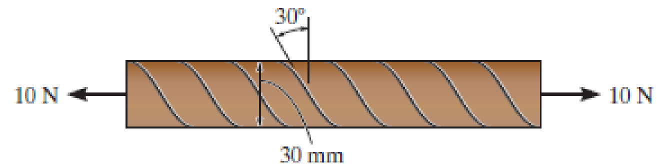
CE 270
Fall 2011
Solution – Homework 16

9–38. A paper tube is formed by rolling a paper strip in a spiral and then gluing the edges together as shown. Determine the shear stress acting along the seam, which is at 30° from the vertical, when the tube is subjected to an axial force of 10 N. The paper is 1 mm thick and the tube has an outer diameter of 30 mm.

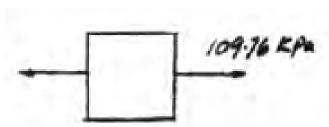


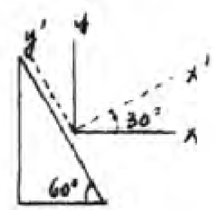
$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.03^2 - 0.028^2)} = 109.76 \text{ kPa}$$

$$\sigma_x = 109.76 \text{ kPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0 \qquad \theta = 30^\circ$$

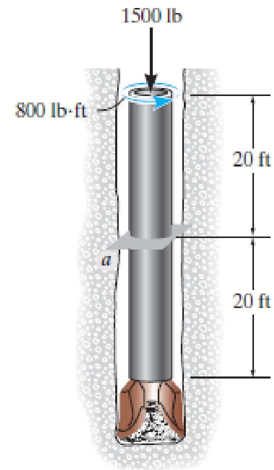
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{106.76 - 0}{2}\sin 60^\circ + 0 = -47.5 \text{ kPa}$$ **Ans.**

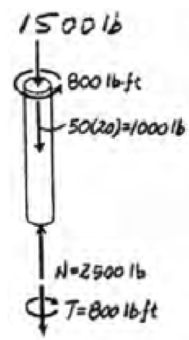




9–42. The drill pipe has an outer diameter of 3 in., a wall thickness of 0.25 in., and a weight of 50 lb/ft. If it is subjected to a torque and axial load as shown, determine (a) the principal stress and (b) the maximum in-plane shear stress at a point on its surface at section *a*.



Internal Forces and Torque: As shown on FBD(a).



Section Properties:

$$A = \frac{\pi}{4}\left(3^2 - 2.5^2\right) = 0.6875\pi \text{ in}^2$$

$$J = \frac{\pi}{2}\left(1.5^4 - 1.25^4\right) = 4.1172 \text{ in}^4$$

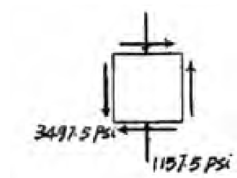
Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-2500}{0.6875\pi} = -1157.5 \text{ psi}$$

Shear Stress: Applying the torsion formula.

$$\tau = \frac{T\,c}{J} = \frac{800(12)(1.5)}{4.1172} = 3497.5 \text{ psi}$$

a) ***In - Plane Principal Stresses:*** $\sigma_x = 0, \sigma_y = -1157.5$ psi and $\tau_{xy} = 3497.5$ psi for any point on the shaft's surface. Applying Eq. 9-5.



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + (-1157.5)}{2} \pm \sqrt{\left(\frac{0 - (-1157.5)}{2}\right)^2 + (3497.5)^2}$$

$$= -578.75 \pm 3545.08$$

$$\sigma_1 = 2966 \text{ psi} = 2.97 \text{ ksi}$$ **Ans.**

$$\sigma_2 = -4124 \text{ psi} = -4.12 \text{ ksi}$$ **Ans.**

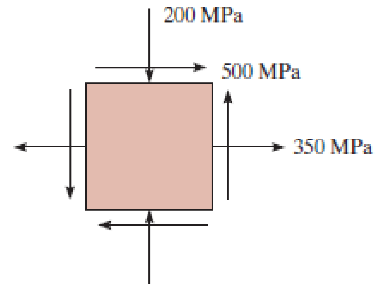
b) ***Maximum In - Plane Shear Stress:*** Applying Eq. 9-7

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{0 - (-1157.5)}{2}\right)^2 + (3497.5)^2}$$

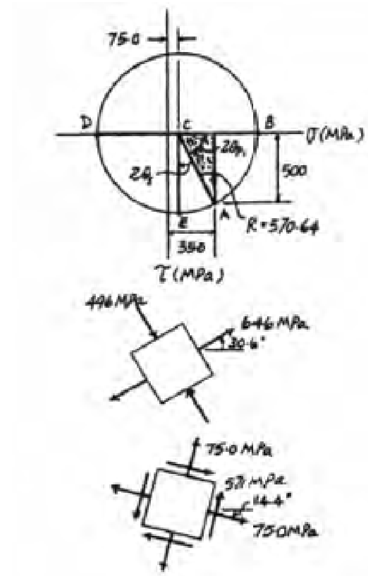
$$= 3545 \text{ psi} = 3.55 \text{ ksi}$$ **Ans.**

9–67. Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 350$ MPa, $\sigma_y = -200$ MPa, and $\tau_{xy} = 500$ MPa. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{350 + (-200)}{2} = 75.0 \text{ MPa}$$ **Ans.**



The coordinates for reference point A and C are

$$A(350, 500) \qquad C(75.0, 0)$$

The radius of the circle is

$$R = \sqrt{(350 - 75.0)^2 + 500^2} = 570.64 \text{ MPa}$$

a)

In - Plane Principal Stresses: The coordinate of points B and D represent σ_1 and σ_2 respectively.

$$\sigma_1 = 75.0 + 570.64 = 646 \text{ MPa}$$ **Ans.**

$$\sigma_2 = 75.0 - 570.64 = -496 \text{ MPa}$$ **Ans.**

Orientaion of Principal Plane: From the circle

$$\tan 2\theta_{P1} = \frac{500}{350 - 75.0} = 1.82$$

$$\theta_{P1} = 30.6° \ (Counterclockwise)$$ **Ans.**

b)

Maximum In - Plane Shear Stress: Represented by the coordinates of point E on the circle.

$$\tau_{\substack{max \\ in\text{-}plane}} = R = 571 \text{ MPa}$$ **Ans.**

Orientation of the Plane for Maximum In - Plane Shear Stress: From the circle

$$\tan 2\theta_s = \frac{350 - 75.0}{500} = 0.55$$

$$\theta_s = 14.4° \quad (Clockwise)$$ **Ans.**