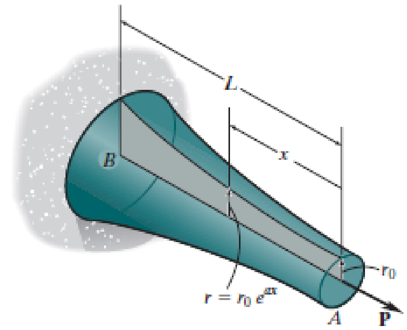
4–27. The circular bar has a variable radius of $r = r_0 e^{ax}$ and is made of a material having a modulus of elasticity of E. Determine the displacement of end A when it is subjected to the axial force $\mathbf{P}$.



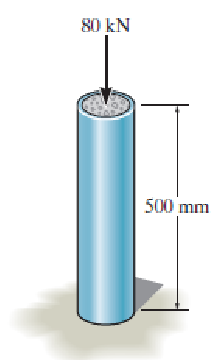
Displacements: The cross-sectional area of the bar as a function of x is $A(x) = \pi r^2 = \pi r_0{}^2 e^{2ax}$. We have

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E} = \frac{P}{\pi r_0{}^2 E}\int_0^L \frac{dx}{e^{2ax}}$$

$$= \frac{P}{\pi r_0{}^2 E}\left[-\frac{1}{2ae^{2ax}}\right]\Bigg|_0^L$$

$$= -\frac{P}{2a\pi r_0{}^2 E}\left(1 - e^{-2aL}\right) \qquad \textbf{Ans.}$$

•**4–33.** The steel pipe is filled with concrete and subjected to a compressive force of 80 kN. Determine the average normal stress in the concrete and the steel due to this loading. The pipe has an outer diameter of 80 mm and an inner diameter of 70 mm. $E_{st} = 200$ GPa, $E_c = 24$ GPa.

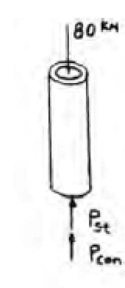


$+\uparrow \Sigma F_y = 0;\qquad P_{st} + P_{con} - 80 = 0 \qquad \textbf{(1)}$

$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}\,L}{\frac{\pi}{4}(0.08^2 - 0.07^2)\,(200)\,(10^9)} = \frac{P_{con}\,L}{\frac{\pi}{4}(0.07^2)\,(24)\,(10^9)}$$

$$P_{st} = 2.5510\,P_{con} \qquad \textbf{(2)}$$



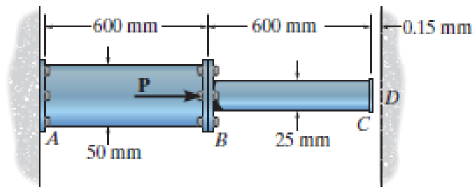
Solving Eqs. (1) and (2) yields

$$P_{st} = 57.47\text{ kN} \qquad P_{con} = 22.53\text{ kN}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{57.47\,(10^3)}{\frac{\pi}{4}\,(0.08^2 - 0.07^2)} = 48.8\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{22.53\,(10^3)}{\frac{\pi}{4}\,(0.07^2)} = 5.85\text{ MPa} \qquad \textbf{Ans.}$$

4–46. If the gap between C and the rigid wall at D is initially 0.15 mm, determine the support reactions at A and D when the force $\mathbf{P} = 200$ kN is applied. The assembly is made of A36 steel.



Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 200(10^3) - F_D - F_A = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. b,

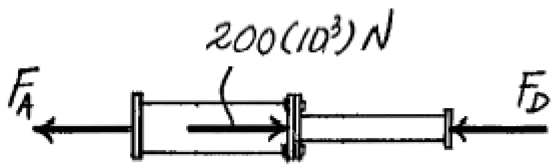
$$(\xrightarrow{+}) \qquad \delta = \delta_P - \delta_{F_D}$$

$$0.15 = \frac{200(10^3)(600)}{\frac{\pi}{4}(0.05^2)(200)(10^9)} - \left[\frac{F_D\,(600)}{\frac{\pi}{4}(0.05^2)(200)(10^9)} + \frac{F_D\,(600)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}\right]$$

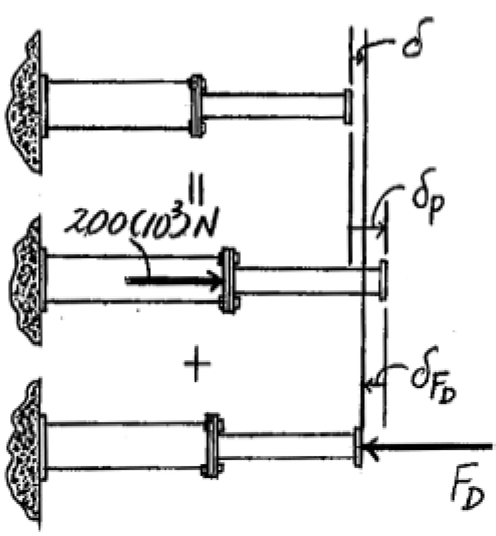
$$F_D = 20\,365.05 \text{ N} = 20.4 \text{ kN} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_A = 179\,634.95 \text{ N} = 180 \text{ kN} \qquad \textbf{Ans.}$$

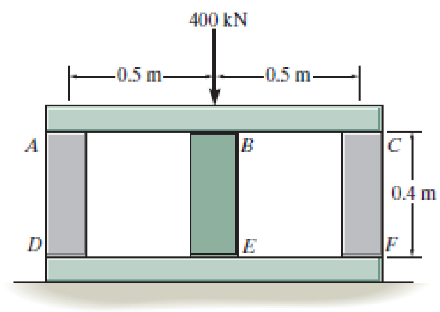


(a)



(b)

*4–60. The assembly consists of two posts AD and CF made of A-36 steel and having a cross-sectional area of 1000 mm^2, and a 2014-T6 aluminum post BE having a cross-sectional area of 1500 mm^2. If a central load of 400 kN is applied to the rigid cap, determine the normal stress in each post. There is a small gap of 0.1 mm between the post BE and the rigid member ABC.



Equation of Equilibrium. Due to symmetry, $\mathbf{F}_{AD} = \mathbf{F}_{CF} = \mathbf{F}$. Referring to the *FBD* of the rigid cap, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{BE} + 2F - 400(10^3) = 0 \qquad \textbf{(1)}$$

Compatibility Equation. Referring to the initial and final position of rods AD (CF) and BE, Fig. *b*,

$$\delta = 0.1 + \delta_{BE}$$

$$\frac{F(400)}{1(10^{-3})\left[200(10^9)\right]} = 0.1 + \frac{F_{BE}(399.9)}{1.5(10^{-3})\left[73.1(10^9)\right]}$$

$$F = 1.8235\,F_{BE} + 50(10^3) \qquad \textbf{(2)}$$

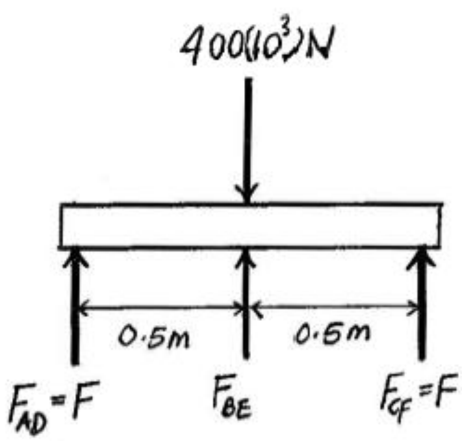
Solving Eqs (1) and (2) yield

$$F_{BE} = 64.56(10^3)\text{ N} \qquad F = 167.72(10^3)\text{ N}$$

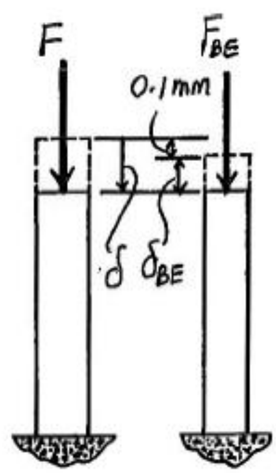
Normal Stress.

$$\sigma_{AD} = \sigma_{CF} = \frac{F}{A_{st}} = \frac{167.72(10^3)}{1(10^{-3})} = 168\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{al}} = \frac{64.56(10^3)}{1.5(10^{-3})} = 43.0\text{ MPa} \qquad \textbf{Ans.}$$

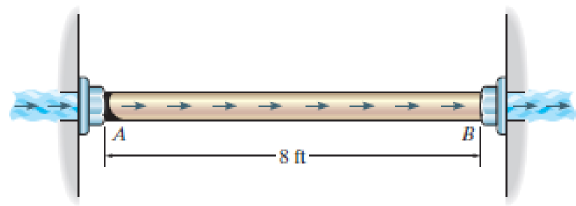


(a)

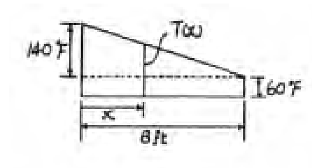


(b)

4–74. The bronze C86100 pipe has an inner radius of 0.5 in. and a wall thickness of 0.2 in. If the gas flowing through it changes the temperature of the pipe uniformly from $T_A = 200°\text{F}$ at A to $T_B = 60°\text{F}$ at B, determine the axial force it exerts on the walls. The pipe was fitted between the walls when $T = 60°\text{F}$.

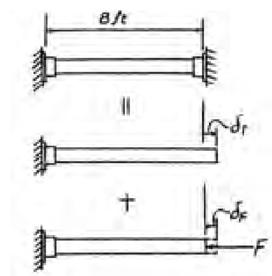


Temperature Gradient:



$$T(x) = 60 + \left(\frac{8 - x}{8}\right)140 = 200 - 17.5x$$

Compatibility:



$$0 = \delta_T - \delta_F \qquad \text{Where} \qquad \delta_T = \int \alpha \Delta T \, dx$$

$$0 = 9.60\left(10^{-6}\right)\int_0^{2\text{ft}} [(200 - 17.5x) - 60]\, dx - \frac{F(8)}{\frac{\pi}{4}(1.4^2 - 1^2)15.0(10^3)}$$

$$0 = 9.60\left(10^{-6}\right)\int_0^{2\text{ft}} (140 - 17.5x)\, dx - \frac{F(8)}{\frac{\pi}{4}(1.4^2 - 1^2)\, 15.0(10^3)}$$

$F = 7.60$ kip **Ans.**