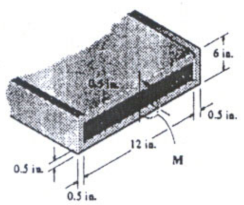
CE 270 – Introductory Structural Mechanics

HW 8 – Solutions

6–121. The top plate made of 2014-T6 aluminum is used to reinforce a Kevlar 49 plastic beam. If the allowable bending stress for the aluminum is $(\sigma_{\text{allow}})_{al} = 40$ ksi and for the Kevlar $(\sigma_{\text{allow}})_k = 8$ ksi, determine the maximum moment M that can be applied to the beam.



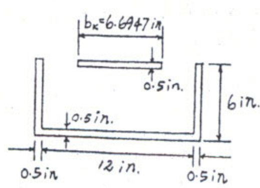
Section Properties :

$$n = \frac{E_{al}}{E_k} = \frac{10.6(10^3)}{19.0(10^3)} = 0.55789$$

$$b_k = n\, b_{al} = 0.55789(12) = 6.6947 \text{ in.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.25(13)(0.5) + 2[(3.25)(5.5)(0.5)] + 5.75(6.}{13(0.5) + 2(5.5)(0.5) + 6.6947(0.5}$$
$$= 2.5247 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(13)(0.5^3) + 13(0.5)(2.5247 - 0.25)^2$$
$$+ \frac{1}{12}(1)(5.5^3) + 1(5.5)(3.25 - 2.5247)^2$$
$$+ \frac{1}{12}(6.6947)(0.5^3) + 6.6947(0.5)(5.75 - 2.$$
$$= 85.4170 \text{ in}^4$$



Maximum Bending Stress : Applying the flexure formula

Assume failure of aluminium

$$(\sigma_{\text{allow}})_{al} = n\,\frac{Mc}{I}$$

$$40 = 0.55789\left[\frac{M(6 - 2.5247)}{85.4170}\right]$$

$$M = 1762 \text{ kip}\cdot\text{in} = 146.9 \text{ kip}\cdot\text{ft}$$

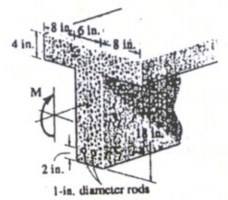
Assume failure of Kevlar 49

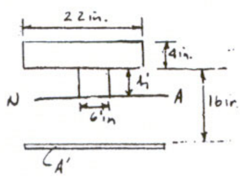
$$(\sigma_{\text{allow}})_k = \frac{Mc}{I}$$

$$8 = \frac{M(6 - 2.5247)}{85.4170}$$

$$M = 196.62 \text{ kip}\cdot\text{in}$$
$$= 16.4 \text{ kip}\cdot\text{ft} \qquad (\textit{Controls!}) \qquad \textbf{Ans}$$

6–125. The reinforced concrete beam is made using two steel reinforcing rods. If the allowable tensile stress for the steel is $(\sigma_{st})_{allow} = 40$ ksi, and the allowable compressive stress for the concrete is $(\sigma_{conc})_{allow} = 3$ ksi, determine the maximum moment M that can be applied to the section. Assume the concrete cannot support a tensile stress. $E_{st} = 29(10^3)$ ksi, $E_{conc} = 3.8(10^3)$ ksi.





$$A_{st} = 2(\pi)(0.5)^2 = 1.5708 \text{ in}^2$$

$$A' = nA_{st} = \frac{29(10^3)}{3.8(10^3)}(1.5708) = 11.9877 \text{ in}^2$$

$$\Sigma \tilde{y}A = 0; \qquad 22(4)(h'+2) + h'(6)(h'/2) - 11.9877(16-h') = 0$$

$$3h'^2 + 99.9877h' - 15.8032 = 0$$

Solving for the positive root :

$$h' = 0.15731 \text{ in.}$$

$$I = [\frac{1}{12}(22)(4)^3 + 22(4)(2.15731)^2] + [\frac{1}{12}(6)(0.15731)^3 + 6(0.15731)(0.15731/2)^2]$$
$$+ 11.9877(16 - 0.15731)^2 = 3535.69 \text{ in}^4$$

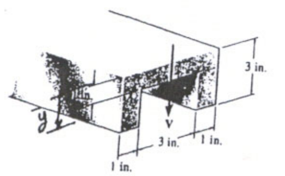
Assume concrete fails :

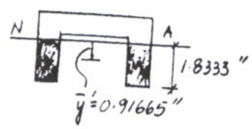
$$(\sigma_{con})_{allow} = \frac{My}{I}; \qquad 3 = \frac{M(4.15731)}{3535.69}$$
$$M = 2551 \text{ kip} \cdot \text{in.}$$

Assume steel fails :

$$(\sigma_{st})_{allow} = n(\frac{My}{I}); \qquad 40 = (\frac{29(10^3)}{3.8(10^3)})(\frac{M(16-0.15731)}{3535.69})$$
$$M = 1169.7 \text{ kip} \cdot \text{in.} = 97.5 \text{ kip} \cdot \text{ft (controls)} \qquad \textbf{Ans}$$

7–10. Determine the largest shear force V that the member can sustain if the allowable shear stress is $\tau_{allow} = 8$ ksi.





$$\bar{y} = \frac{(0.5)(1)(5) + 2\,[(2)(1)(2)]}{1\,(5) + 2\,(1)(2)} = 1.1667 \text{ in.}$$

$$I = \frac{1}{12}(5)(1^3) + 5\,(1)(1.1667 - 0.5)^2$$

$$+\, 2\,(\frac{1}{12})(1)(2^3) + 2\,(1)(2)(2 - 1.1667)^2 = 6.75 \text{ in}^4$$

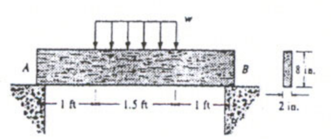
$$Q_{max} = \Sigma \bar{y}'A' = 2\,(0.91665)(1.8333)(1) = 3.3611 \text{ in}^3$$

$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$8\,(10^3) = \frac{V(3.3611)}{6.75\,(2)(1)}$$

$V = 32132$ lb $= 32.1$ kip **Ans**

7–21. The supports at A and B exert vertical reactions on the wood beam. If the allowable shear stress is $\tau_{allow} = 400$ psi, determine the intensity w of the largest distributed load that can be applied to the beam.



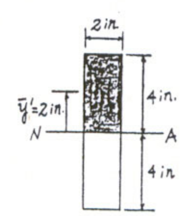
Support Reactions : As shown on FBD.

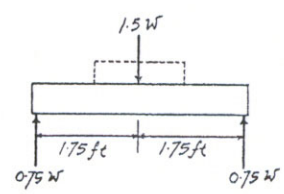
Internal Shear Force : The maximum shear force occurs at the region $0 \leq x < 1$ft where $V_{max} = 0.750w$.

Section Properties :

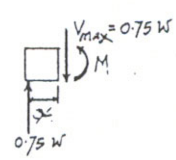
$$I_{NA} = \frac{1}{12}(2)\left(8^3\right) = 85.333 \text{ in}^4$$

$$Q_{max} = \bar{y}'A' = 2(4)(2) = 16.0 \text{ in}^3$$





Allowable Shear Stress : Maximum shear stress occurs at the point where the neutral axis pass through the section. Applying shear formula

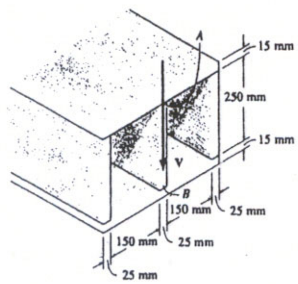


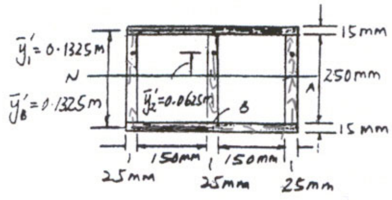
$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$400 = \frac{0.750w\,(16.0)}{85.333\,(2)}$$

$w = 5689$ lb/ft $= 5.69$ kip/ft **Ans**

7–55. The box girder is subjected to a shear of $V = 15$ kN. Determine (*a*) the shear flow developed at point B and (*b*) the maximum shear flow in the girder's web AB.





Q_B =

$$I = \frac{1}{12}(0.375)(0.28^3) - \frac{1}{12}(0.3)(0.25^3) = 0.295375(10^{-3})\ \text{m}^4$$

$$Q_B = \bar{y}_B' A' = 0.1325(0.375)(0.015) = 0.7453125(10^{-3})\ \text{m}^3$$

$$Q_{max} = \Sigma \bar{y}' A' = 0.1325(0.375)(0.015) + 3[(0.0625)(0.125)(0.025)] = 1.33125(10^{-3})\ \text{m}^3$$

a) $q_B = \frac{1}{3}[\frac{VQ_B}{I}] = \frac{1}{3}[\frac{15(10^3)(0.7453125)(10^{-3})}{0.295375(10^{-3})}] = 12.6$ kN/m **Ans**

b) $q_{max} = \frac{1}{3}[\frac{VQ_{max}}{I}] = \frac{1}{3}[\frac{15(10^3)(1.33125)(10^{-3})}{0.295375(10^{-3})}] = 22.5$ kN/m **Ans**