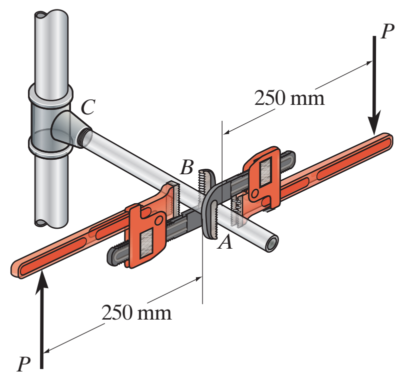
5–19. Two wrenches are used to tighten the pipe. If $P = 300$ N is applied to each wrench, determine the maximum torsional shear stress developed within regions AB and BC. The pipe has an outer diameter of 25 mm and inner diameter of 20 mm. Sketch the shear stress distribution for both cases.



Internal Loadings: The internal torque developed in segments AB and BC of the pipe can be determined by writing the moment equation of equilibrium about the x axis by referring to their respective free - body diagrams shown in Figs. a and b.

$\Sigma M_x = 0;\ T_{AB} - 300(0.25) = 0 \qquad T_{AB} = 75\,\text{N}\cdot\text{m}$

And

$\Sigma M_x = 0;\ T_{BC} - 300(0.25) - 300(0.25) = 0 \qquad T_{BC} = 150\,\text{N}\cdot\text{m}$

Allowable Shear Stress: The polar moment of inertia of the pipe is $J = \frac{\pi}{2}\left(0.0125^4 - 0.01^4\right) = 22.642(10^{-9})\text{m}^4$.

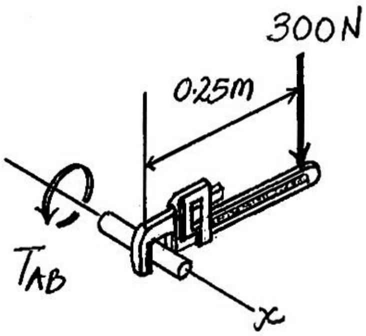
$$\left(\tau_{\max}\right)_{AB} = \frac{T_{AB}\,c}{J} = \frac{75(0.0125)}{22.642(10^{-9})} = 41.4\text{ MPa} \qquad \textbf{Ans.}$$

$$\left(\tau_{AB}\right)_{\rho=0.01\text{ m}} = \frac{T_{AB}\,\rho}{J} = \frac{75(0.01)}{22.642(10^{-9})} = 33.1\text{ MPa}$$

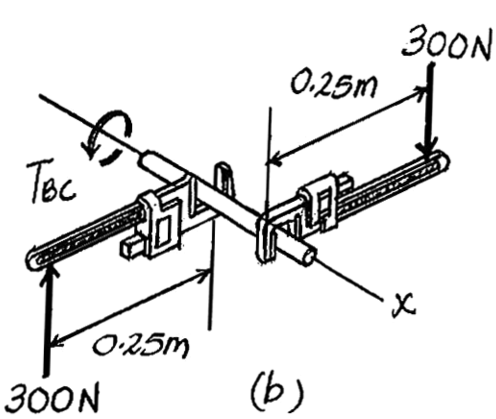
$$\left(\tau_{\max}\right)_{BC} = \frac{T_{BC}\,c}{J} = \frac{150(0.0125)}{22.642(10^{-9})} = 82.8\text{ MPa} \qquad \textbf{Ans.}$$

$$\left(\tau_{BC}\right)_{\rho=0.01\text{ m}} = \frac{T_{BC}\,\rho}{J} = \frac{150(0.01)}{22.642(10^{-9})} = 66.2\text{ MPa}$$

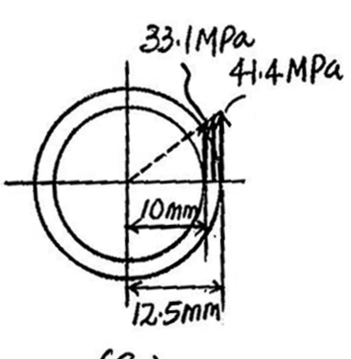
The shear stress distribution along the radial line of segments AB and BC of the pipe is shown in Figs. c and d, respectively.



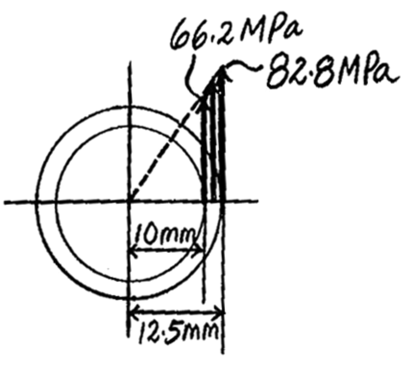
(a)



(b)

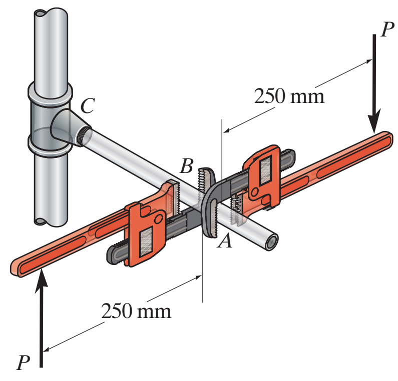


(c)



(d)

***5–20.** Two wrenches are used to tighten the pipe. If the pipe is made from a material having an allowable shear stress of $\tau_{allow} = 85$ MPa, determine the allowable maximum force **P** that can be applied to each wrench. The pipe has an outer diameter of 25 mm and inner diameter of 20 mm.



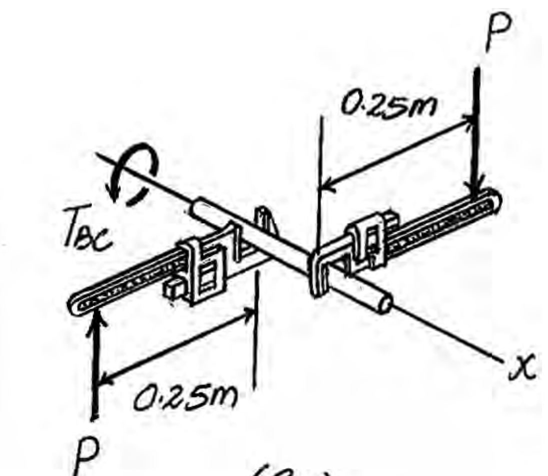
Internal Loading: By observation, segment BC of the pipe is critical since it is subjected to a greater internal torque than segment AB. Writing the moment equation of equilibrium about the x axis by referring to the free-body diagram shown in Fig. a, we have

$$\Sigma M_x = 0; \quad T_{BC} - P(0.25) - P(0.25) = 0 \qquad T_{BC} = 0.5P$$

Allowable Shear Stress: The polar moment of inertia of the pipe is $J = \frac{\pi}{2}\left(0.0125^4 - 0.01^4\right) = 22.642(10^{-9})\text{m}^4$

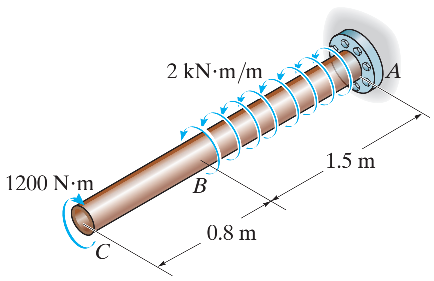
$$\tau_{allow} = \frac{T_{BC}\,c}{J}; \qquad 85(10^6) = \frac{0.5P(0.0125)}{22.642(10^{-9})}$$

$$P = 307.93\text{N} = 308\text{N}$$ **Ans.**



(a)

5–22. The solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the required diameter d of the shaft to the nearest mm if the allowable shear stress for the material is $\tau_{\text{allow}} = 50$ MPa.



The internal torque for segment BC is constant $T_{BC} = 1200$ N · m, Fig. a. However, the internal torque for segment AB varies with x, Fig. b.

$$T_{AB} - 2000x + 1200 = 0 \quad T_{AB} = (2000x - 1200)\ \text{N}\cdot\text{m}$$

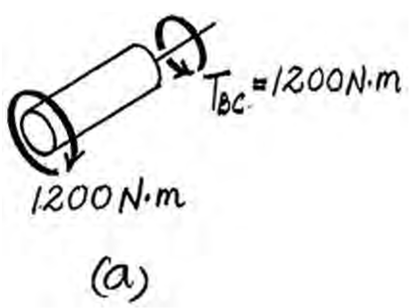
For segment AB, the maximum internal torque occurs at fixed support A where $x = 1.5$ m. Thus,

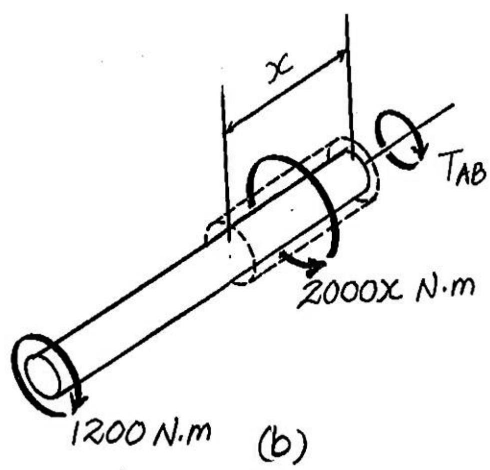
$$\left(T_{AB}\right)_{\max} = 2000(1.5) - 1200 = 1800\ \text{N}\cdot\text{m}$$

Since $\left(T_{AB}\right)_{\max} > T_{BC}$, the critical cross-section is at A. The polar moment of inertia of the rod is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$. Thus,

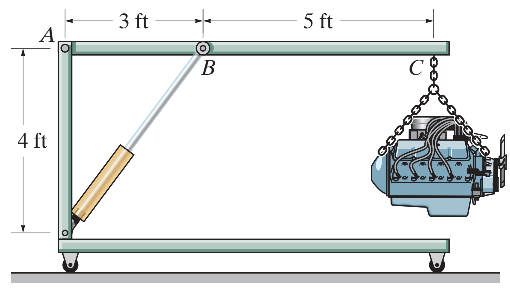
$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 50(10^6) = \frac{1800(d/2)}{\pi d^4/32}$$

$$d = 0.05681\ \text{m} = 56.81\ \text{mm} = 57\ \text{mm} \qquad \textbf{Ans.}$$





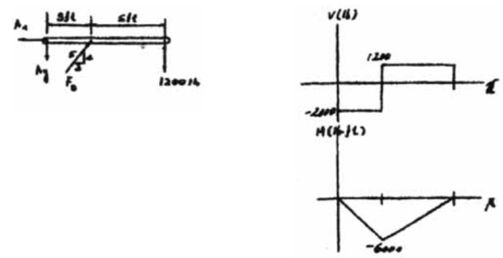
6–3. The engine crane is used to support the engine, which has a weight of 1200 lb. Draw the shear and moment diagrams of the boom *ABC* when it is in the horizontal position shown.



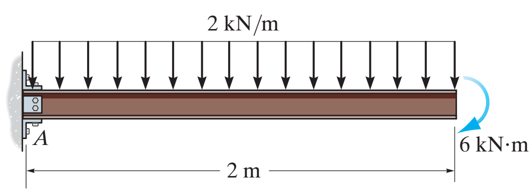
$\circlearrowleft + \Sigma M_A = 0; \quad \frac{4}{5} F_A(3) - 1200(8) = 0; \qquad F_A = 4000 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad -A_y + \frac{4}{5}(4000) - 1200 = 0; \qquad A_y = 2000 \text{ lb}$

$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad A_x - \frac{3}{5}(4000) = 0; \qquad A_x = 2400 \text{ lb}$



***6–4.** Draw the shear and moment diagrams for the cantilever beam.



The free-body diagram of the beam's right segment sectioned through an arbitrary point shown in Fig. *a* will be used to write the shear and moment equations of the beam.

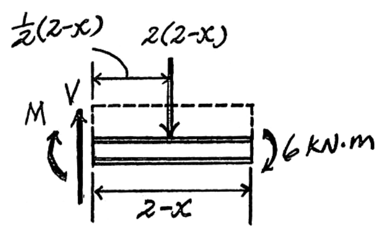
$+\uparrow \Sigma F_y = 0; \qquad V - 2(2 - x) = 0 \qquad V = \{4 - 2x\} \text{ kN}, \quad (1)$

$\circlearrowleft + \Sigma M = 0; \; -M - 2(2 - x)\left[\frac{1}{2}(2 - x)\right] - 6 = 0 \quad M = \{-x^2 + 4x - 10\} \text{kN} \cdot \text{m}, (2)$

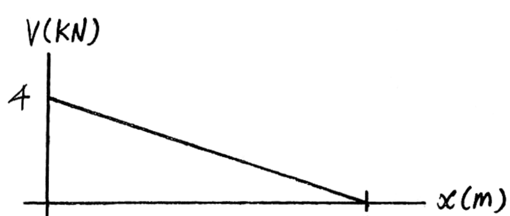
The shear and moment diagrams shown in Figs. *b* and *c* are plotted using Eqs. (1) and (2), respectively. The value of the shear and moment at $x = 0$ is evaluated using Eqs. (1) and (2).

$$V\big|_{x=0} = 4 - 2(0) = 4 \text{ kN}$$

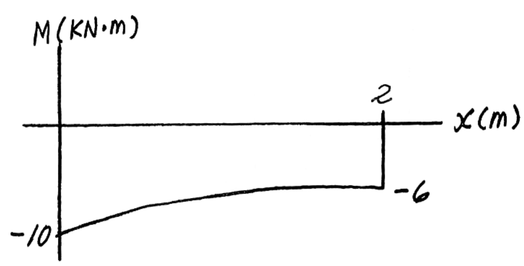
$$M\big|_{x=0} = \left[-0 + 4(0) - 10\right] = -10 \text{kN} \cdot \text{m}$$



(a)

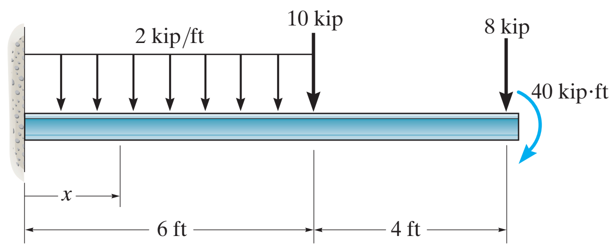


(b)



(c)

6–18. Draw the shear and moment diagrams for the beam, and determine the shear and moment throughout the beam as functions of x.



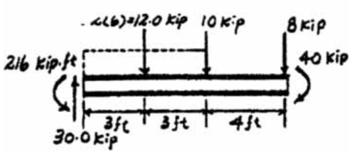
Support Reactions: As shown on FBD.

Shear and Moment Function:

For $0 \leq x < 6$ ft:

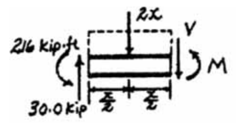
$$+\uparrow \Sigma F_y = 0; \qquad 30.0 - 2x - V = 0$$

$$V = \{30.0 - 2x\} \text{ kip} \qquad \textbf{Ans.}$$



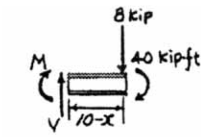
$$\circlearrowleft +\Sigma M_{NA} = 0; \quad M + 216 + 2x\left(\frac{x}{2}\right) - 30.0x = 0$$

$$M = \{-x^2 + 30.0x - 216\} \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$



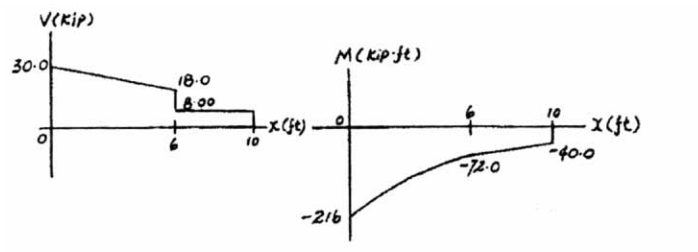
For 6 ft $< x \leq$ 10 ft:

$$+\uparrow \Sigma F_y = 0; \qquad V - 8 = 0 \qquad V = 8.00 \text{ kip} \qquad \textbf{Ans.}$$

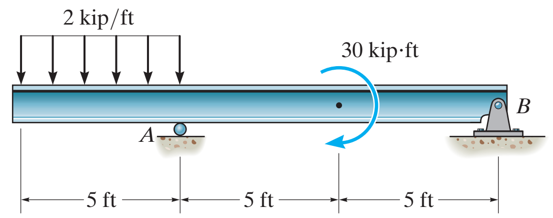


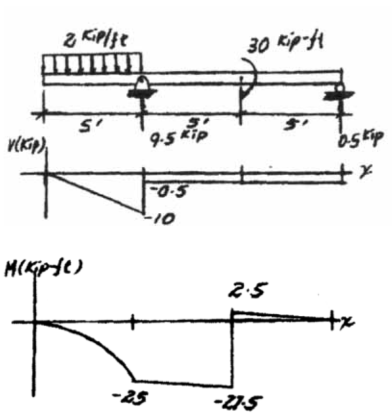
$$\circlearrowleft +\Sigma M_{NA} = 0; \qquad -M - 8(10 - x) - 40 = 0$$

$$M = \{8.00x - 120\} \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

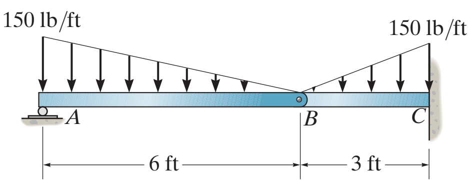


6–19. Draw the shear and moment diagrams for the beam.





6–30. Draw the shear and moment diagrams for the compound beam.



Support Reactions:

From the FBD of segment AB

$$\circlearrowleft + \Sigma M_B = 0; \quad 450(4) - A_y(6) = 0 \quad A_y = 300.0 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad B_y - 450 + 300.0 = 0 \quad B_y = 150.0 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad B_x = 0$$

From the FBD of segment BC

$$\circlearrowleft + \Sigma M_C = 0; \quad 225(1) + 150.0(3) - M_C = 0$$

$$M_C = 675.0 \text{ lb} \cdot \text{ft}$$

$$+\uparrow \Sigma F_y = 0; \quad C_y - 150.0 - 225 = 0 \quad C_y = 375.0 \text{ lb}$$

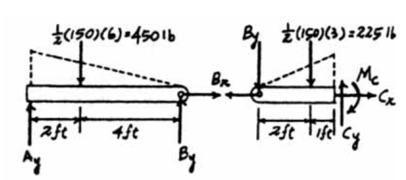
$$\xrightarrow{+} \Sigma F_x = 0; \quad C_x = 0$$

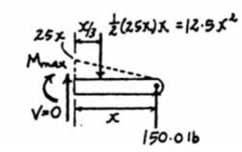
Shear and Moment Diagram: The maximum positive moment occurs when $V = 0$.

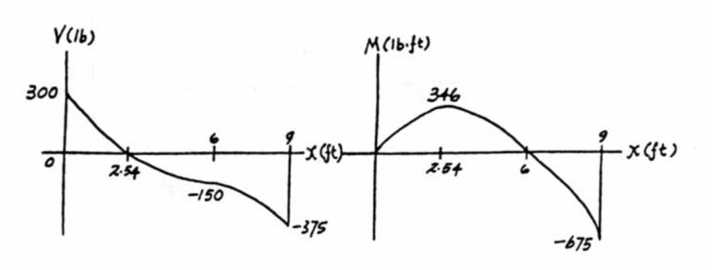
$$+\uparrow \Sigma F_y = 0; \quad 150.0 - 12.5x^2 = 0 \quad x = 3.464 \text{ ft}$$

$$\circlearrowleft + \Sigma M_{NA} = 0; \quad 150(3.464) - 12.5\left(3.464^2\right)\left(\frac{3.464}{3}\right) - M_{max} = 0$$

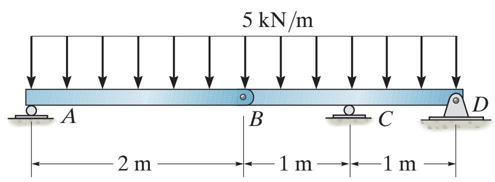
$$M_{max} = 346.4 \text{ lb} \cdot \text{ft}$$







6–42. Draw the shear and moment diagrams for the compound beam.



Support Reactions:

From the FBD of segment AB

$$\zeta + \Sigma M_A = 0; \qquad B_y(2) - 10.0(1) = 0 \qquad B_y = 5.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 10.0 + 5.00 = 0 \qquad A_y = 5.00 \text{ kN}$$

From the FBD of segment BD

$$\zeta + \Sigma M_C = 0; \qquad 5.00(1) + 10.0(0) - D_y(1) = 0$$

$$D_y = 5.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad C_y - 5.00 - 5.00 - 10.0 = 0$$

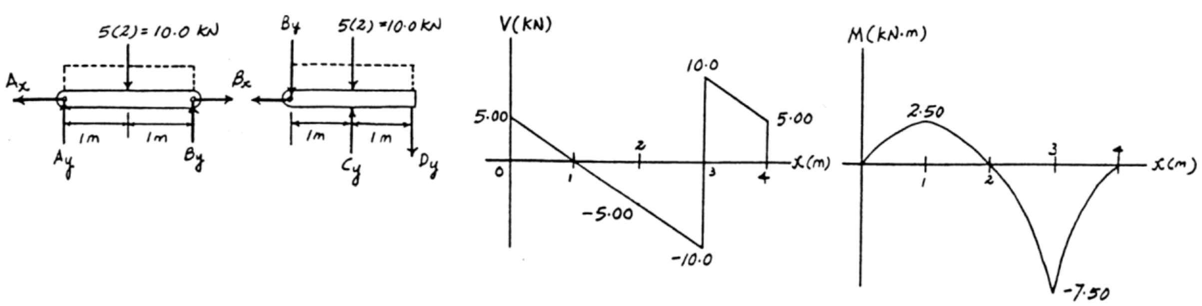
$$C_y = 20.0 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$$

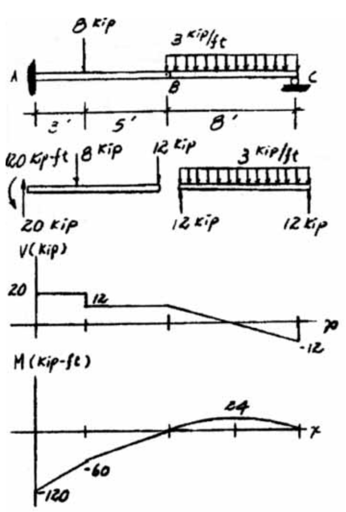
From the FBD of segment AB

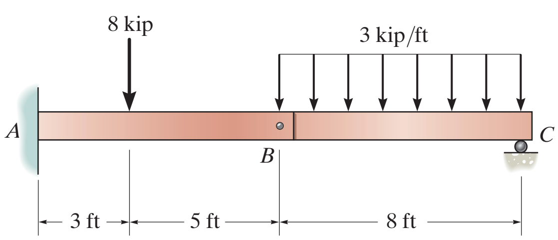
$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$$

Shear and Moment Diagram:

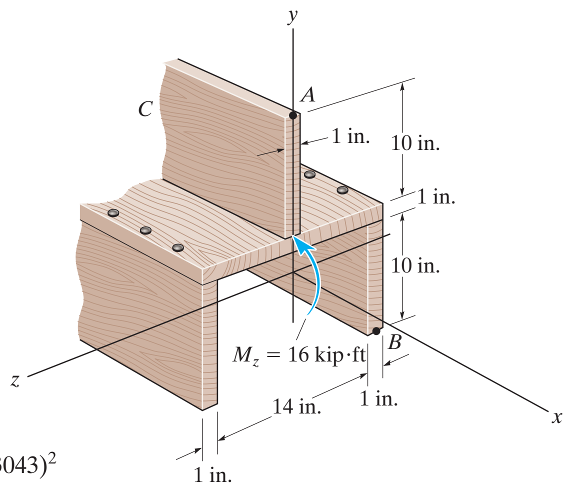


6–43. Draw the shear and moment diagrams for the beam. The two segments are joined together at B.





***6–60.** The beam is constructed from four boards as shown. If it is subjected to a moment of $M_z = 16\text{ kip}\cdot\text{ft}$, determine the stress at points A and B. Sketch a three-dimensional view of the stress distribution.



$$\bar{y} = \frac{2[5(10)(1)] + 10.5(16)(1) + 16(10)(1)}{2(10)(1) + 16(1) + 10(1)}$$

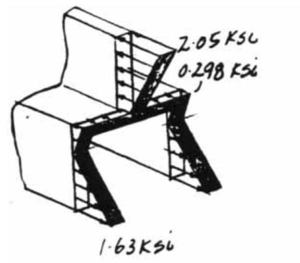
$$= 9.3043\text{ in.}$$

$$I = 2\left[\frac{1}{12}(1)(10^3) + 1(10)(9.3043 - 5)^2\right] + \frac{1}{12}(16)(1^3) + 16(1)(10.5 - 9.3043)^2$$

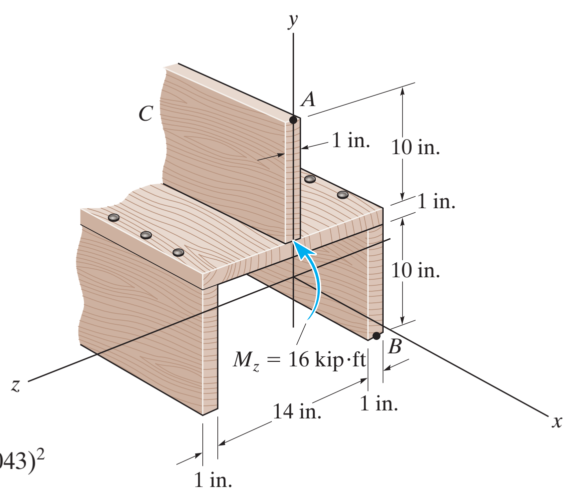
$$+ \frac{1}{12}(1)(10^3) + 1(10)(16 - 9.3043)^2 = 1093.07\text{ in}^4$$

$$\sigma_A = \frac{Mc}{I} = \frac{16(12)(21 - 9.3043)}{1093.07} = 2.05\text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{My}{I} = \frac{16(12)(9.3043)}{1093.07} = 1.63\text{ ksi} \qquad \textbf{Ans.}$$



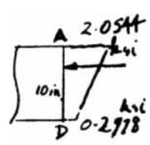
•6–61. The beam is constructed from four boards as shown. If it is subjected to a moment of $M_z = 16\text{ kip}\cdot\text{ft}$, determine the resultant force the stress produces on the top board C.



$$\bar{y} = \frac{2[5(10)(1)] + 10.5(16)(1) + 16(10)(1)}{2(10)(1) + 16(1) + 10(1)} = 9.3043\text{ in.}$$

$$I = 2\left[\frac{1}{12}(1)(10^3) + (10)(9.3043 - 5)^2\right] + \frac{1}{12}(16)(1^3) + 16(1)(10.5 - 9.3043)^2$$

$$+ \frac{1}{12}(1)(10^3) + 1(10)(16 - 9.3043)^2 = 1093.07\text{ in}^4$$

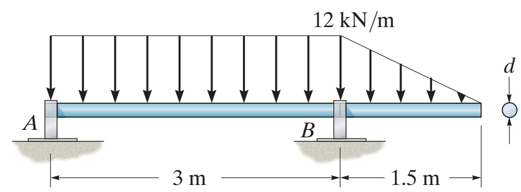


$$\sigma_A = \frac{Mc}{I} = \frac{16(12)(21 - 9.3043)}{1093.07} = 2.0544\text{ ksi}$$

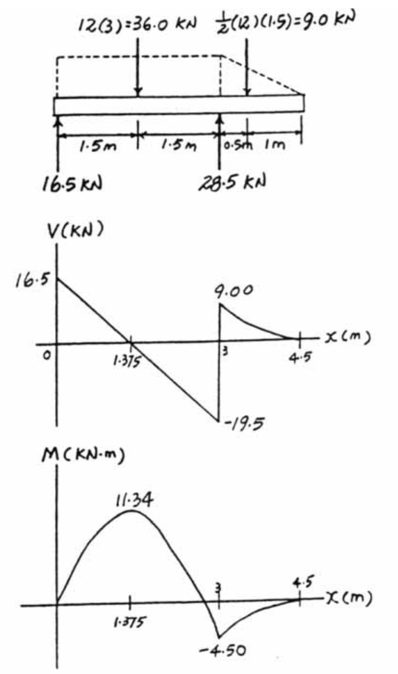
$$\sigma_D = \frac{My}{I} = \frac{16(12)(11 - 9.3043)}{1093.07} = 0.2978\text{ ksi}$$

$$(F_R)_C = \frac{1}{2}(2.0544 + 0.2978)(10)(1) = 11.8\text{ kip} \qquad \textbf{Ans.}$$

***6–68.** The rod is supported by smooth journal bearings at A and B that only exert vertical reactions on the shaft. Determine its smallest diameter d if the allowable bending stress is $\sigma_{\text{allow}} = 180$ MPa.



Allowable Bending Stress: The maximum moment is $M_{\max} = 11.34$ kN·m as indicated on the moment diagram. Applying the flexure formula

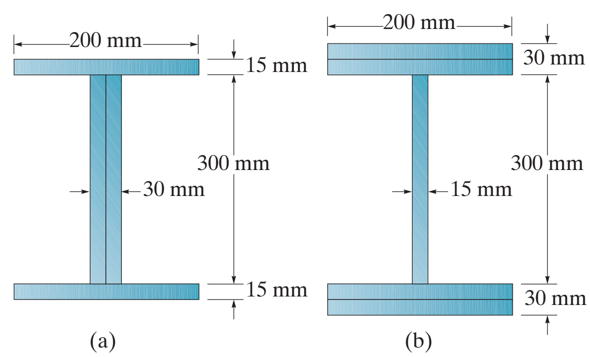


$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{M_{\max}\, c}{I}$$

$$180\left(10^6\right) = \frac{11.34(10^3)\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 0.08626 \text{ m} = 86.3 \text{ mm}$$ **Ans.**

•6–69. Two designs for a beam are to be considered. Determine which one will support a moment of $M =$ 150 kN·m with the least amount of bending stress. What is that stress?



Section Property:

For section (a)

$$I = \frac{1}{12}(0.2)\left(0.33^3\right) - \frac{1}{12}(0.17)(0.3)^3 = 0.21645(10^{-3}) \text{ m}^4$$

For section (b)

$$I = \frac{1}{12}(0.2)\left(0.36^3\right) - \frac{1}{12}(0.185)\left(0.3^3\right) = 0.36135(10^{-3}) \text{ m}^4$$

Maximum Bending Stress: Applying the flexure formula $\sigma_{\max} = \dfrac{Mc}{I}$

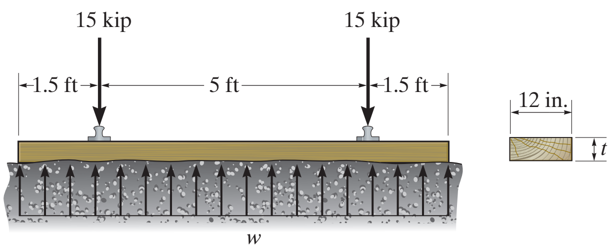
For section (a)

$$\sigma_{\max} = \frac{150(10^3)(0.165)}{0.21645(10^{-3})} = 114.3 \text{ MPa}$$

For section (b)

$$\sigma_{\max} = \frac{150(10^3)(0.18)}{0.36135(10^{-3})} = 74.72 \text{ MPa} = 74.7 \text{ MPa}$$ **Ans.**

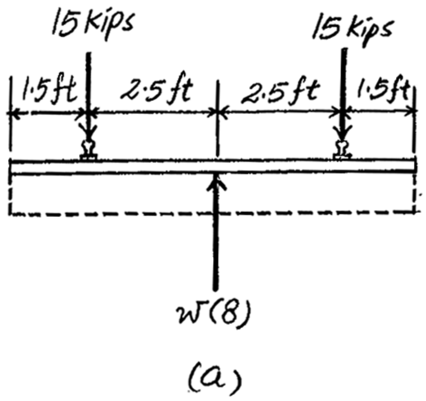
6–82. The reaction of the ballast on the railway tie can be assumed uniformly distributed over its length as shown. If the wood has an allowable bending stress of $\sigma_{allow} = 1.5$ ksi, determine the required minimum thickness t of the rectangular cross sectional area of the tie to the nearest $\frac{1}{8}$ in.



Support Reactions: Referring to the free-body diagram of the tie shown in Fig. *a*, we have

$$+\uparrow \Sigma F_y = 0; \qquad w(8) - 2(15) = 0$$

$$w = 3.75 \text{ kip/ft}$$



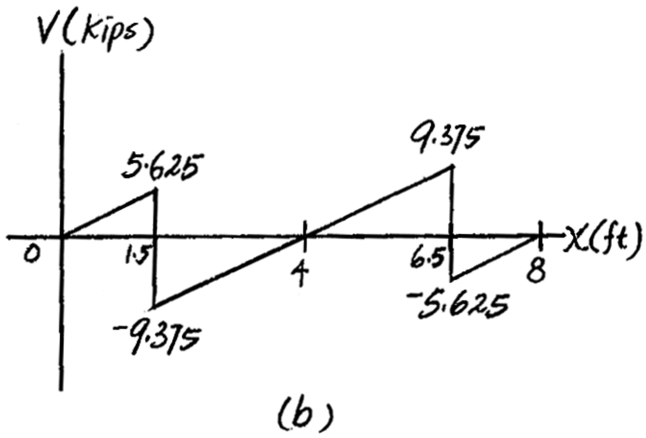
Maximum Moment: The shear and moment diagrams are shown in Figs. *b* and *c*. As indicated on the moment diagram, the maximum moment is $|M_{max}| = 7.5 \text{ kip} \cdot \text{ft}$.

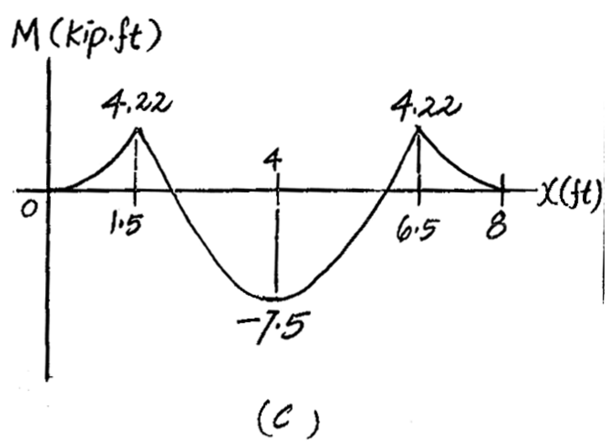
Absolute Maximum Bending Stress:

$$\sigma_{max} = \frac{Mc}{I}; \qquad 1.5 = \frac{7.5(12)\left(\frac{t}{2}\right)}{\frac{1}{12}(12)t^3}$$

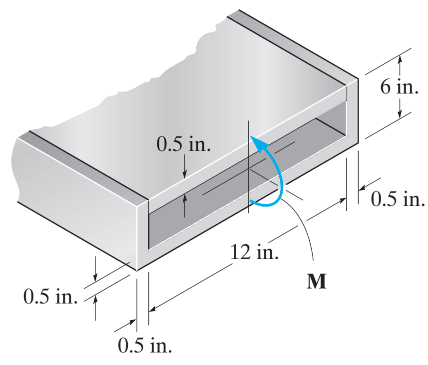
$$t = 5.48 \text{ in.}$$

Use $\qquad t = 5\frac{1}{2}$ in. **Ans.**

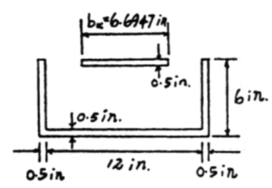




***6–132.** The top plate is made of 2014-T6 aluminum and is used to reinforce a Kevlar 49 plastic beam. Determine the maximum stress in the aluminum and in the Kevlar if the beam is subjected to a moment of $M = 900 \text{ lb} \cdot \text{ft}$.



Section Properties:



$$n = \frac{E_{al}}{E_k} = \frac{10.6(10^3)}{19.0(10^3)} = 0.55789$$

$$b_k = n\, b_{al} = 0.55789(12) = 6.6947 \text{ in.}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.25(13)(0.5) + 2[(3.25)(5.5)(0.5)] + 5.75(6.6947)(0.5)}{13(0.5) + 2(5.5)(0.5) + 6.6947(0.5)}$$

$$= 2.5247 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(13)\left(0.5^3\right) + 13(0.5)(2.5247 - 0.25)^2$$

$$+ \frac{1}{12}(1)\left(5.5^3\right) + 1(5.5)(3.25 - 2.5247)^2$$

$$+ \frac{1}{12}(6.6947)\left(0.5^3\right) + 6.6947(0.5)(5.75 - 2.5247)^2$$

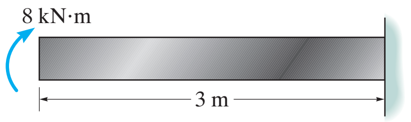
$$= 85.4170 \text{ in}^4$$

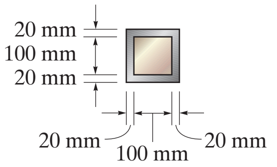
Maximum Bending Stress: Applying the flexure formula

$$(\sigma_{\max})_{al} = n\frac{Mc}{I} = 0.55789\left[\frac{900(12)(6 - 2.5247)}{85.4170}\right] = 245 \text{ psi}$$ **Ans.**

$$(\sigma_{\max})_k = \frac{Mc}{I} = \frac{900(12)(6 - 2.5247)}{85.4168} = 439 \text{ psi}$$ **Ans.**

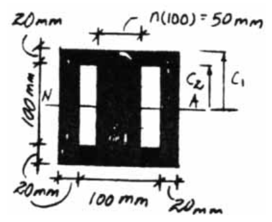
6–134. The member has a brass core bonded to a steel casing. If a couple moment of 8 kN · m is applied at its end, determine the maximum bending stress in the member. $E_{br} = 100$ GPa, $E_{st} = 200$ GPa.





$$n = \frac{E_{br}}{E_{st}} = \frac{100}{200} = 0.5$$

$$I = \frac{1}{12}(0.14)(0.14)^3 - \frac{1}{12}(0.05)(0.1)^3 = 27.84667(10^{-6})\text{m}^4$$



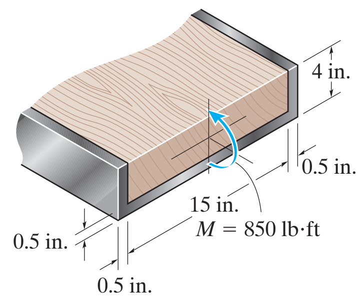
Maximum stress in steel:

$$(\sigma_{st})_{max} = \frac{Mc_1}{I} = \frac{8(10^3)(0.07)}{27.84667(10^{-6})} = 20.1 \text{ MPa} \qquad \text{(max)}$$ **Ans.**

Maximum stress in brass:

$$(\sigma_{br})_{max} = \frac{nMc_2}{I} = \frac{0.5(8)(10^3)(0.05)}{27.84667(10^{-6})} = 7.18 \text{ MPa}$$

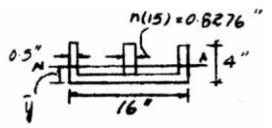
6–135. The steel channel is used to reinforce the wood beam. Determine the maximum stress in the steel and in the wood if the beam is subjected to a moment of $M = 850$ lb · ft. $E_{st} = 29(10^3)$ ksi, $E_w = 1600$ ksi.



$$\bar{y} = \frac{(0.5)(16)(0.25) + 2(3.5)(0.5)(2.25) + (0.8276)(3.5)(2.25)}{0.5(16) + 2(3.5)(0.5) + (0.8276)(3.5)} = 1.1386 \text{ in.}$$

$$I = \frac{1}{12}(16)(0.5^3) + (16)(0.5)(0.8886^2) + 2\left(\frac{1}{12}\right)(0.5)(3.5^3) + 2(0.5)(3.5)(1.1114^2)$$

$$+ \frac{1}{12}(0.8276)(3.5^3) + (0.8276)(3.5)(1.1114^2) = 20.914 \text{ in}^4$$



Maximum stress in steel:

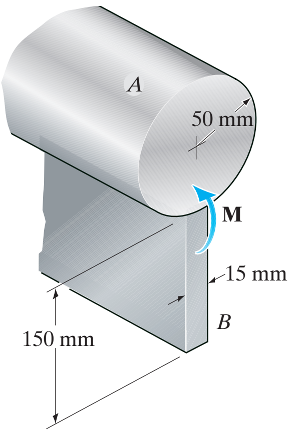
$$(\sigma_{st}) = \frac{Mc}{I} = \frac{850(12)(4 - 1.1386)}{20.914} = 1395 \text{ psi} = 1.40 \text{ ksi}$$ **Ans.**

Maximum stress in wood:

$$(\sigma_w) = n(\sigma_{st})_{max}$$

$$= 0.05517(1395) = 77.0 \text{ psi}$$ **Ans.**

•6–137. If the beam is subjected to an internal moment of $M = 45\ \text{kN}\cdot\text{m}$, determine the maximum bending stress developed in the A-36 steel section A and the 2014-T6 aluminum alloy section B.



Section Properties: The cross section will be transformed into that of steel as shown in Fig. *a*. Here, $n = \frac{E_{al}}{E_{st}} = \frac{73.1(10^9)}{200(10^9)} = 0.3655$. Thus, $b_{st} = nb_{al} = 0.3655(0.015) = 0.0054825$ m. The location of the transformed section is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.075(0.15)(0.0054825) + 0.2\left[\pi\left(0.05^2\right)\right]}{0.15(0.0054825) + \pi\left(0.05^2\right)}$$

$$= 0.1882 \text{ m}$$

The moment of inertia of the transformed section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2 = \frac{1}{12}(0.0054825)\left(0.15^3\right) + 0.0054825(0.15)(0.1882 - 0.075)^2$$

$$+ \frac{1}{4}\pi\left(0.05^4\right) + \pi\left(0.05^2\right)(0.2 - 0.1882)^2$$

$$= 18.08\left(10^{-6}\right) \text{m}^4$$

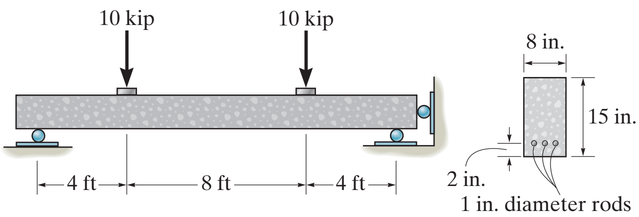
Maximum Bending Stress: For the steel,

$$(\sigma_{\max})_{st} = \frac{Mc_{st}}{I} = \frac{45\left(10^3\right)(0.06185)}{18.08\left(10^{-6}\right)} = 154 \text{ MPa} \qquad \textbf{Ans.}$$

For the aluminum alloy,

$$(\sigma_{\max})_{al} = n\frac{Mc_{al}}{I} = 0.3655\left[\frac{45\left(10^3\right)(0.1882)}{18.08\left(10^{-6}\right)}\right] = 171 \text{ MPa} \qquad \textbf{Ans.}$$

•6–141. The reinforced concrete beam is used to support the loading shown. Determine the absolute maximum normal stress in each of the A-36 steel reinforcing rods and the absolute maximum compressive stress in the concrete. Assume the concrete has a high strength in compression and yet neglect its strength in supporting tension.



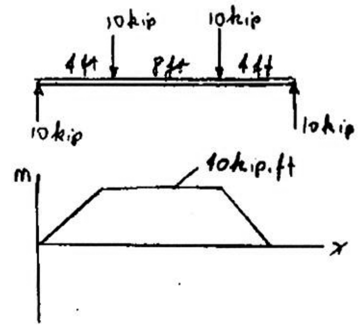
$$M_{max} = (10\text{ kip})(4\text{ ft}) = 40\text{ kip}\cdot\text{ft}$$

$$A_{st} = 3(\pi)(0.5)^2 = 2.3562\text{ in}^2$$

$$E_{st} = 29.0(10^3)\text{ ksi}$$

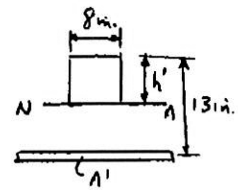
$$E_{con} = 4.20(10^3)\text{ ksi}$$

$$A' = nA_{st} = \frac{29.0(10^3)}{4.20(10^3)}(2.3562) = 16.2690\text{ in}^2$$



$$\Sigma\bar{y}A = 0; \qquad 8(h')\left(\frac{h'}{2}\right) - 16.2690(13 - h') = 0$$

$$h'^2 + 4.06724h - 52.8741 = 0$$



Solving for the positive root:

$$h' = 5.517\text{ in.}$$

$$I = \left[\frac{1}{12}(8)(5.517)^3 + 8(5.517)(5.517/2)^2\right] + 16.2690(13 - 5.517)^2$$

$$= 1358.781\text{ in}^4$$

$$(\sigma_{con})_{max} = \frac{My}{I} = \frac{40(12)(5.517)}{1358.781} = 1.95\text{ ksi} \qquad \textbf{Ans.}$$

$$(\sigma_{st})_{max} = n\left(\frac{My}{I}\right) = \left(\frac{29.0(10^3)}{4.20(10^3)}\right)\left(\frac{40(12)(13 - 5.517)}{1358.781}\right) = 18.3\text{ ksi} \qquad \textbf{Ans.}$$