A&AE 613
Spring 2000
Professor Steve Schneider
Problem Set 3
Handed Out: Friday, 11 February
Due: Friday, 18 February

The following 2 problems, sans numerics, formed a 3-hour exam in Fall 1996.

1. Consider heat transfer from a uniformly heated (or cooled) flar plate in low speed flow. The velocity profile is of the form $u = Uf(\eta)$, where $\eta = y\sqrt{U/\nu x}$. There is zero pressure gradient. The enthalpy distribution can be written as

$$h_w - h = (h_w - h_\infty)g(\eta),$$

so $h = h(\eta)$.

   (a) Derive the differential equations for f and g from the continuity, momentum, and energy equations. Find solutions and plot some of the results.

   (b) Find the dimensionless heat flux (the Nusselt number) N as a function of Pr and Re. Using a first approximation to the Nusselt number solution, show that N is proportional to powers of Pr and Re, and determine the powers.

Note that low speed flow means that the kinetic energy is small compared to the enthalpy, and that shear dissipation is small compared to heat conduction. The fluid properties can be taken as constants. The Nusselt number $N = c_p q_w x / k(h_w - h_\infty)$. The usual boundary-layer assumptions are to be made.

(over)

2. Consider a cone of half-angle $\beta\pi/2$, and a related 2D wedge of the same half-angle. From inviscid theory used for the Falkner-Skan derivations, the velocity distribution is $U = Kx^m$ for the 2D flow, where $\beta = 2m/(m+1)$. From Evans (1968, see text), we can see that for a cone of half-angle $\phi, U = Kx^n$. Assume the two $K's$ are the same, and assume that the two half-angles are small. You also need that $\phi \simeq 6.7n$, for small n, where ϕ is in radians. The Rott-Crabtree equation for axisymmetric boundary layers is

$$\theta^2 = \frac{0.45\nu}{[r_0(x)]^2\,[U(x)]^6} \int_0^x [r_0(s)]^2\,[U(s)]^5\ ds;$$

this is derived and used the same as Thwaites method, and differs only by addition of the r_0 terms (White, 2nd ed., Sec. 4-10.2). Using Thwaites method and this method, solve for the momentum thickness on the cone and flat plate.

Determine the relation between the momentum thicknesses for the two cases, to first approximation. Discuss the meaning of the result. Can you give any physical explanation for your result?

Remember, x is the coordinate along the body, y is the coordinate normal to the surface, r_0 is the radius from the body centerline to the surface at a given x, and U is the inviscid outer flow velocity, which depends on the streamwise coordinate. To get the lowest order approximation, use a Taylor series:

$$f(x+\epsilon) \simeq f(x) + f'(x)\epsilon,$$

for $\epsilon \ll x$. For example, this gives that

$$\frac{1}{1+\epsilon} \simeq 1 - \epsilon$$

for $\epsilon \ll 1$.