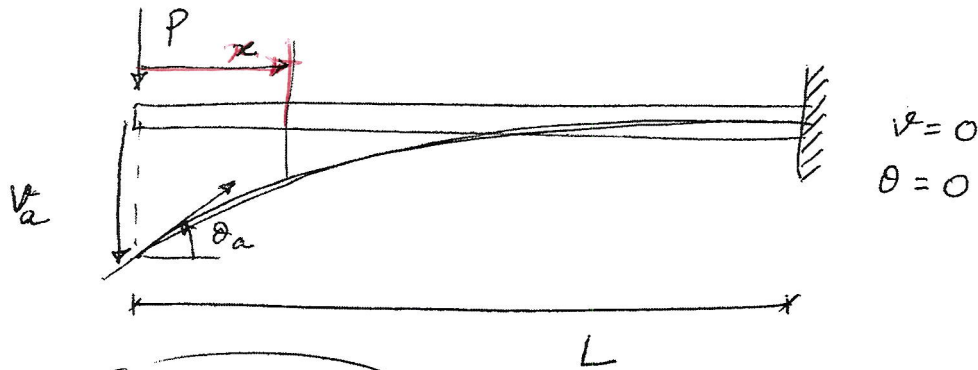
$$v''' = 0 \quad V = 0$$

$v' \rightarrow$ continuous



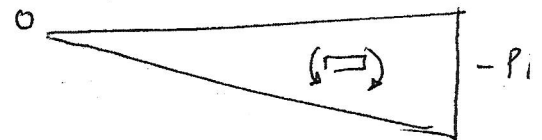
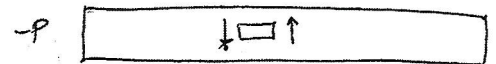


$$EI \vartheta'' = M(x)$$



$$\therefore M(x) = -Px$$

$$EI \times \frac{d^2 v}{dx^2} = -Px$$



Obj $\rightarrow v(x)$ then sub $x = -$
 $v(x = \text{---})$

$$EI \frac{d^2 v}{dx^2} = -Px$$

$$\therefore EI \frac{dv}{dx} = -P \frac{x^2}{2} + C_1$$

$$EI v(x) = -P \frac{x^3}{6} + C_1 x + C_2$$

$$\therefore v(x) = -\frac{P}{EI} \times \frac{x^3}{6} + \frac{C_1}{EI} x + \frac{C_2}{EI}$$

Use the boundary conditions to find C_1 & C_2

$$v(x) = -\frac{Px^3}{EI \times 6} + \frac{C_1}{EI}x + \frac{C_2}{EI}$$

$$(a) \quad x=L \quad v(L) = 0$$

$$\theta(L) = 0$$

$$\therefore \cancel{v(L)} = \theta(x) = \frac{dv}{dx} = -\frac{Px^2}{2EI} + \frac{C_1}{EI}$$

$$\theta(x=L) = -\frac{PL^2}{2EI} + \frac{C_1}{EI} = 0$$

$$C_1 = PL^2/2$$

$$v(x=L) = -\frac{PxL^3}{6EI} + \left(\frac{PL^2}{2EI}\right) \times L + \frac{C_2}{EI} = 0$$

$$= \frac{-PL^3 + 3PL^3}{6EI} + \frac{C_2}{EI} = 0$$

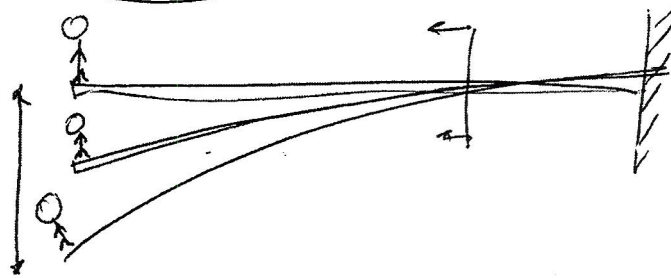
$$\therefore \frac{2PL^3}{3EI} + \frac{C_2}{EI} = 0$$

$$C_2 = -\frac{PL^3}{3}$$

$$\therefore \boxed{v(x) = -\frac{Px^3}{6EI} + \frac{PL^2}{2EI}x - \frac{PL^3}{3EI}}$$

(a) $x = 0$

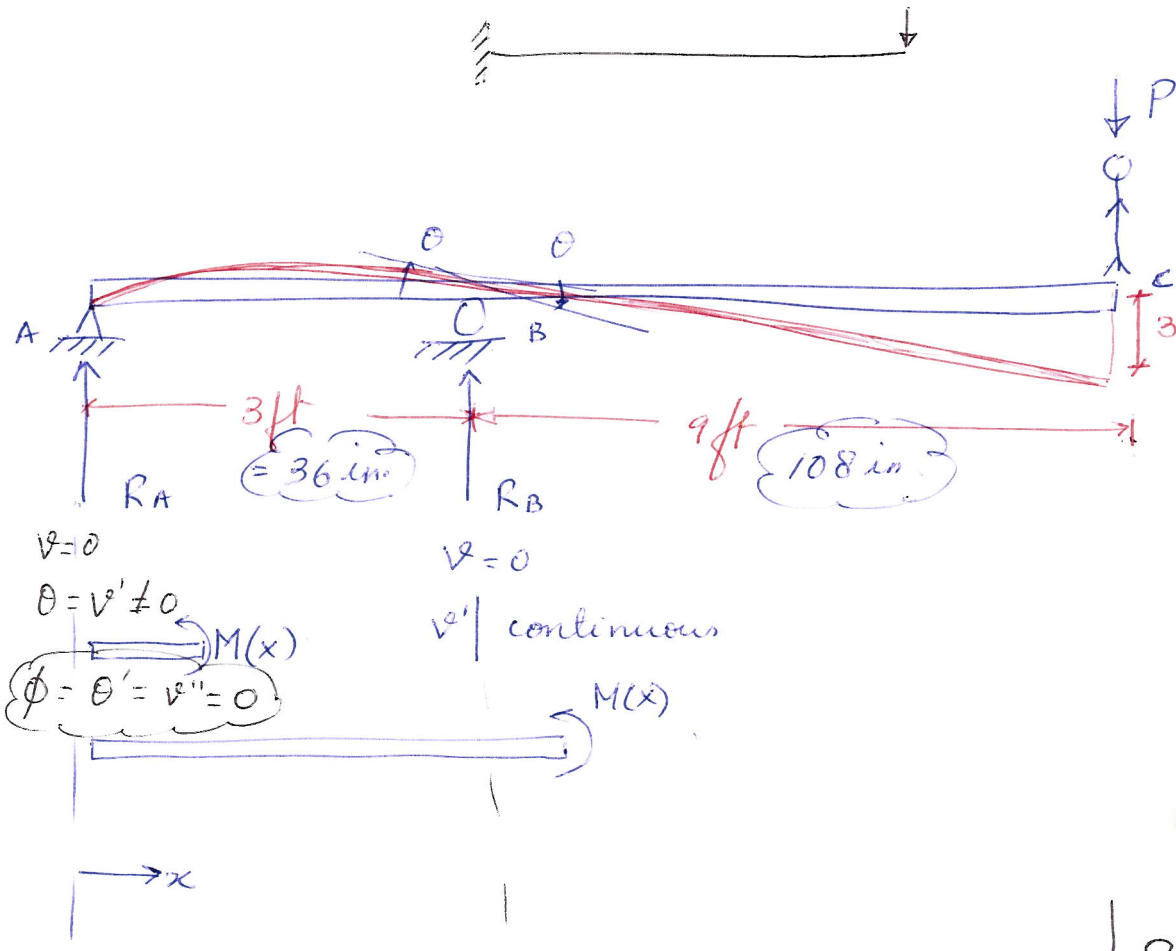
$$v(x=0) = -\frac{PL^3}{3EI}$$



$$I = \frac{bh^3}{12}$$

Serviceability $\rightarrow$ deflections

$$\& \quad \frac{M}{EI} = \frac{d^2v}{dx^2}$$



$E = 1500 \text{ ksi}$

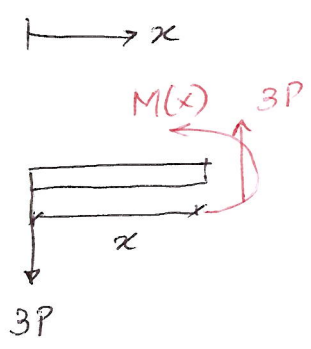
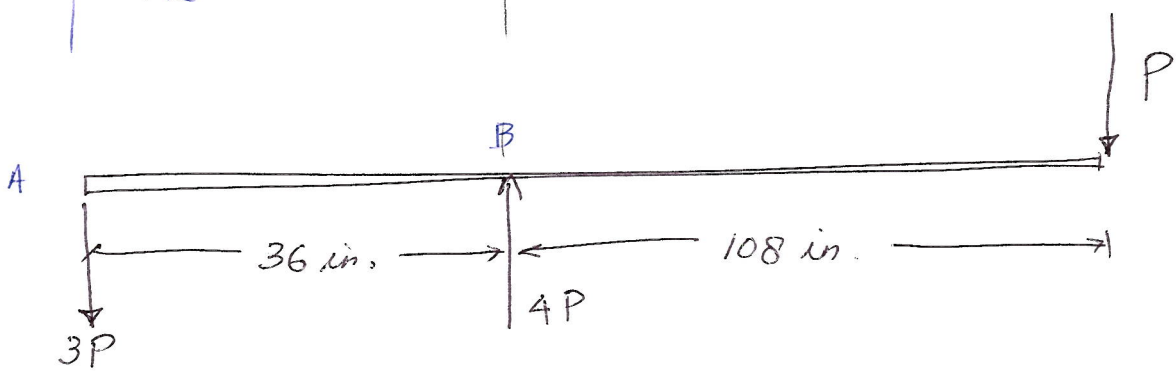
18×2

$I = 18 \times \frac{2^3}{12} \text{ in}^4$

$I = 12 \text{ in}^4$

$EI = 1500 \times 12$

$= 18000 \text{ (k-in}^2\text{)}$



$M(x) + 3Px = 0$

$\therefore M(x) = -3Px$

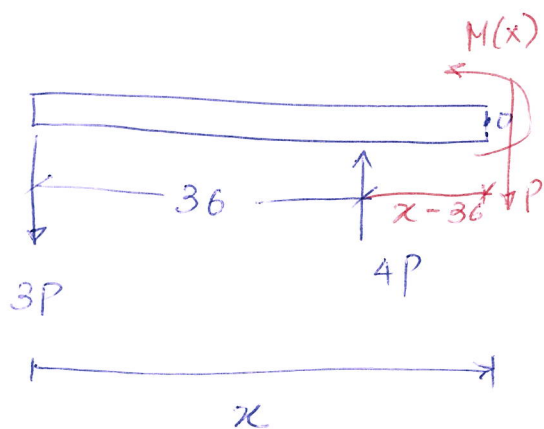
$\forall 0 \leq x \leq 36$

$EI \frac{d^2 v}{dx^2} = M(x)$

$\therefore v(x) = -\frac{3P}{EI} \frac{x^3}{6} + C_1 x + C_2$

$v'(x) = -\frac{3P}{EI} \times \frac{3x^2}{6} + C_1$

$v''(x) = -\frac{3P}{EI} \times \frac{6x}{6}$



$$(+ \sum M_0 = 0$$

$$\therefore M(x) - 4P(x-36) + 3Px = 0$$

$$\therefore M(x) = Px - 144P$$

$$= P(x - 144)$$

$$\forall 36 < x \leq 144$$

$$EI \frac{d^2 v}{dx^2} = M(x) = Px - 144P$$

$$v(x) = \frac{P}{EI} \left(\frac{x^3}{6} - 144 \frac{x^2}{2} \right) + C_3 x + C_4$$

_____ x _____ x _____ x _____

Calculate constants C_1, C_2, C_3, C_4

$$v(0) = 0 \Rightarrow C_2 = 0$$

$$v''(0) = 0 \Rightarrow \text{no constants evaluated}$$

$$v(36) = 0 \Rightarrow -\frac{3P}{EI} \times \frac{36^2}{6} + C_1 \times 36 = 0$$

$$C_1 = 648 \frac{P}{EI}$$

$$\forall 0 \leq x \leq 36 \quad v(x) = -\frac{3P}{EI} \times \frac{x^3}{6} + 648 \frac{P}{EI} \times x + 0$$

_____ x _____ x _____

(169)

$$\forall 36 \leq x \leq 144 \quad v(x) = \frac{P}{EI} \left(\frac{x^3}{6} - 144 \frac{x^2}{2} \right) + C_3 x + C_4$$

$$@ x = 36 \rightarrow v = 0$$

$$v(36) = -85536 \frac{P}{EI} + C_3 \cdot 36 + C_4 = 0$$

$$\text{no } C_3 \text{ or } C_4 \quad \therefore C_4 = 85536 \frac{P}{EI} - 36 C_3$$

$$v'(36) \text{ from this equation} = v'(36) \text{ from previous equation}$$

$$v'(36) \Big|_{\text{this Eq.}} = \frac{P}{EI} \left(\frac{3x^2}{6} - 144x \right) + C_3$$

$$v'(36) \Big|_{\text{prev. eq.}} = -\frac{3P}{EI} \left(\frac{3x^2}{6} \right) + 648 \frac{P}{EI}$$

Equal @
 $x = 36$

$$\therefore \frac{P}{EI} \left(\frac{3 \times 36^2}{6} - 144 \times 36 \right) + C_3 = -\frac{3P}{EI} \left(\frac{3 \times 36^2}{6} \right) + 648 \frac{P}{EI}$$

$$\therefore C_3 = 3240 \frac{P}{EI}$$

$$C_4 = -31,104 \frac{P}{EI}$$

$$\forall 36 \leq x \leq 144$$

$$\therefore v(x) \rightarrow \frac{P}{EI} \left(\frac{x^3}{6} - \frac{144x^2}{2} \right) + 3240 \frac{P}{EI} x - 31,104 \frac{P}{EI}$$

$$\therefore v(x) @ x = 144 \text{ in.}$$

$$v(x) = \frac{P}{EI} (-995328) + \frac{P}{EI} (466560) - \frac{P}{EI} \times 31104$$

$$v(144) = \frac{P}{EI} \times (-559872)$$

$$\text{But, } EI = 1500 \text{ ksi} \times 12 \text{ in}^4 = 18000 \text{ k-in}^2$$

$$\therefore v(144) = \frac{P}{18000} \times (-559872) \text{ in.}$$

$$v(144) = -31.1 P \times \text{in.}$$



$$= -3.5 \text{ in.}$$

$$\therefore -31.1 P = -3.5 \text{ in.}$$

$$\therefore P = \frac{3.5}{31.1}$$

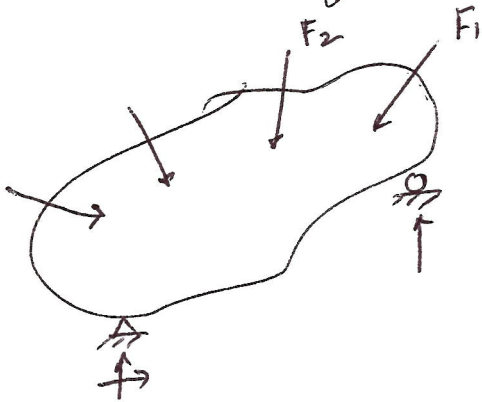
$$P = 0.1125 \text{ kips}$$

$$\therefore P = 112.5 \text{ lbs.}$$



CE 270 → last topic

internal forces produced by loads.



$$\sum \vec{F} = m \times \vec{a}$$

↓
calculate reactions! ✓

Loads $\rightarrow$ Deformations $\rightarrow$ Strains $\rightarrow$ Stresses

w, P δ, Δ ϵ, γ σ, τ

Internal Forces
 N, V, M, τ

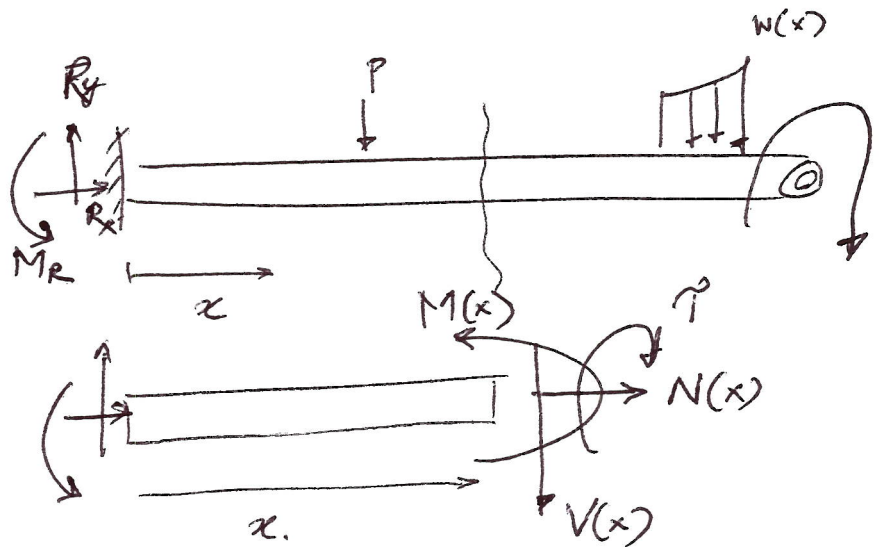
_____ x _____ x _____

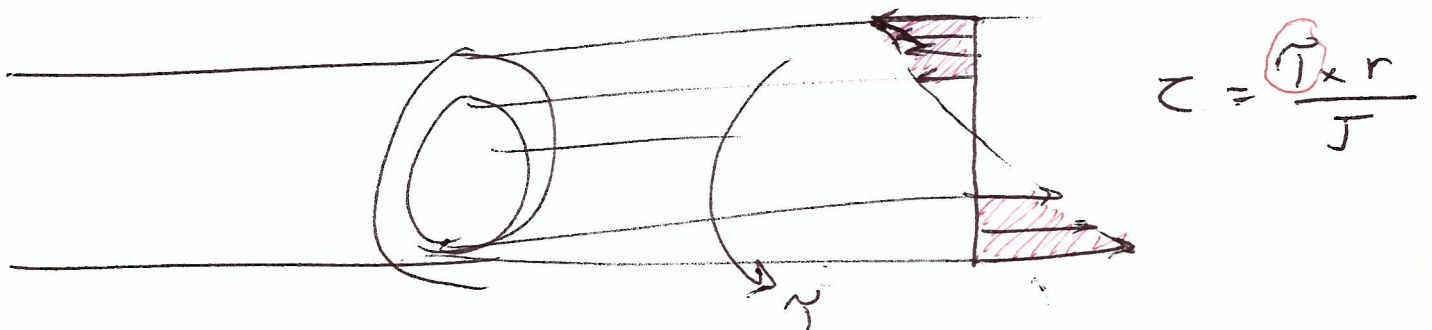
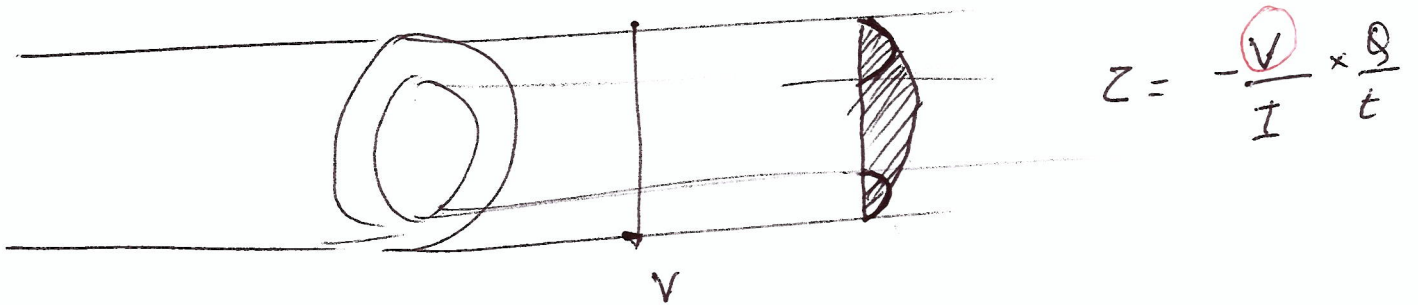
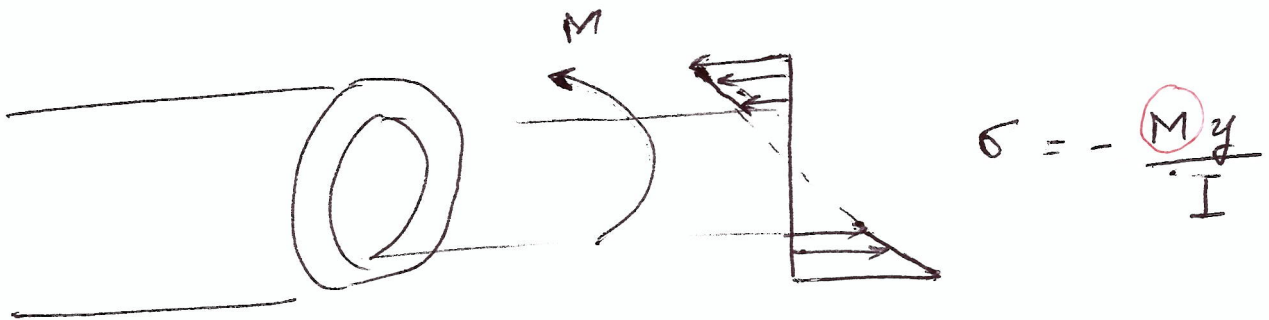
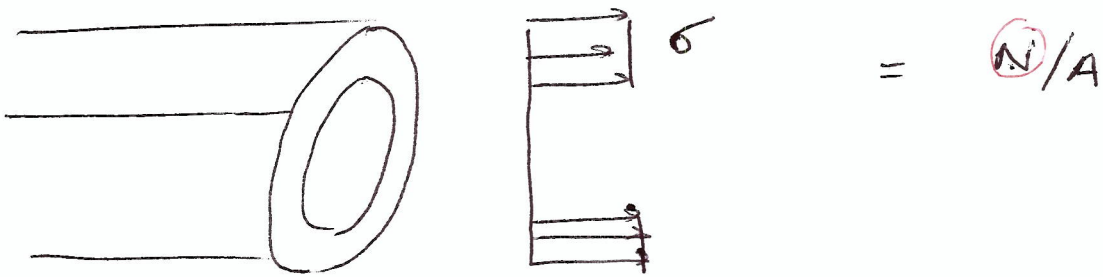
$$\sigma = \frac{N}{A} \quad \checkmark$$

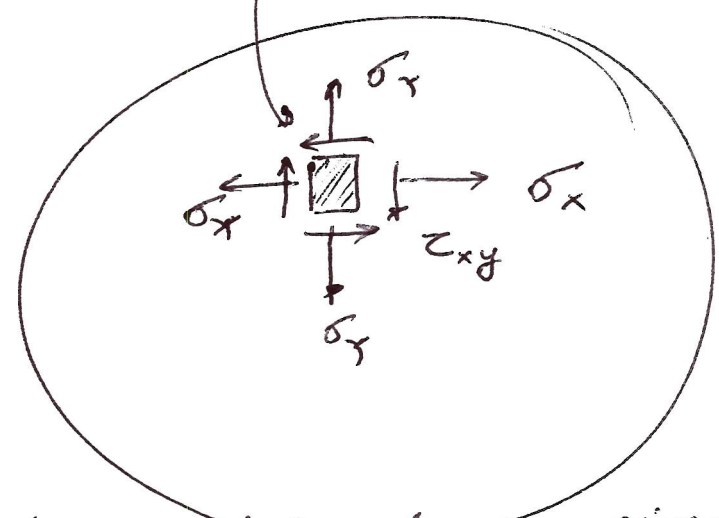
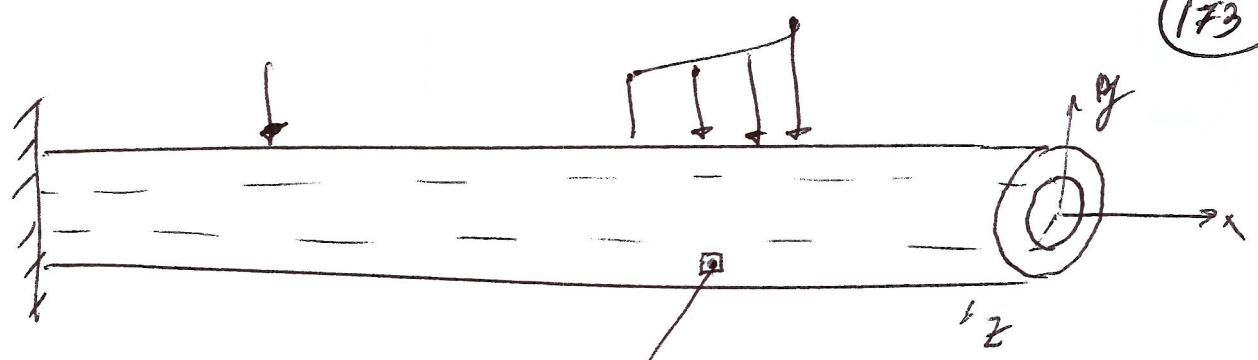
$$\sigma = - \frac{M \times y}{I} \quad \checkmark$$

$$\rightarrow \tau = \frac{V}{I} \times \frac{Q}{t} \quad \checkmark$$

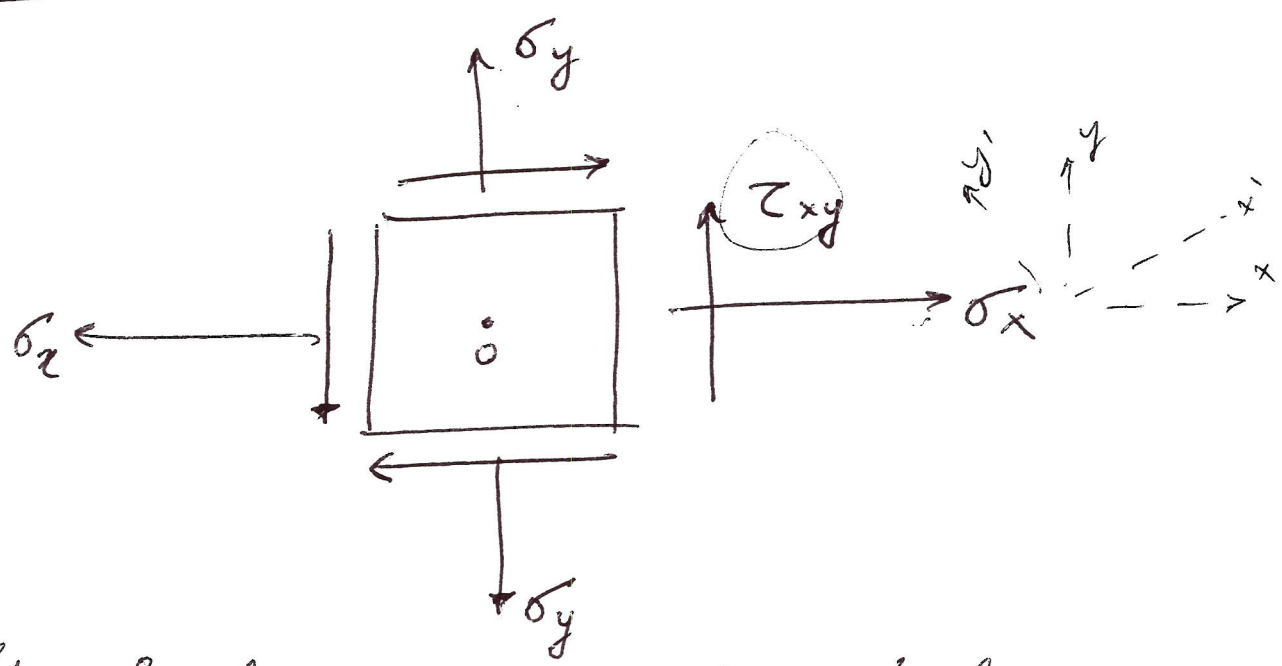
$$\tau = \frac{\gamma \times r}{J} \quad \checkmark$$





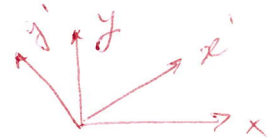
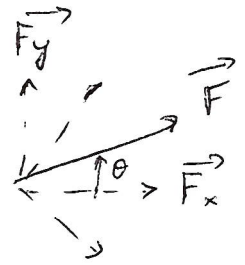
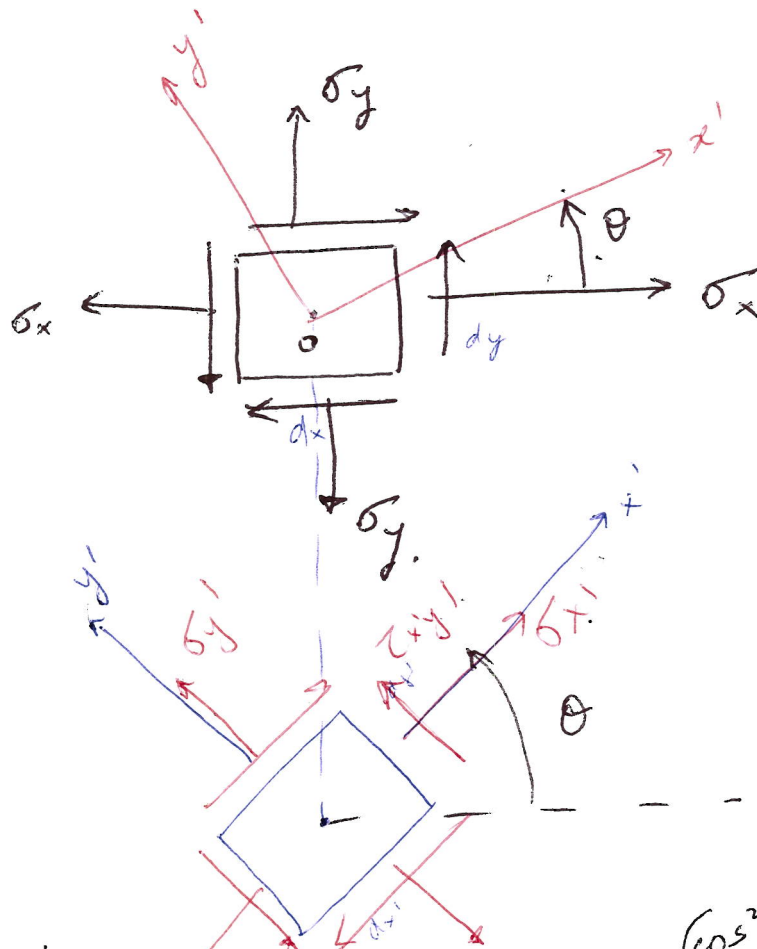


Combined stress state at any given point in the body



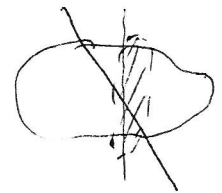
Strength of material = 50 ksi steel
5000 psi concrete

Need to know something important about stresses.



$$\sigma = \frac{F}{A}$$

$$\tau = F/A$$



$$(\cos^2 \theta - \sin^2 \theta)$$

$$2 \sin \theta \cos \theta$$

Stress Transformation

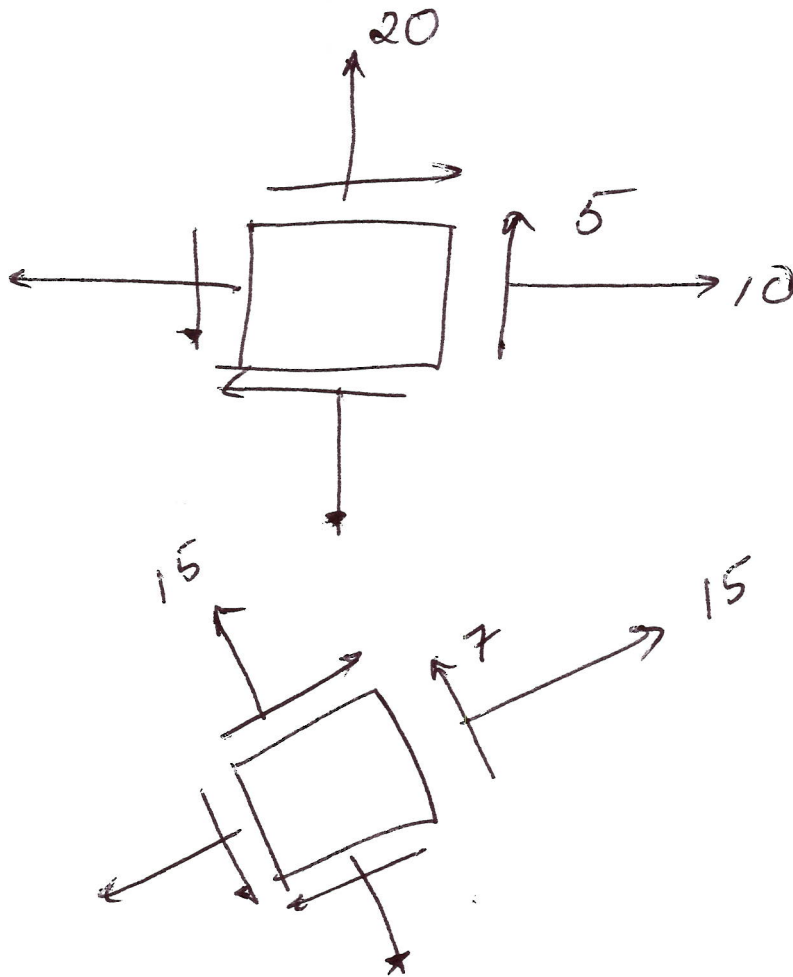
$$\sigma_{x'} = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \left(\frac{\sigma_x + \sigma_y}{2} \right) - \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = - \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

When $\tau_{x'y'} = 0$

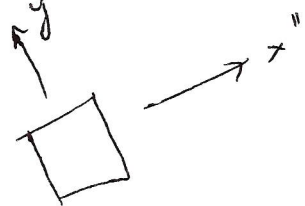
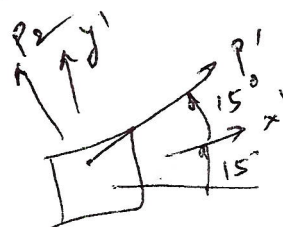
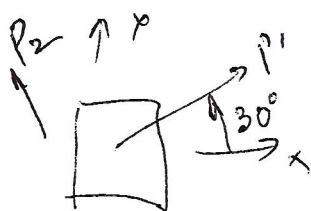
$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$



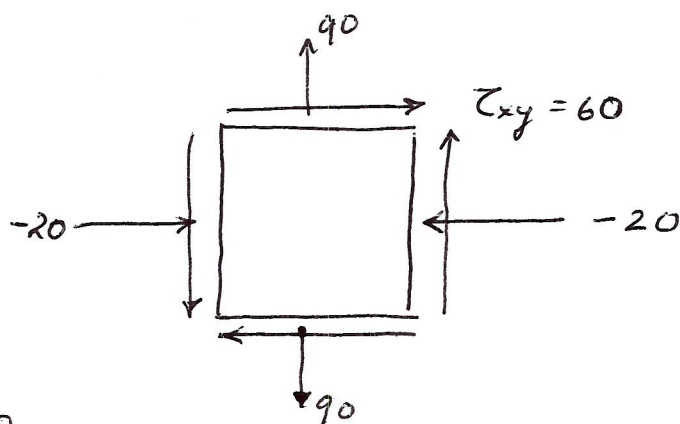
There is unique stress @ θ

that is obtained when $\tau_{x'y'} = 0$

$$\sigma_{p_{1,2}} = \left(\frac{\sigma_x + \sigma_y}{2} \right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Example ①

$$\sigma_x = -20$$

$$\sigma_y = +90$$

$$\tau_{xy} = +60$$

} Calculate the principal stresses & orientation

consistent units.

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} = \begin{bmatrix} -20 & 60 \\ 60 & 90 \end{bmatrix} \text{ matrix}$$

The principal stresses

$$\sigma_{p1, p2} = \left(\frac{\sigma_x + \sigma_y}{2} \right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + (\tau_{xy})^2}$$

$$= \frac{-20 + 90}{2} \pm \sqrt{\left(\frac{-20 - 90}{2} \right)^2 + (60)^2}$$

$$= 35 \pm 81.4$$

$$\sigma_{p1, 2} = \underline{\underline{116 \text{ MPa}}} \quad \text{or} \quad \underline{\underline{-46.4 \text{ MPa}}}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{120}{-20 - 90}$$

$$2\theta_p = -47.5^\circ$$

$$\therefore \theta_p = -23.7^\circ$$

$$\text{or} \quad -47.5^\circ + 180^\circ$$

$$\text{or} \quad 66.3^\circ \rightarrow 90^\circ \text{ apart}$$

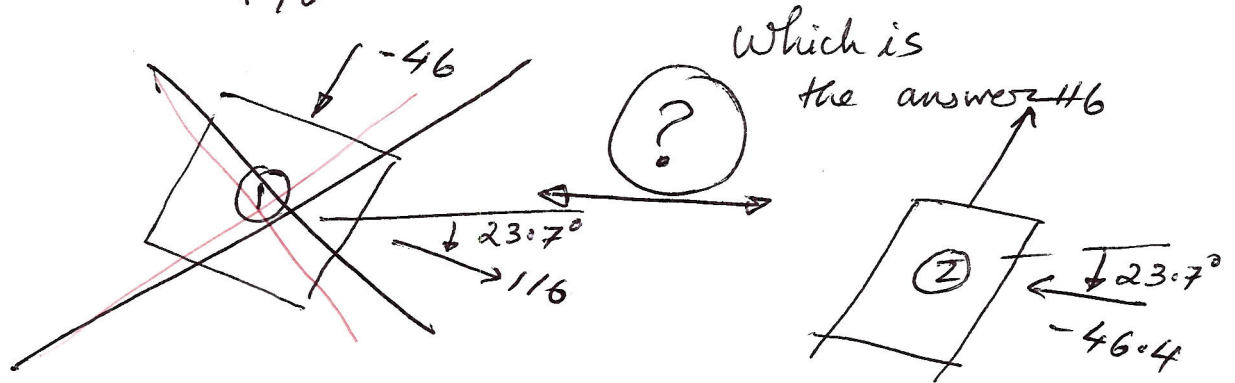
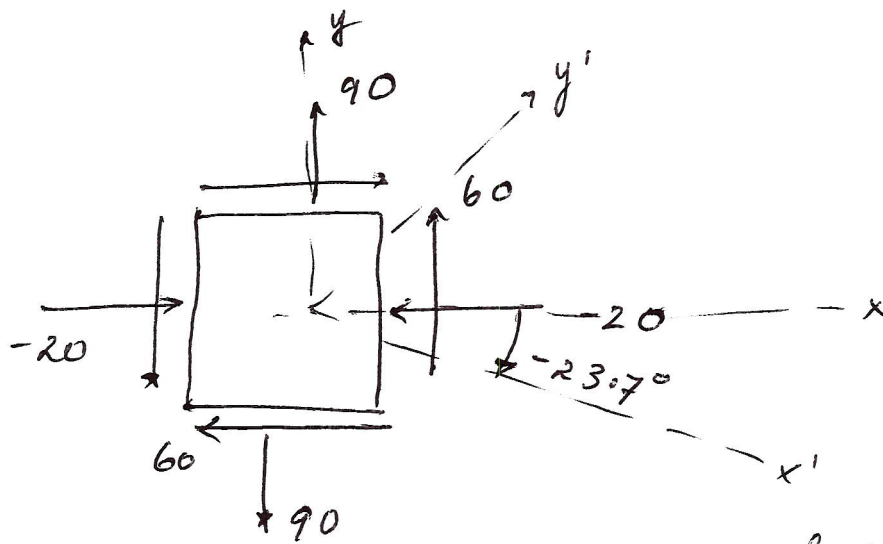
$$\begin{Bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{Bmatrix}$$

$$\{b_0 \ b_1 \ b_2 \ b_3\} \text{ array}$$

$$\{x \hat{i} \ y \hat{j} \ z \hat{k}\}$$

Vectors as arrays

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$$\sigma_{p1,2} = 116 \text{ MPa} \text{ or } -46.4 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} (\sin 2\theta)$$

Substitute $\theta = \theta_p = -23.7^\circ$

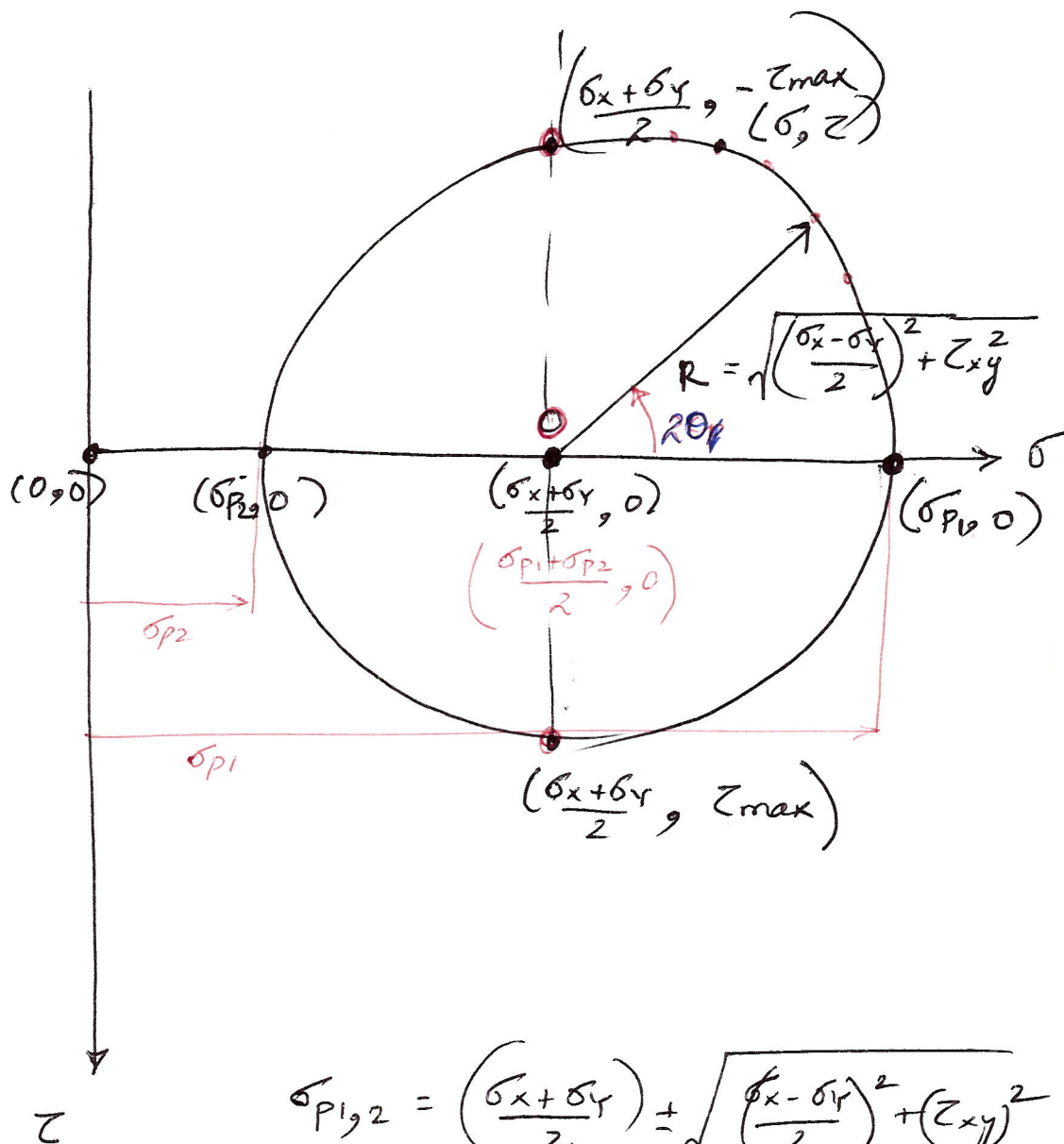
$$= \frac{-20 + 90}{2} + \frac{-20 - 90}{2} \cos(-47.5^\circ) + 60 \sin(47.5^\circ)$$

✓ = -46.4 MPa

$|\sigma_{p1} - \sigma_{p2}| \rightarrow \text{For} \rightarrow \text{then failure occurs}$

$|\sigma_{p1}| \rightarrow \text{For} \rightarrow \text{then failure occurs}$

Mohr Circle.



$$\sigma_{p1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (z_{xy})^2}$$

↓
center

$$\sigma_x + \sigma_y = \sigma_{p1} + \sigma_{p2}$$

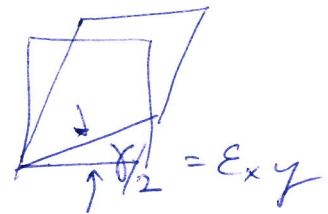
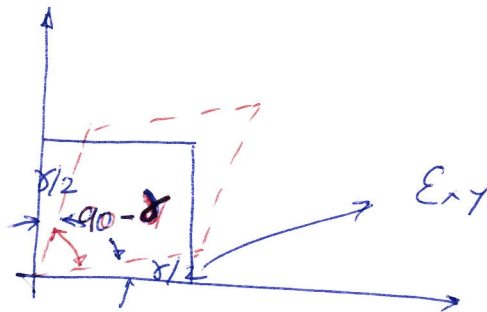
Mohr circle & transformation equations for strain

$$3 \text{ Tensors} \rightarrow [\sigma] \quad [\epsilon] \quad [I]$$

same transformation equations.

no need to rewrite.

$$[\epsilon] \text{ as a tensor} \quad \begin{matrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \rightarrow \gamma/2 \end{matrix}$$



$$\tau_{xy} = G \times \gamma$$

$$= 2G \times \gamma/2$$

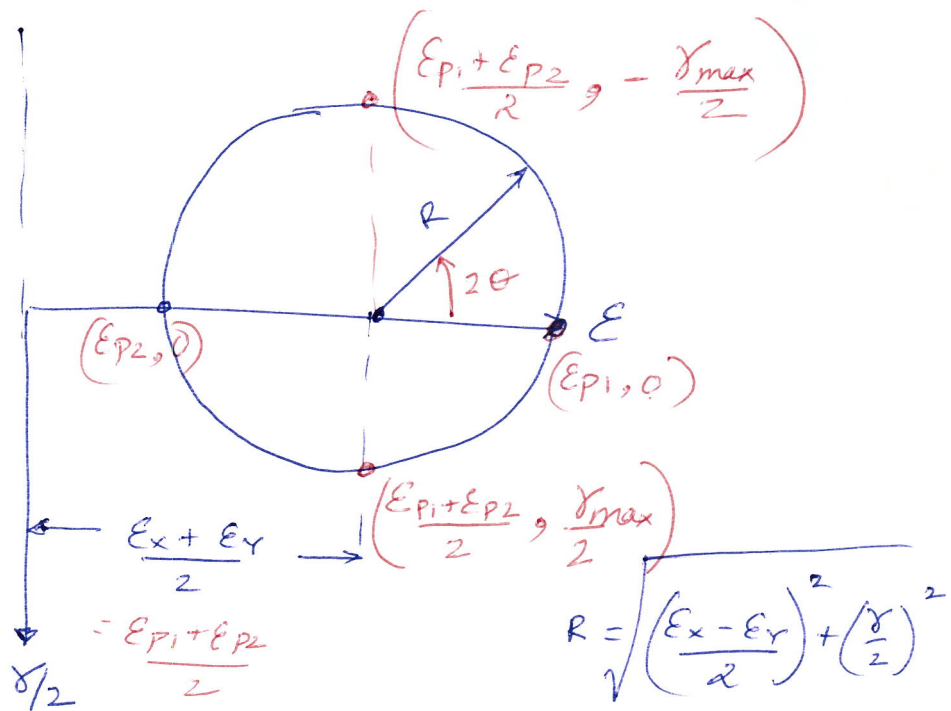
$$= 2G \times \epsilon_{xy}$$

When writing the strain
transformation equations

don't replace τ_{xy} with γ (engineering shear strain)

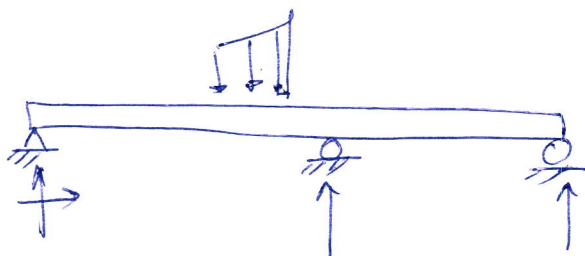
replace τ_{xy} with $\epsilon_{xy} = \frac{\gamma}{2}$ (mechanical shear strain)

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Material Failure Criteria are defined uniquely in terms of $(\sigma_{p1}, \sigma_{p2})$ or $(\epsilon_{p1}, \epsilon_{p2})$

CE 270 Promise $\longrightarrow$ Delivered!



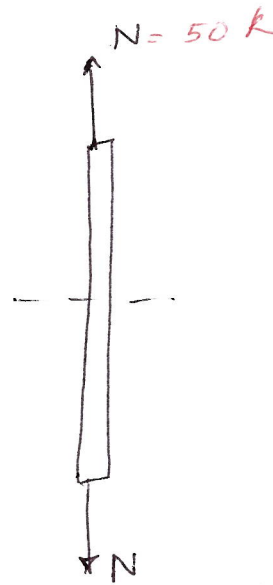
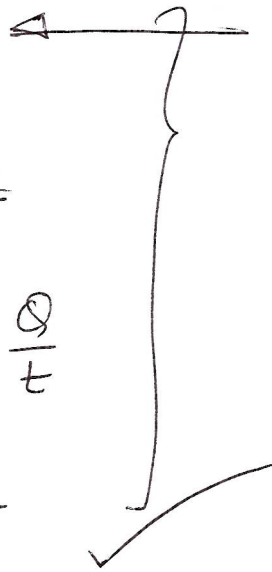
_____ x _____

$$\sigma = \frac{N}{A}$$

$$\sigma = -\frac{M \times y}{I}$$

$$\tau = \frac{V}{I} \times \frac{Q}{t}$$

$$\tau = \gamma \times \frac{r}{J}$$



$$\sigma = N/A$$

$$\sigma = N/A$$

$$\sigma \rightarrow F_y$$

material yield stress

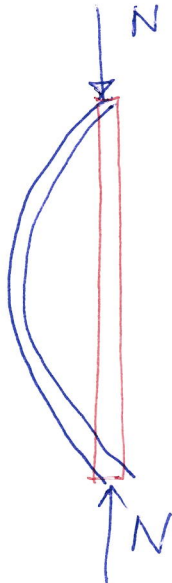
$$N_{\max} = \sigma_{\max} \times A$$

$$= \underline{\underline{F_y \times A}}$$

$$A = 1 \text{ in}^2$$

$$F_y = 50 \text{ ksi}$$

$$N_{\max} = 50 \text{ kip}$$



$$\sigma = -N/A$$

will $N_{\max} = \sigma_{\max} \times A$ | comp.

buckling sometimes before reaching the yield strength



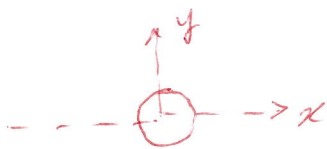
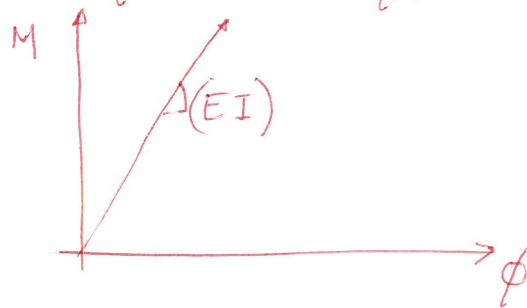
What is the buckling strength
in compression of
a column?

Leonhard Euler

d.e. & solve it



$$P \rightarrow P_{cr} = \frac{\pi^2 (EI)}{(KL)^2}$$

 $EI \rightarrow$ flexural stiffness of the member


flex about x-axis
flex about y-axis.

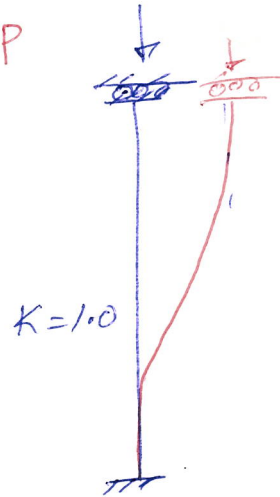
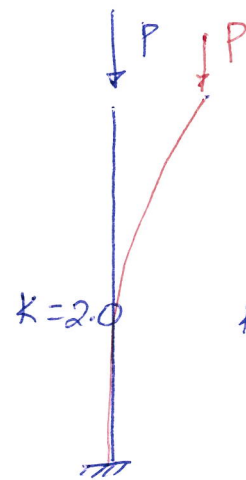
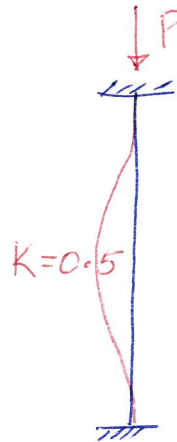
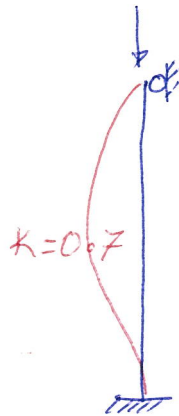
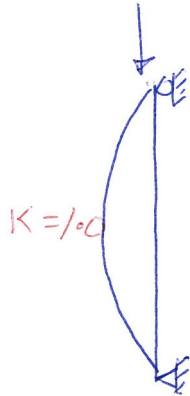
$$P_{cr-x} = \frac{\pi^2 (EI_x)}{(KL)^2}$$

$$\& \quad P_{cr-y} = \frac{\pi^2 EI_y}{(KL)^2}$$

Smaller of two will govern

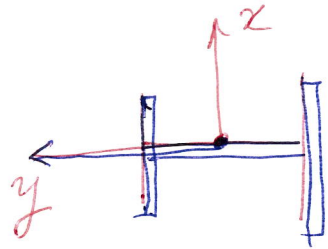
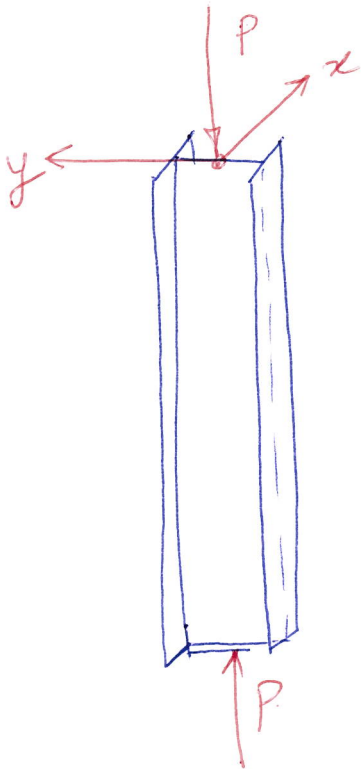
$(KL) \rightarrow$ effective length of the column.

K a/c for different end conditions.



$$P_{cr} = \frac{\pi^2 (EI)}{(KL)^2}$$

flexural stiffness.



I_x

I_y

$$P_{cr-x} = \frac{\pi^2 (EI_x)}{(KL)^2}$$

$$P_{cr-y} = \frac{\pi^2 (EI_y)}{(KL)^2}$$

smaller will govern

major larger I
minor smaller I