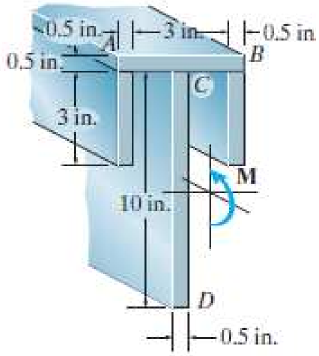
CE 270
Fall 2011
Problem Set 5 (Bending Stress and Strain) Solution

•6–49. Determine the maximum tensile and compressive bending stress in the beam if it is subjected to a moment of $M = 4 \text{ kip}\cdot\text{ft}$.



Section Properties:

$$\bar{y} = \frac{\Sigma\,\bar{y}\,A}{\Sigma A}$$

$$= \frac{0.25(4)(0.5) + 2[2(3)(0.5)] + 5.5(10)(0.5)}{4(0.5) + 2[(3)(0.5)] + 10(0.5)} = 3.40 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(4)\left(0.5^3\right) + 4(0.5)(3.40 - 0.25)^2$$

$$+ 2\left[\frac{1}{12}(0.5)(3^3) + 0.5(3)(3.40 - 2)^2\right]$$

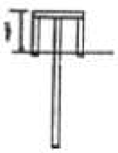
$$+ \frac{1}{12}(0.5)\left(10^3\right) + 0.5(10)(5.5 - 3.40)^2$$

$$= 91.73 \text{ in}^4$$

Maximum Bending Stress: Applying the flexure formula $\sigma_{\max} = \dfrac{Mc}{I}$

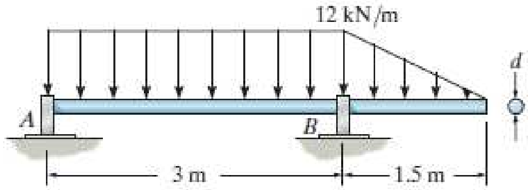
$$(\sigma_t)_{\max} = \frac{4(10^3)(12)(10.5 - 3.40)}{91.73} = 3715.12 \text{ psi} = 3.72 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\sigma_c)_{\max} = \frac{4(10^3)(12)(3.40)}{91.73} = 1779.07 \text{ psi} = 1.78 \text{ ksi} \qquad \textbf{Ans.}$$



CE 270
Fall 2011
Problem Set 5 (Bending Stress and Strain) Solution

*6–68. The rod is supported by smooth journal bearings at A and B that only exert vertical reactions on the shaft. Determine its smallest diameter d if the allowable bending stress is $\sigma_{allow} = 180$ MPa.

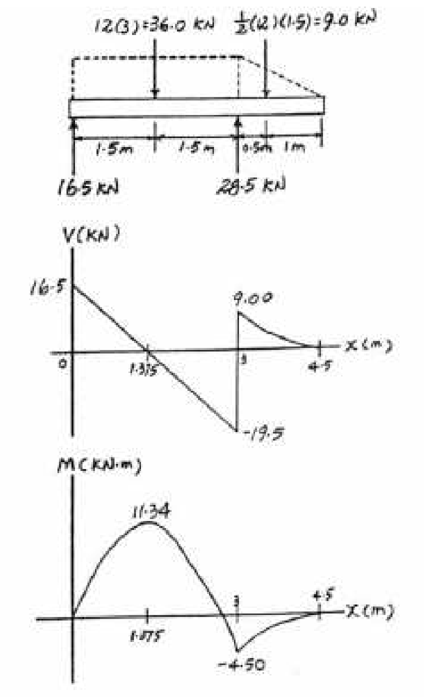


Allowable Bending Stress: The maximum moment is $M_{max} = 11.34$ kN · m as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\,c}{I}$$

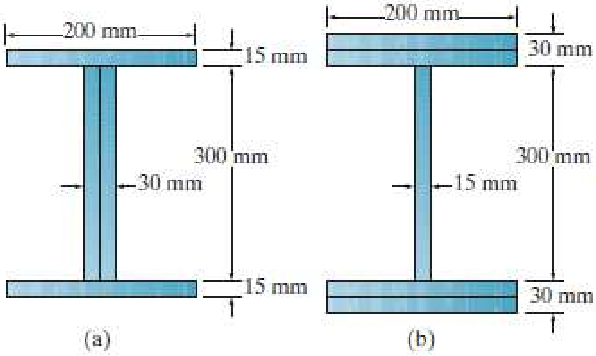
$$180(10^6) = \frac{11.34(10^3)\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 0.08626 \text{ m} = 86.3 \text{ mm}$$ **Ans.**



CE 270
Fall 2011
Problem Set 5 (Bending Stress and Strain) Solution

•6–69. Two designs for a beam are to be considered. Determine which one will support a moment of $M = 150\ \text{kN} \cdot \text{m}$ with the least amount of bending stress. What is that stress?



Section Property:

For section (a)

$$I = \frac{1}{12}(0.2)(0.33^3) - \frac{1}{12}(0.17)(0.3)^3 = 0.21645(10^{-3})\ \text{m}^4$$

For section (b)

$$I = \frac{1}{12}(0.2)(0.36^3) - \frac{1}{12}(0.185)(0.3^3) = 0.36135(10^{-3})\ \text{m}^4$$

Maximum Bending Stress: Applying the flexure formula $\sigma_{\max} = \dfrac{Mc}{I}$

For section (a)

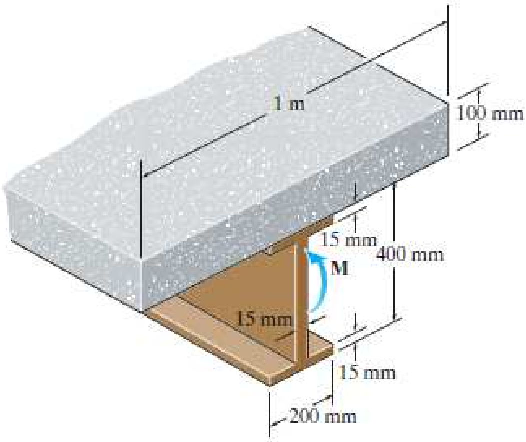
$$\sigma_{\max} = \frac{150(10^3)(0.165)}{0.21645(10^{-3})} = 114.3\ \text{MPa}$$

For section (b)

$$\sigma_{\max} = \frac{150(10^3)(0.18)}{0.36135(10^{-3})} = 74.72\ \text{MPa} = 74.7\ \text{MPa}$$ **Ans.**

CE 270
Fall 2011
Problem Set 5 (Bending Stress and Strain) Solution

***6–140.** The low strength concrete floor slab is integrated with a wide-flange A-36 steel beam using shear studs (not shown) to form the composite beam. If the allowable bending stress for the concrete is $(\sigma_{allow})_{con} = 10$ MPa, and allowable bending stress for steel is $(\sigma_{allow})_{st} = 165$ MPa, determine the maximum allowable internal moment **M** that can be applied to the beam.



Section Properties: The beam cross section will be transformed into that of steel. Here, $n = \dfrac{E_{con}}{E_{st}} = \dfrac{22.1}{200} = 0.1105$. Thus, $b_{st} = nb_{con} = 0.1105(1) = 0.1105$ m. The location of the transformed section is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.0075(0.015)(0.2) + 0.2(0.37)(0.015) + 0.3925(0.015)(0.2) + 0.45(0.1)(0.1105)}{0.015(0.2) + 0.37(0.015) + 0.015(0.2) + 0.1(0.1105)}$$

$$= 0.3222 \text{ m}$$

The moment of inertia of the transformed section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2 = \frac{1}{12}(0.2)\left(0.015^3\right)$$
$$+ 0.2(0.015)(0.3222 - 0.0075)^2$$
$$+ \frac{1}{12}(0.015)\left(0.37^3\right) + 0.015(0.37)(0.3222 - 0.2)^2$$
$$+ \frac{1}{12}(0.2)\left(0.015^3\right) + 0.2(0.015)(0.3925 - 0.3222)^2$$
$$+ \frac{1}{12}(0.1105)\left(0.1^3\right) + 0.1105(0.1)(0.45 - 0.3222)^2$$
$$= 647.93\left(10^{-6}\right) \text{ m}^4$$

Bending Stress: Assuming failure of steel,

$$(\sigma_{allow})_{st} = \frac{Mc_{st}}{I}; \quad 165\left(10^6\right) = \frac{M(0.3222)}{647.93\left(10^{-6}\right)}$$

$$M = 331\,770.52 \text{ N} \cdot \text{m} = 332 \text{ kN} \cdot \text{m}$$

Assuming failure of concrete,

$$(\sigma_{allow})_{con} = n\frac{Mc_{con}}{I}; \qquad 10\left(10^6\right) = 0.1105\left[\frac{M(0.5 - 0.3222)}{647.93\left(10^{-6}\right)}\right]$$

$$M = 329\,849.77 \text{ N} \cdot \text{m} = 330 \text{ kN} \cdot \text{m (controls)} \quad \textbf{Ans.}$$

6-49, 6-