A&AE 613
Spring 2000
Professor Steve Schneider
Problem Set 5
Handed Out: Monday, 6 March
Due: Friday, 24 March

1. For a 1-D shock wave with a Pr of 1, in a perfect gas with constant specific heats, we derived the following equation:

$$\dot{m}\frac{du}{dx}\frac{\gamma+1}{2\gamma}\left(1-\frac{(a^*)^2}{u^2}\right)=\frac{d}{dx}\left(\tilde{\mu}\frac{du}{dx}\right),$$

where $u \to u_1$ as $x \to -\infty$ and $u \to u_2$ as $x \to \infty$, and where $\tilde{\mu} = \lambda + \frac{4}{3}\mu$.

   (a) Nondimensionalize this equation using the mean free path l, the sound speed at Mach 1, a^*, and the mass flux $\dot{m}$; non-dimensionalize $\tilde{\mu}$ in terms of its value at Mach 1, $\tilde{\mu}^*$ (we will define various representations for $\tilde{\mu}/\tilde{\mu}^*$). All the remaining parameters can be grouped into one shock-Reynolds number parameter, *Re*. What is this parameter?

   (b) Use Stoke's hypothesis that $\lambda = -\frac{2}{3}\mu$, and Chapman's expression for the mean free path in dilute non-polar gases, $\mu \approx 0.67\rho l a$, (see White eqn. 1-32) to simplify the expression for *Re*.

   (c) Reduce the second order ODE to a quadrature (meaning an integral) using analytical methods. Discuss the meaning of any integration constants which may appear, and provide a means of evaluating these constants. Hint: Remember that as $x \to \pm\infty$, $\frac{du}{dx} \to 0$. If $u \to u_1$ upstream, what must u go to downstream? (see part (1f) below). What is the only reasonable choice for u/a^* at $x = 0$?

   (d) Assume that the viscosity μ is linear in the temperature T. Simplify the integral of part (1c). For air with $\gamma = 1.4$ and for upstream velocities $u_1/a_1 = 1.1, 2, \text{and } 10$, perform the numerical integration, and plot the non-dimensional velocity profile through the shock. How many mean-free paths is the shock width? Hint: What is the behavior of the integral as $u \to u_1$ or u_2? What does

$u(x)$ look like? How can you change the usual method of integration in order to capture the behavior of the solution near $x = 0$?

(e) Assume that the viscosity μ follows a Sutherland law,

$$\mu = 10^7 \frac{AT^{3/2}}{T + B},$$

where $A = 145.8$ and $B = 110.4$ for air, and T is measured in Kelvin and μ in poise (see Hilsenrath et al., NBS circular 564, 1955). Repeat the evaluations of part (1d) for this more accurate viscosity law. Are the differences significant?

(f) Compare your results for the downstream velocities to those expected using the jump conditions for shock waves (in particular, the Prandtl-Meyer relation $u_1 u_2 = (a^*)^2$).

2. White, problem 7-8.