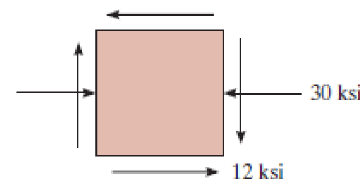
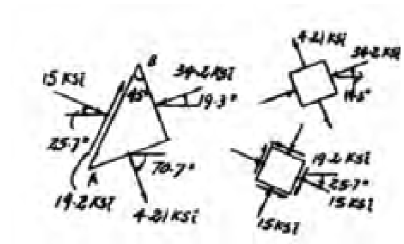
CE 270
Fall 2011
Solution – Homework 15

9–14. The state of stress at a point is shown on the element. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case. Show the results on each element.



$\sigma_x = -30$ ksi $\qquad \sigma_y = 0 \qquad \tau_{xy} = -12$ ksi



a)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \frac{-30 + 0}{2} \pm \sqrt{\left(\frac{-30 - 0}{2}\right)^2 + (-12)^2}$$

$\sigma_1 = 4.21$ ksi **Ans.**

$\sigma_2 = -34.2$ ksi **Ans.**

Orientation of principal stress:

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-12}{(-30-0)/2} = 0.8$$

$$\theta_P = 19.33^\circ \quad \text{and} \quad -70.67^\circ$$

Use Eq. 9-1 to determine the principal plane of σ_1 and σ_2.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\theta = 19.33^\circ$$

$$\sigma_{x'} = \frac{-30 + 0}{2} + \frac{-30 - 0}{2}\cos 2(19.33^\circ) + (-12)\sin 2(19.33^\circ) = -34.2 \text{ ksi}$$

Therefore $\theta_{P_2} = 19.3^\circ$ **Ans.**

and $\theta_{P_1} = -70.7^\circ$ **Ans.**

b)

$$\tau_{\max_{\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{-30 - 0}{2}\right)^2 + (-12)^2} = 19.2 \text{ ksi}$$ **Ans.**

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-30 + 0}{2} = -15 \text{ ksi}$$ **Ans.**

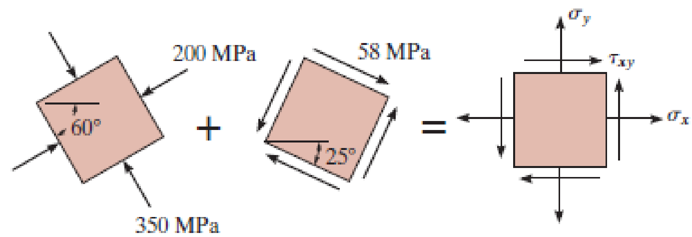
Orientation of max, in - plane shear stress:

$$\tan 2\theta_P = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(-30 - 0)/2}{-12} = -1.25$$

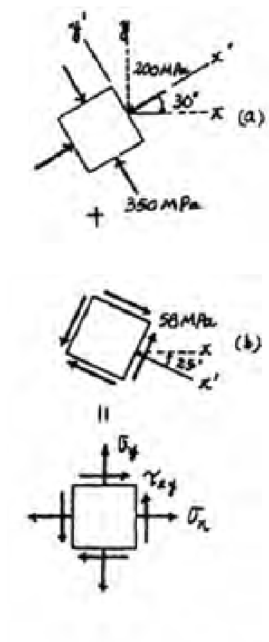
$\theta_P = -25.2^\circ$ and 64.3° **Ans.**

By observation, in order to preserve equllibrium along AB, $\tau_{\max}$ has to act in the direction shown in the figure.

9–18. A point on a thin plate is subjected to the two successive states of stress shown. Determine the resultant state of stress represented on the element oriented as shown on the right.



Stress Transformation Equations: Applying Eqs. 9-1, 9-2, and 9-3 to element (a) with $\theta = -30°$, $\sigma_{x'} = -200$ MPa, $\sigma_{y'} = -350$ MPa and $\tau_{x'y'} = 0$.



$$(\sigma_x)_a = \frac{\sigma_{x'} + \sigma_{y'}}{2} + \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta + \tau_{x'y'}\sin 2\theta$$

$$= \frac{-200 + (-350)}{2} + \frac{-200 - (-350)}{2}\cos(-60°) + 0$$

$$= -237.5 \text{ MPa}$$

$$(\sigma_y)_a = \frac{\sigma_{x'} + \sigma_{y'}}{2} - \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta - \tau_{x'y'}\sin 2\theta$$

$$= \frac{-200 + (-350)}{2} - \frac{-200 - (-350)}{2}\cos(-60°) - 0$$

$$= -312.5 \text{ MPa}$$

$$(\tau_{xy})_a = -\frac{\sigma_{x'} - \sigma_{y'}}{2}\sin 2\theta + \tau_{x'y'}\cos 2\theta$$

$$= -\frac{-200 - (-350)}{2}\sin(-60°) + 0$$

$$= 64.95 \text{ MPa}$$

For element (b), $\theta = 25°$, $\sigma_{x'} = \sigma_{y'} = 0$ and $\sigma_{x'y'} = 58$ MPa.

$$(\sigma_x)_b = \frac{\sigma_{x'} + \sigma_{y'}}{2} + \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta + \tau_{x'y'}\sin 2\theta$$

$$= 0 + 0 + 58 \sin 50°$$

$$= 44.43 \text{ MPa}$$

$$(\sigma_y)_b = \frac{\sigma_{x'} + \sigma_{y'}}{2} - \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta - \tau_{x'y'}\sin 2\theta$$

$$= 0 - 0 - 58 \sin 50°$$

$$= -44.43 \text{ MPa}$$

$$(\tau_{xy})_b = -\frac{\sigma_{x'} - \sigma_{y'}}{2}\sin 2\theta + \tau_{x'y'}\cos 2\theta$$

$$= -0 + 58 \cos 50°$$

$$= 37.28 \text{ MPa}$$

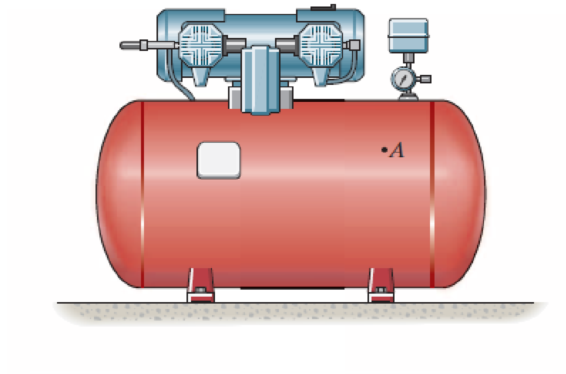
Combining the stress components of two elements yields

$$\sigma_s = (\sigma_x)_a + (\sigma_x)_b = -237.5 + 44.43 = -193 \text{ MPa}$$ **Ans.**

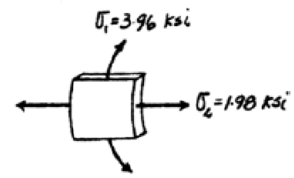
$$\sigma_y = (\sigma_y)_a + (\sigma_y)_b = -312.5 - 44.43 = -357 \text{ MPa}$$ **Ans.**

$$\tau_{xy} = (\tau_{xy})_a + (\tau_{xy})_b = 64.95 + 37.28 = 102 \text{ MPa}$$ **Ans.**

*8–4. The tank of the air compressor is subjected to an internal pressure of 90 psi. If the internal diameter of the tank is 22 in., and the wall thickness is 0.25 in., determine the stress components acting at point A. Draw a volume element of the material at this point, and show the results on the element.



Hoop Stress for Cylindrical Vessels: Since $\frac{r}{t} = \frac{11}{0.25} = 44 > 10$, then *thin wall* analysis can be used. Applying Eq. 8–1

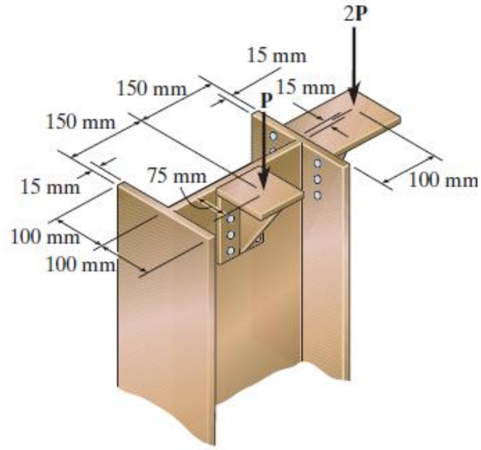


$$\sigma_1 = \frac{pr}{t} = \frac{90(11)}{0.25} = 3960 \text{ psi} = 3.96 \text{ ksi} \qquad \textbf{Ans.}$$

Longitudinal Stress for Cylindrical Vessels: Applying Eq. 8–2

$$\sigma_2 = \frac{pr}{2t} = \frac{90(11)}{2(0.25)} = 1980 \text{ psi} = 1.98 \text{ ksi} \qquad \textbf{Ans.}$$

*8–60. Determine the maximum allowable force **P**, if the column is made from material having an allowable normal stress of $\sigma_{allow} = 100$ MPa.



Equivalent Force System: Referring to Fig. *a*,

$$+\uparrow \Sigma F_x = (F_R)_x; \qquad -P - 2P = -F$$

$$F = 3P$$

$$\Sigma M_y = (M_R)_y; \qquad -P(0.075) = -M_y$$

$$M_y = 0.075\,P$$

$$\Sigma M_z = (M_R)_z; \qquad -2P(0.25) = -M_z$$

$$M_z = 0.5P$$

Section Properties: The cross-sectional area and the moment of inertia about the *y* and *z* axes of the cross section are

$$A = 0.2(0.3) - 0.185(0.27) = 0.01005\ \text{m}^2$$

$$I_z = \frac{1}{12}(0.2)\left(0.3^3\right) - \frac{1}{12}(0.185)\left(0.27^3\right) = 0.14655\left(10^{-3}\right)\text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.15)\left(0.2^3\right)\right] + \frac{1}{12}(0.27)\left(0.015^3\right) = 20.0759\left(10^{-6}\right)\text{m}^4$$

Normal Stress: The normal stress is the combination of axial and bending stress. Here, **F** is negative since it is a compressive force. Also, $\mathbf{M}_y$ and $\mathbf{M}_z$ are negative since they are directed towards the negative sense of their respective axes. By inspection, point *A* is subjected to a maximum normal stress, which is in compression. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$-100\left(10^6\right) = -\frac{3P}{0.01005} - \frac{(-0.5P)(-0.15)}{0.14655\left(10^{-3}\right)} + \frac{-0.075P(0.1)}{20.0759\left(10^{-6}\right)}$$

$$P = 84470.40\ \text{N} = 84.5\ \text{kN}$$ **Ans.**